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Probabilistic Model for Estimation of Monthly Cost for Mobile Phone

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Abstract: Managers and their decisions can be successful only if they research with correct and relevant information. Because it is not possible to implement an experiment with institutions that are to be managed, it is necessary to provide artificial experiments within an appropriate model. This is the way how simulation models are introduced into managerial research. The aim of this paper is to introduce a simulation model and its implementation in MATLAB that can be used for estimation of total monthly cost for a selected mobile phone tariff. Finally the properties for different parameters and other tariff are discussed as well.

Key words: Model, simulation, cost, decision making, MATLAB

INTRODUCTION

The ubiquitous features of the business world are change and uncertainty. Uncertainty therefore plays an important role in analyzing most managerial decisions. Unlike natural sciences, management science cannot effectively plan and realized experiments (Pidd, 2004). Instead of that managers rather learn from their experiences or models and uncertainty is an important aspect in their considerations (Balakrishnan *et al*., 2007; Turban *et al*., 2011; Prazak, 2015, 2014). With regard to the managerial practice, a model can be considered as simplified representation of a real object or organization of the interest (Prazak, 2015). Models are usually used to study properties of the given organizing processes in two ways: either to reveal the effect of different policies impact on that organization, or to investigate possible improvements in the organization (Pidd, 2004). Typical models constructed by business analysts are used in decision support systems and for comprehension of business rules. Decision support systems can be considered as a part of business analytics component of business intelligence (Turban *et al*., 2011). Since a lot of variables within the considered model can have a probability character, computer simulations are widely used to study their properties. In this study we would like to present a simulation model for cost analysis of using mobile phones. Cost exploration of different processes belongs to important area of managerial interest. Most employees of different firms use mobile phone to be able to communicate with their colleagues or their customers. Mobile phones are common in many areas of business life but they are used irregularly if required. It is not possible to plan the number of calls between employees or the number of calls to customers (usually outside the firm). To be able to estimate monthly cost of using mobile phones within the given firm it is necessary to know key factors as the tariff and the time of using mobile phone per unit time (e.g., 1 month). The tariff is usually given by an agreement and all its parameters are fixed. On the other side the time of using mobile phone is uncertain; it can be considered as a random variable with a given distribution function or probability density function (Prazak, 2015). The paper is organized as follows. At first the essential exogenous parameters and settings of the model are introduced. Then the own model and its implementation in MATLAB is described as Cho and Martinez (2015). After that the results of simulation and their statistical analysis are introduced. Finally a discussion that deal with another properties of the simulation models and compare two different tariffs is presented.

Problem description: As a problem to solve it is considered the following situation: an entrepreneur wants to find his possible mobile phone costs for a given tariff. It is considered that the tariff is characterized by unit prices given in Table 1.

To be able to find costs of calling per month, one needs data that are related to the time of calling. For different entrepreneurs we find different input data and therefore, we can gain different results. These data can be

Table 1: Settings of mobile phone tariff

Variables	Setting of mobile phones
m	Monthly fee in CZK
F	Free credit in minutes
E	Extra free credit in minutes, calling to T-Mobile net
q	Price in CZK per minute, calling inside the company
p	Price in CZK per minute
r	Price per SMS in CZK

Table 2: Random variables of the model and their distributions (own arrangements)

Variables	Distribution
Time of calling within the firm/minutes	$T_1 \sim N(\mu_1, \sigma_1^2)$
Time of calling to T-Mobile/minutes	$T_2 \sim N(\mu_2, \sigma_2^2)$
Time of calling outside of T-Mobile/minutes	$T_3 \sim N(\mu_3, \sigma_3^2)$
Number of SMS	$X \sim R(a, b)$

collected from entrepreneur's records from his past invoices for calling. In general, it is possible to consider the following probability distributions in Table 2.

