

## **Adaptive Technology of Pupils' Mathematics Teaching That Considers the Specific Features of Pupils' Subject-Matter Giftedness**

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**Abstract:** The study gives content-methodological characteristics of the development project for the pupils' adaptive automated mathematics teaching system considering the nature of pupils' subject-matter giftedness. The project is based on the original concept of mathematical giftedness, according to which this giftedness is seen as a complex hierarchical structure. Revealing this condition at a particular pupil allows identifying appropriate measures for its further teaching and development.

**Key words:** Adaptive teaching technology, mathematical giftedness, pupil's search activity for solving mathematical problems, giftedness

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### **INTRODUCTION**

**Setting a problem:** The problem of working with gifted children in teaching mathematics has always been quite a difficult and ambiguously perceived problem for the mass education system due to a number of subjective and objective circumstances. These circumstances can, in particular include a still wide spread misconception that a properly organized teaching process implies self-actualization and self-development of giftedness. Meanwhile, many psychological studies have proven the need to take into account an individual character of the child's giftedness. Such accounting can be done in a natural way by means of appropriate adaptive technologies, which include a wide range of hardware and software solutions; they allow adapting ways of transmission and presentation of different types of information to the user's characteristics in the automatic mode.

Currently, many teachers are engaged in the problem of adaptive teaching technology as a means of adequate diagnostics and further formation of the pupils' subject-matter competence both in the theoretical and in practical terms. However, one of the most discussed and by no means till the end solved issues of methodological nature is an objective choice of latent parameters of the level of training, structure of cognitive activity, as well as

personal characteristics that should underlie the development of diagnostic and forming procedures (Rodionov *et al.*, 2016).

### **MATERIALS AND METHODS**

In solving the problem outlined above, we have selected the following individual learner's characteristics as a unit to analyze the formation of the giftedness phenomenon that underlies its adequate diagnostic and formation system.

A competence component (S) representing the learned subject-matter competences of a learner determines the composition and structure of individual human experience and the level of training. The second Component (C) describes the learner's cognitive activity and, in particular, the nature of his/her meaning and goal formation at various stages of teaching and search activity.

Finally, the Third cognitive component (T) characterizes the pupil's level of mathematical intuition and knowledge of heuristic methods of mathematical activity in solving any research problems. The above components can be presented in the pupil's giftedness structure at different levels, and one or another combination of conditions predetermines a possibility of his/her teaching and development.

However, the corresponding adaptive teaching technology should be directed at identifying a type and level of learner's giftedness and the creation of favorable conditions for the development of "underachieved" components.

The special features of our proposed methodological solution are, on one hand, relying on the concept of diagnostics and development of gifted children in mathematics, and, on the other "binding" to the existing program in mathematics that allows supplementing the learnt course with the search and research material. In particular, such character of the material can be provided by the targeted substantial and structural transformation of a problem-related situation leading to the construction of a new problem being in one way or another related to the original one. The significance of this work stems from the fact that the teacher's need and target setting originally created for learners does not fade at the end of solving a problem; it gains an extra "sound" at its transfer to a new subject-matter content and simultaneously facilitates its subsequent adoption (Rodionov and Velmisova, 2008); Victorovana *et al.*, 2016; Rodionov and Akimova, 2010; Rodionov and Marina, 2006b).

However, the greatest developmental effect occurs when an initiative of a task transformation originates directly from the learners themselves in the form of conjectures and hypotheses. If they have in this case a conscious right of free choice regarding the search direction, it ensures comprehension of the obtained results and the search work techniques leading to them as "their", "hard-won" products, and they are completely satisfied with the successfully implemented creative search (Victorovna *et al.*, 2016); (Rodionov and Marina, 2006b).

**Description of testing:** Currently, a pupils' adaptive automated teaching system is being developed, so it could take into account a type and extent of pupils' mathematical giftedness. Jointly with IT-Service, Ivanovo, the authors conduct their development activity supported by the Foundation for Assistance to Small Innovative Enterprises in Science and Technology as part of the MOST project (Modernization of Education with Modern Technologies).

The project is intended for pupils of grades 5-6 and includes 16 modules corresponding to the topics of a course in mathematics reflected in the program. In the framework of a module, the pupils master an appropriate method of heuristic search, while passing through three stages of work with electronic content: diagnostic, forming and reflexive. To manage the educational materials, a system of programs for the educational process organization is used (Rodionov *et al.*, 2016).

**Diagnostic stage:** A test, which consists of problems of different level of complexity, is suggested to a pupil at this stage. Each problem is characterized by certain indicators T, C and S (one of three parameters) of various level of complexity. The "heuristic potential" of a considered problem-related material determines the total number of tasks.

A range of procedures that make up this potential is specified by the special features of all main aspects of the mathematical thinking to the extent accessible for secondary school. Corresponding to the very nature of creative mathematical thinking, these procedures are reflected (both individually and in various combinations with each other) in the subject-matter search activity. However, the relation of a subject of activity to these procedures is different at various levels. If at the initial level a pupil regards the use of a method as an act of sudden inspiration (conjecture, insight), when transiting to a higher level such use becomes more and more conscious, and it is realized as a result of conscious choice of a number of meaningful alternatives. However, the larger a range of procedures a pupil is able to use and the more generalized nature they have, the more effectively such choice is made (Victorovana *et al.*, 2016); (Rodionov and Marina, 2006b); (Rodionov and Khranova, 2007).

General characteristics of the tasks: Each task includes three parts.

- The first (mandatory) part suggests solving a typical problem which checks the formation of the pupils' specific subject-matter skills and knowledge (Criterion S)
- The second (optional) part includes questions, for which the pupils do not have any clear guidelines to answer them. The purpose of stating these questions is to give to pupils a certain "hint" at the possibility of developing an original typical problem-related situation. Attempts made to find answers in the second part of the task with the involvement of certain heuristic procedures mentioned above indicate some motivational significance of the proposed material for pupils (Criterion C)
- The third part is, in fact, a complicated task that develops the topic of the previous stage and it is realized using the mathematical subject-matter instruments available in the pupil's creative arsenal (Criterion T)

According to the results of testing, the educational process management system generates a report showing the levels of formation of the individual pupil's diagnosed components (S, C and T), for example, Fig. 1. The total number of points ranges approximately from 0-8.

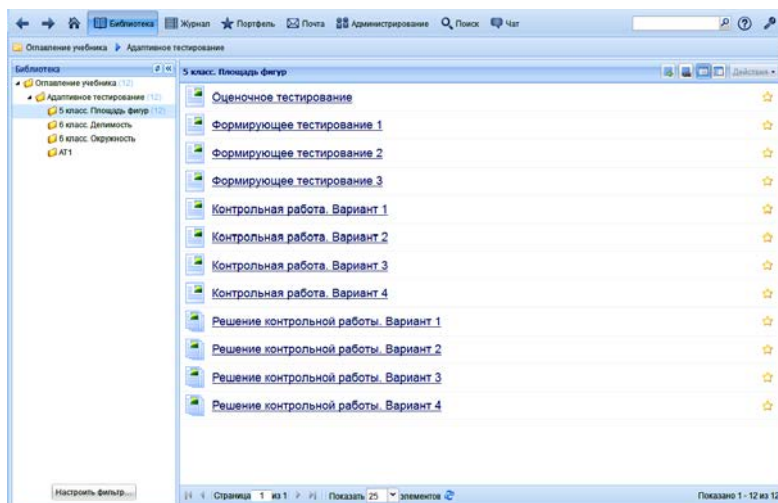


Fig. 1: Structure of educational materials on the topic “Area of geometric figures” for the 6th grade

#### Forming stage:

After diagnostic testing and identification of the giftedness profile, the educational process management system suggests to a pupil the following educational materials corresponding to the results of diagnostics:

- Theoretical materials (text, algorithms, tables, charts, graphs)
- Practical tasks of the basic and advanced level being common to all pupils to master the studied subject matter on the level provided for in the Federal State Educational Standards (number of such tasks depends on the structure of corresponding algorithmic order - 5-6 tasks on the average)
- Individual tasks that increase a level of “underachieved” substructures (motivational and cognitive) in the profile of pupil’s mathematical giftedness (when drafting and analyzing them, the medium of dynamic mathematics “1C:Mathematical Constructor” is widely used).