MATERIALS AND METHODS

Suppose that FC is the fixed monthly fee of using mobile phone tariff. Further, suppose that MC are the variable costs of using mobile phone calls within the firm, VC are the variable costs of using mobile phone calls outside the firm. Finally, suppose that XC are variable costs of using mobile phone for SMS. The total costs TC of using mobile phone are the sum of the fixed costs and the variable costs which can be formally written as:

$$TC = FC + MC + VC + XC \quad (1)$$

First, let us consider the variable costs of using mobile phone calls outside the firm. Since, there are free minutes to use mobile tariff in this mode, it is necessary to determine total time T which has to be charged in this mode. According to Table 1, the time T_{MOB} that has to be paid for calling to T-Mobile net is:

$$T_{MOB} = \begin{cases} 0 & T_2 \le E \\ T_2 - E & T_2 > E \end{cases} \quad (2)$$

Similarly, the total time T that has to be paid for calling outside the firm is:

$$T = \begin{cases} 0 & T_{MOB} + T_3 \le F \\ T_{MOB} + T_3 - F & T_{MOB} + T_3 > F \end{cases} \quad (3)$$

Equation 2 and 3 can be simplified and summarized as follows:

$$T = \max\{0, \max\{0, T_2 - E\} + T_3 - F\} \quad (4)$$

Now it is possible to express the relation for the total costs. According to Eq. 1 and Table 1:

$$TC = m + q \cdot T_1 + p \cdot T + r \cdot X \quad (5)$$

Equation 4 and 5 represent the final mathematical model of our problem. Although, we found simple expressions for total monthly costs of using mobile phone, we could realize that the quantity TC is a random variable. When we want to better characterize it we need to carry out simulations. To implement simulation model the software MATLAB was used, (manual). After of loading essential parameters given in Table 1 and 2 into the data file (parameters.mat) model Eq. 4 and 5 can be shortly written as the following MATLAB function:

```
01 function [TC] = phone(n, parameters)
03
02 load(parameters)
04
05 O=zeros(n, 1)
06 T1=normrnd(mu1, sqrt(var1), n, 1)
07 T2=normrnd(mu2, sqrt(var2), n, 1)
08 T3=normrnd(mu3, sqrt(var3), n, 1)
09 X=randi([a, b], n, 1)
10
11 T=max([O, max([O, T2-E], [], 2) + T3-F], [], 2)
12 TC=m+q*T1+p*T+r*X
13
14 end
```

Lines 06-09 of the above given function determine sample of n attempts of the simulation It is worthwhile to emphasize that lines 11 and 12 compute results for all n attempts at once that is the advantage of matrix computations of the used software. All these results can be subject to statistical analysis. Inferences about gained results can be used for decision which mobile phone tariffs is better to choose.

RESULTS AND DISCUSSION

To present a particular use of the introduced model it is necessary to calibrate it. Real values of parameters will be used, Table 3. Units of the given parameters are given in Table 1. Probability distributions that characterize behavior of the user of mobile phone can be found in Table 4. These functions were gained by an approximation of probability distributions of a particular entrepreneur that does his business in the field of civil engineering.

The final step to assign input parameters and settings is to determine the number n of replications. It can be determined in such a way that the relative error of the mean E(TC) of total costs is >2%. Since,the margin of the error e for 95% confidence interval is approximated by:

$$e = 1.96 \frac{\delta}{\sqrt{n}} \quad (6)$$

where, δ is the standard deviation and n is the sample size (Groebner *et al*., 2013) (in the context of simulation it is the

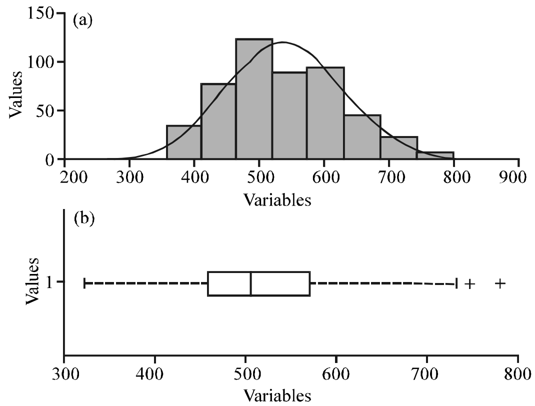

Fig. 1: Histogram and box-plot diagram of random variable TC (computation in MATLAB)