Each of these individual tasks is “responsible” for one or two giftedness components and corresponds to the basic or advanced level of complexity.

For example, if a giftedness profile is characterized as S1ÑÒ1 (insufficient knowledge of the basic material, respectively, he/she can not solve the problems of both the basic and advanced level, nevertheless the search motivation and “desire to experimentation” with numbers and figures are sufficiently expressed), the following types of problems are suggested to a pupil:

- S1 - all common problems are suggested
- Ò1 - all problems Ò are suggested

**Reflexive stage in block:** According to the results of solved problems, in the forming stage the educational process management system re-generates a report displaying summary levels of the diagnosed components of pupil’s giftedness, as well as an individual success index that represents an integral estimate of the pupil’s performance.

**Organization of teaching:** To organize a teaching process, the educational process management systems can be used; they support integration with the adaptive content, for example, “1C:Education 5. School” (<http://obrazovanie.1c.ru/>).

The section “Library” contains educational materials for the organization of adaptive teaching. As noted above, each topic comprises three types of tasks for pupils - evaluation testing to construct an initial profile of giftedness, forming tests for the correction of “underachieved” substructures of the giftedness profile and four options of the review work (Fig. 1).

The tasks are assigned to pupils in the section “Register”. In this section, adaptive educational materials can be assigned both to a group of pupils and individually, and a date interval, during which a task should be performed, can be set (Fig. 2).

The task to be performed will be available to pupils in the section “Register”. As a pupil progresses in the task he/she receives the following material (question, hint, theoretical materials, etc.) depending on how the previous task has been performed (educational materials are presented by pressing the button “Forward” and to complete a task it is necessary to click on the “Finish” button, Fig. 3).

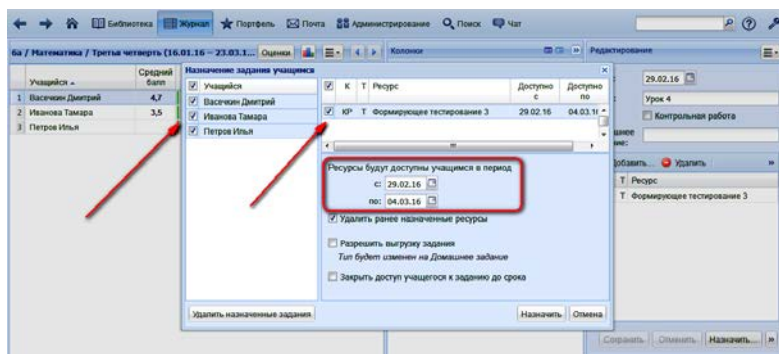


Fig. 2: Assignment of adaptive educational materials to pupils

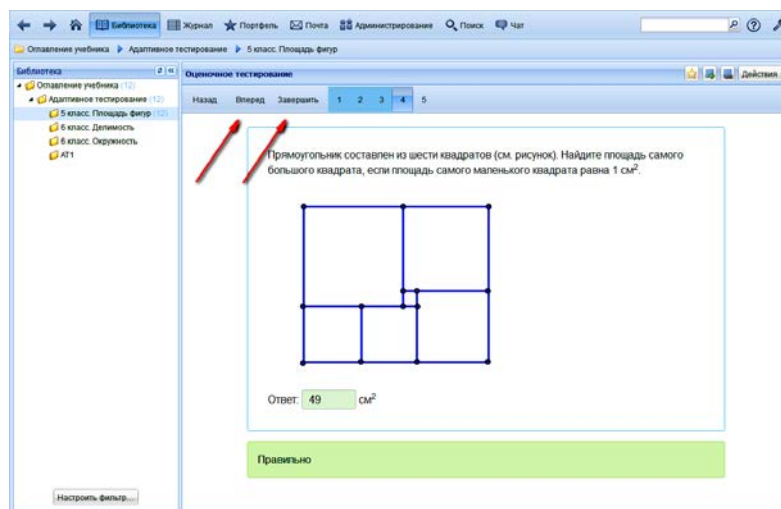


Fig. 3: Implementation of the evaluation testing on the topic “Area of geometric figures”

ба / Математика / Третья четверть (16.01.16 – 23.03.16) Оценки										
Учащийся	Средний балл	19 февр	22 февр	26 февр	29 февр	4 Март	7 Март	11 Март	14 Март	18 Март
1 Васечкин Дмитрий	4,7	5	5	4						
2 Иванова Тамара	3,5	4	3							
3 Петров Илья										

Fig. 4: Displaying grades in the section “Register”

Once a pupil completes the task, a corresponding cell in the register will be marked green (if the task is not performed in time, then the cell will be marked red, Fig. 4). After the evaluation and control tasks are performed, an

automatically given grade with “?” sign will also appear in the register cell; performance of forming tests is not evaluated automatically, but a teacher may assign a grade to a pupil for the performed task at his/her own discretion.

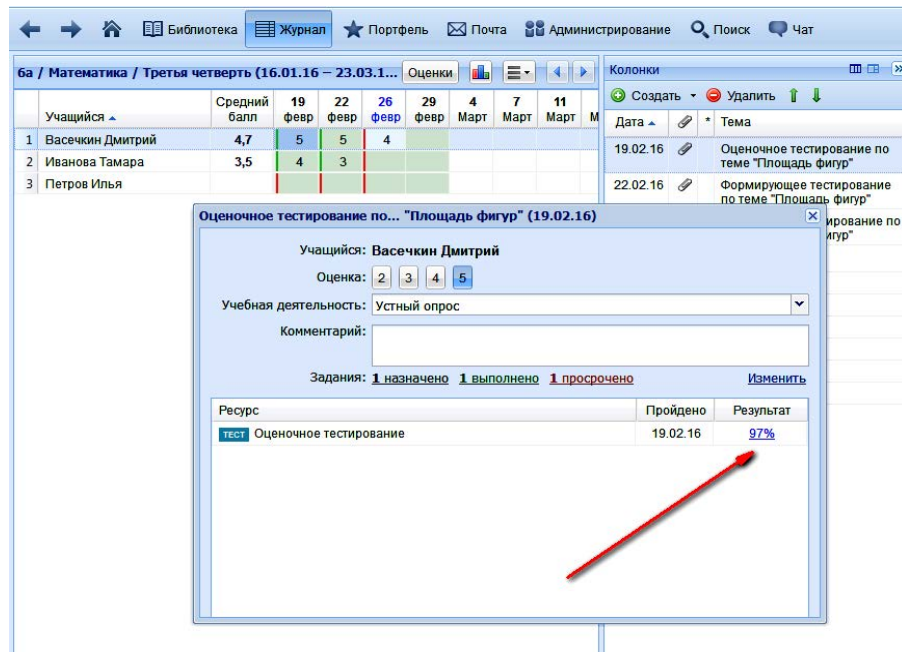


Fig. 5: Short form of the test task report

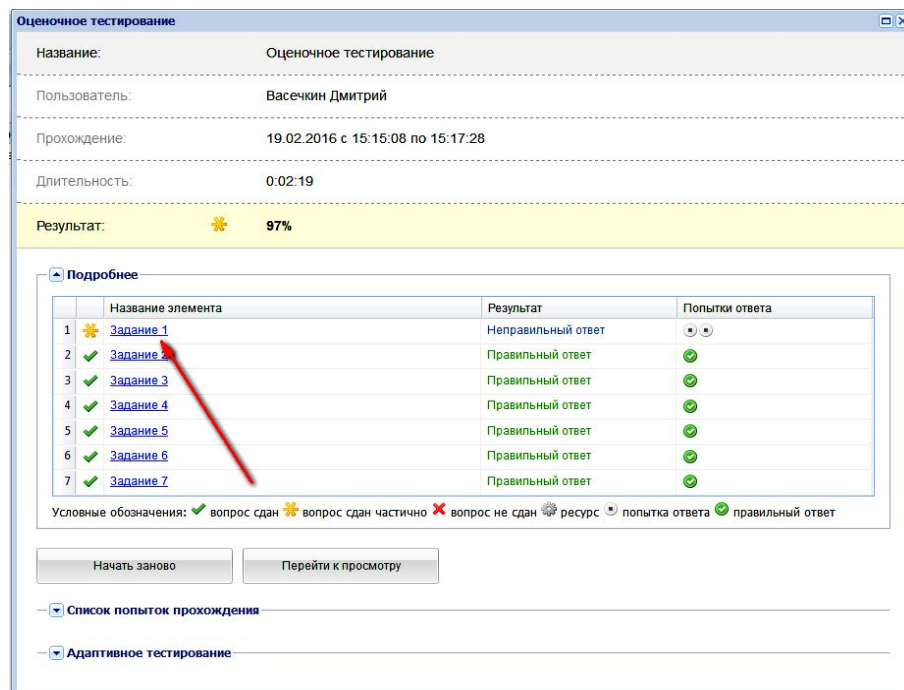


Fig. 6: Detailed test task report