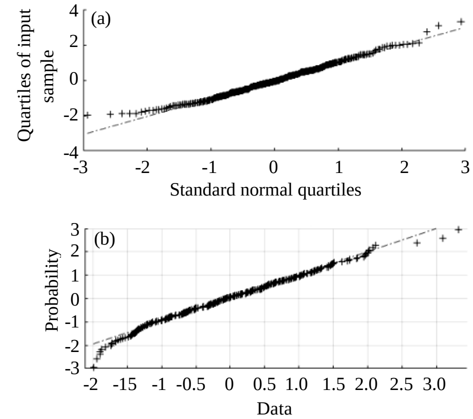

Fig. 2: The Q-Q Plot and Normal probability plot of random variable TC (computation in MATLAB)

Table 3: Settings of mobile phone tariff Plus 250 (T-Mobile, CZ, 2015)

m = 250 CZK	F = 58 min	E = 25 min
q = 0.3 CZK/min	p = 4.3 CZK/min	r = 1.0 CZK/SMS

Table 4: Parameters of probability distribution in CZK (own observation)

$\mu_1 = 180$	$\mu_2 = 80$	$\mu_3 = 50$	a = 0
$\sigma_1^2 = 900$	$\sigma_2^2 = 400$	$\sigma_3^2 = 100$	b = 50

Table 5: Descriptive statistics of total monthly cost TC for mobile phone with tariff plus 250. Symbols Q1-Q3 stand for 1st quartile, median and 3rd quartile. Results are rounded to units of CZK (own computation)

μ	δ	min	Q1	Q2	Q3	Max.
531	95	325	465	520	594	827

number of replications), the relative error can be estimated as $e/\mu \leq 0.02$. Using the given relations, the number n of replications can be estimated as:

$$n \geq \left(1.96 \cdot \frac{\delta}{0.02 \cdot \mu}\right)^2 \tag{7}$$

The unknown values of standard deviation and means can be only approximated at the beginning of the simulation. For instance, it can be used the first 100 replications of our model. In this way it was found that $\mu \approx 540$, $\delta \approx 90$. Using these values and Eq. 7 we find that $n \geq 267$. Particularly the number n = 300 of replications has been selected. Now it is possible to use the simulation model with the given settings. Summary statistics for the simulation trials of total cost TC can be found in Table 5 and Fig. 1.

It can be shown that the 95% confidence interval estimate for mean μ of total cost TC is (520.2, 542.0). Since, $Q2 < \mu$ we can observe that the probability distribution is slightly right-skewed. This observation can be supported by computation of skewness, $m_3 = 0.53$. Although, the skewness is positive, its value is not very large and there

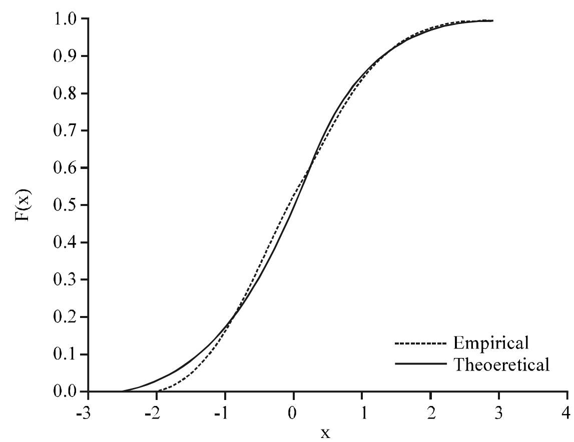

Fig. 3: Cumulated distribution function of random variable TC (computation in MATLAB)

could arise the question whether the true distribution of the random variable TC can be approximated by the normal distribution $N(\mu, \delta^2)$ (Fig. 2 and 3).

To finish this consideration goodness of fit test with the following hypothesis is conducted:

- H_0: Distribution of the random variable TC is normal
- H_1: Distribution of TC is not normal

The hypothesis was tested with Pearson χ^2 goodness of fit test at the 5% significance level. In MATLAB the function: $[h,p] = \chi^2$ gof (TC, alpha, 0.05) can be used. It has been found that p-value of this test is 0.0512 which means that it is not possible to reject the null hypothesis H_0 and the above given normal distribution can be considered as a good model for distribution of total monthly cost TC.