The following information is also available to a teacher: assigned, performed and not performed tasks by a pupil, percentage of performance of a specific test, full report on the performance of a task with

saved pupil's responses, task success index, giftedness diagram (for evaluation testing) and guidance for a teacher on the organization of future activities with this pupil (Fig. 5-7).

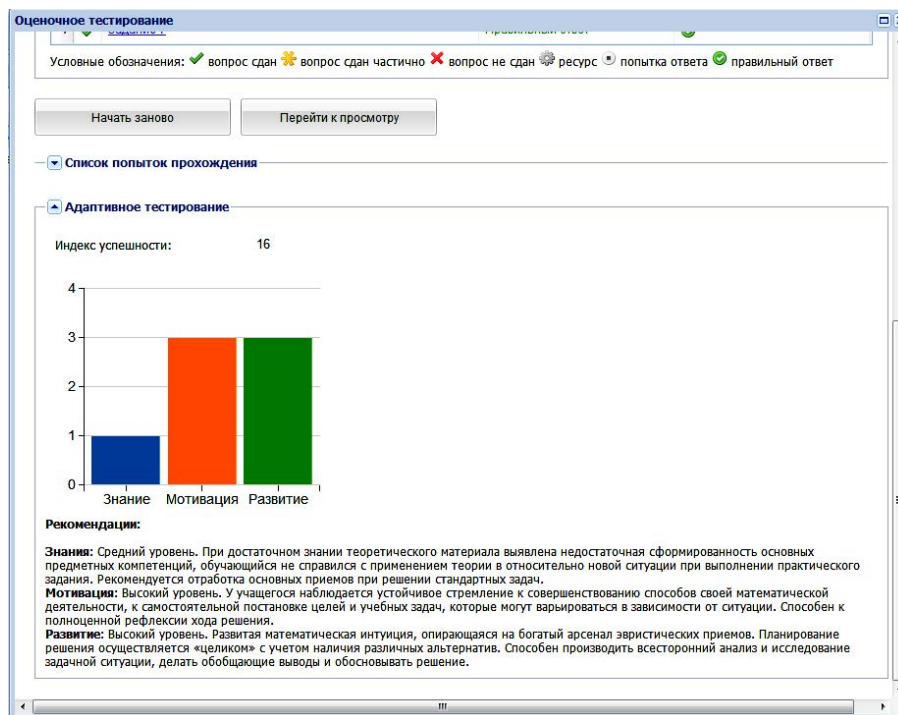


Fig. 7: Giftedness diagram and guidance for teachers on the organization of educational activities with pupils

## RESULTS AND DISCUSSION

The proposed system of educational materials was preliminary approved in a number of Penza schools; the obtained results indicate the availability of the developed adaptive content for pupils of grades 5-6 and its effectiveness in the framework of the declared functionality. In spring 2016, the system was approved in a number of schools in Moscow, Penza and Moscow Region. Approbation was conducted with pupils of sixth grades; a set of educational materials on the topics “Equations” and “Divisibility of Numbers” was proposed for approbation (Fig. 8).

The adaptive automated teaching system was developed on the basis of “1C:Education”; access to the system was organized via the Internet. To work with the tests in schools, classrooms equipped with interactive whiteboard/ multimedia projector; stationary computer classes; mobile computer classes; home computers of pupils were used.

Analyzing the approbation results presented in the used databases and summarizing the approbation results presented in the teachers' reports suggest the following conclusions:

The content and level of complexity of the task material contained in the System correspond to the

existing curriculum in mathematics for grades 5-6; the System includes a sufficient number of search and research tasks; the tasks are quite diverse, by level of complexity - from the simplest to the Olympiad tasks. The teachers who participated in the approbation also mentioned originality, author's approach and a high methodical level of presented tasks (Fig. 9-11).

The system allows to objectively identify an individual character of the pupil's mathematical giftedness and the problems affecting the successful study of the subject matter: inadequate training, lack of motivation and low level of cognitive activity, insufficient mathematical intuition and inadequate knowledge of heuristic methods of mathematical activity.

As a rule, the profiles of pupils' mathematical giftedness obtained in the course of testing coincided with the teachers' observations (Fig. 10).

The system allows creating favorable conditions for the development of “underachieved” components of the pupil's mathematical giftedness; it can be traced back to a change in the giftedness profile as far as a complex of tests on the topic (evaluation, forming and control, Fig. 13a-c) is performed. In a number of schools, the use of the System resulted in increase in the certain pupils' level of training, cognitive activity and knowledge of heuristic methods of mathematical educational activity on



The screenshot shows a software interface for a register page. On the left, there is a list of pupils with their names hidden. The main area displays a grid of test results for various topics, including 'Equations' and 'Divisibility of Numbers'. The grid has columns for different tests and rows for individual pupils. On the right, there is a sidebar with additional information, including a date selector and a list of tests.

Fig. 8: General view of a register page with the test results on the topics “Equations” and “Divisibility of Numbers” (names of pupils are hidden)

The screenshot shows a multi-component standard problem (type S) titled "Оценочное тестирование по теме 'Делимость'". The problem asks the user to select numbers from a given set that satisfy certain criteria. The criteria are listed in a table, and the user is asked to select the correct numbers from a set of options.

Выберите из чисел 1, 2, 3, 4, 5, 9, 12, 13, 17, 27, 30, 31, 108, 125, 285, 713 те, которые являются:	1	2	3	4	5	9	12	13	17	27	30	31	108	125	285	713	
простыми																	
наименьшим и наибольшим составными																	
простыми делителями числа 30																	
кратными 5																	
наибольшим натуральным числом, дающим при делении на 4 с остатком частное 7																	
наибольшим общим делителем чисел 36, 60 и 72																	
наименьшим общим кратным чисел 27 и 36																	

Выберите число и ячейку, куда его нужно переместить.

Правильно

Fig. 9: Multi-component standard problem (type S) that verifies knowledge relating to criteria for divisibility and an ability to find the greatest common divisor and least common multiple of several numbers

The screenshot shows a task (type C) titled "Оценочное тестирование по теме 'Делимость'". The task asks the user to find the greatest common divisor of several numbers and apply it to a word problem. The user has entered the answer "12" for the greatest common divisor and "4" for the number of liters of sunflower oil.

Каково наибольшее количество торговых точек, в которые можно поровну распределить 60л подсолнечного и 48 л кукурузного масла? Сколько литров каждого вида при этом получит одна торговая точка.

Ответ: Торговых точек: 12. Каждая торговая точка получит 4 л кукурузного масла и 5 л подсолнечного масла.