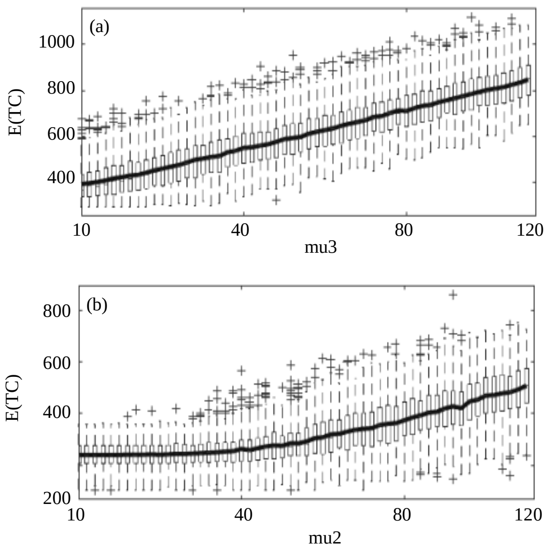

Fig. 4: Dependence of E(TC) on μ2 and μ3 (computation in MATLAB): a) Dependence of E(TC) on mu3; b) Dependence of E(TC) on mu2

As has been already mentioned outputs of the simulation model depend on the selected tariff and the particular behavior of a given user. At first let us consider Eq. 5:

$$E(TC) = m + q \cdot E(T_1) + p \cdot E(T) + r \cdot E(X)$$

Due to this observation we can find out some other features of the model. For instance if $E(T_1)$ is N times as large as the value μ_1 in Table 4 the mean of total cost E(TC) increases by $(N\text{-}1)q\mu_1$. Similarly we can proceed with random variable X. Random variable T is not a linear function of random variables T_2 and T_3. To study a possible change of the mean of total cost E(TC) with respect to these random variables, we make trials with the simulation model. Let us consider tariff Plus 250 given in Table 3, parameters in Table 4 and let $\mu_2 \in \{10, 11, ..., 120\}$ then the relationship of E(TC) on variable μ_2 can be simulated, Fig. 4. Similar trials can be provided with parameters given in Table 4 and $\mu_3 \in \{10, 11, ..., 120\}$. The dependence of E(TC) on variable μ_2 is also given at Fig. 4. Now, it can be observed that E(TC) is more sensitive on the change of variable μ_2.

At second, we consider the same user as in section 3 but we change the tariff. This new tariff is characterized in Table 6. The simulation model provide us essential numerical measures that are summarized in Table 7. In this discussion we concentrate on comparison of two means[5]: $\mu_{250} = E(TC_{250})$ and $\mu_{450} = E(TC_{450})$ where $E(TC_{250})$ is the mean of the random variable TC for tariff Plus 250 and $E(TC_{450})$ is the mean of the random variable TC for tariff Plus 450.

Table 6: Settings of mobile phone tariff Plus 450

m = 450 CZK	F = 136 min	E = 45 min
q = 0.3 CZK/min	p = 3.3 CZK/min	r = 1.0 CZK/SMS

Table 7: Descriptive statistics of total monthly cost TC for mobile phone with tariff Plus 450. Results are rounded to units of CZK (own computation)

μ	δ	min	Q1	Q2	Q3	max
550	31	495	527	545	571	645

For further discussion we will consider the following hypotheses H_0: $\mu_1 = \mu_2$ and H_1: $\mu_{250} < \mu_{450}$. The test will be conducted with the significance level 5%. The simulation has been adapted in such a way that both values TC_{250} and TC_{450} are computed simultaneously, so t-test for paired samples can be used (Cho and Martinez, 2014). In MATLAB the function: [h,p] = t-test (TC250, TC450, α = 0.05) can be used. It has been found that p-value of this test is 0.0039 which means that we reject null hypothesis H_0. This observation allows us to formulate a conclusion: based on the input data and parameters and on the simulation data, a sufficient statistical evidence exists to conclude that there is a difference between the tariff Plus 250 and the tariff Plus 450.

CONCLUSION

Each model can be enriched, modified and particularized in many ways (Pidd, 2004). The model that was presented in this study can be modified too. Instead of the monthly costs of phone calls we could compute e.g. the annual costs. Moreover, the statistical analysis can be enriched and can focus on describing of another feature of the output data, for instance that of mapping the difference in the variety of the output data distributions. Our future research will concentrate on comparison of more tariffs for different mobile phone operators within the Czech Republic and in the process use similar models like the one introduced in this study.

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