Правильно

Fig. 10: Task (type C) that verifies an ability to apply the concept of the greatest common divisor of several numbers in an irregular situation

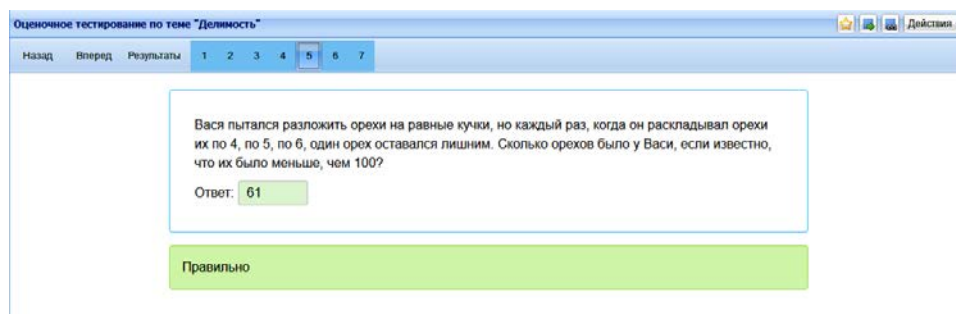


Fig. 11: Task (type T) that verifies an ability to put forward an idea of solving a problem (heuristics) and to apply the concept of the least common multiple of several numbers in an irregular situation

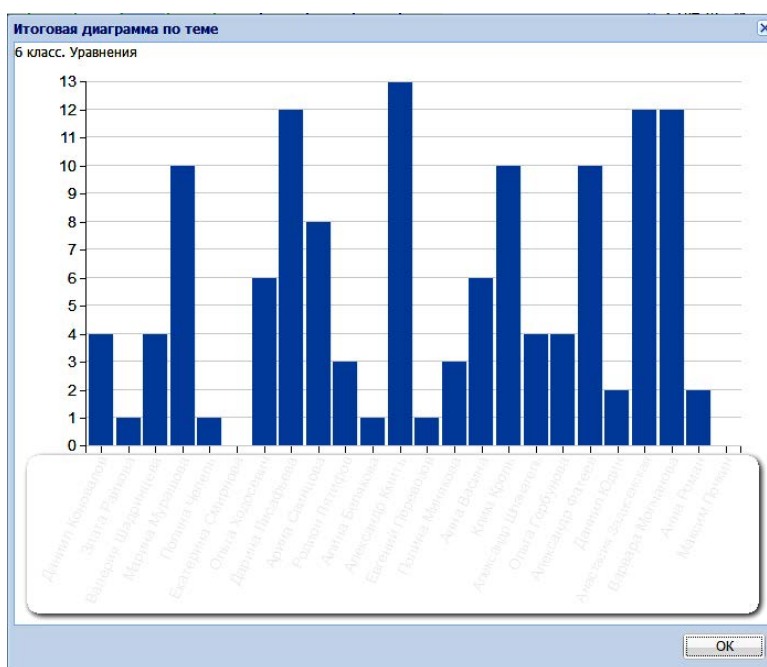


Fig. 12: Summary diagram on the topic “Equations” - pupils with the highest development level of all diagnosed components in the giftedness profile can be clearly seen (names of pupils are hidden)

the topic “Equations” and “Divisibility”; improved results of the annual review work for the 6th grade course in mathematics, as many tasks in such review work were associated with the topic “Equations” brought up for the approbation.

An interesting and unexpected result of the System approbation was the fact that communication and regulatory competence of pupils developed during the work with the System: when working with the forming tests the pupils on their own united in pairs and groups, discussed solutions, developed a common opinion, consulted each other and a teacher.

The system is convenient for teacher’s application. The approbation participants noted the following among

the convenience of use: system’s ease of use; high rate of operations on the task assignment and check of task performance; visual representation of the pupils’ results, an opportunity to be more flexible and accurate in specifying the learners’ knowledge and identifying the topics that a learner knows poorly; an opportunity to set an individual teaching trajectory for addressing the gaps.

Among the learner’s advantages, the approbation participants indicated: user friendly interface for a pupil; understandable and accessible tasks wordings that allow saving time at a lesson and using the System for assigning homework to pupils; interactive operating mode when after making mistakes a pupil has an opportunity to review the information: where a mistake was made, how to



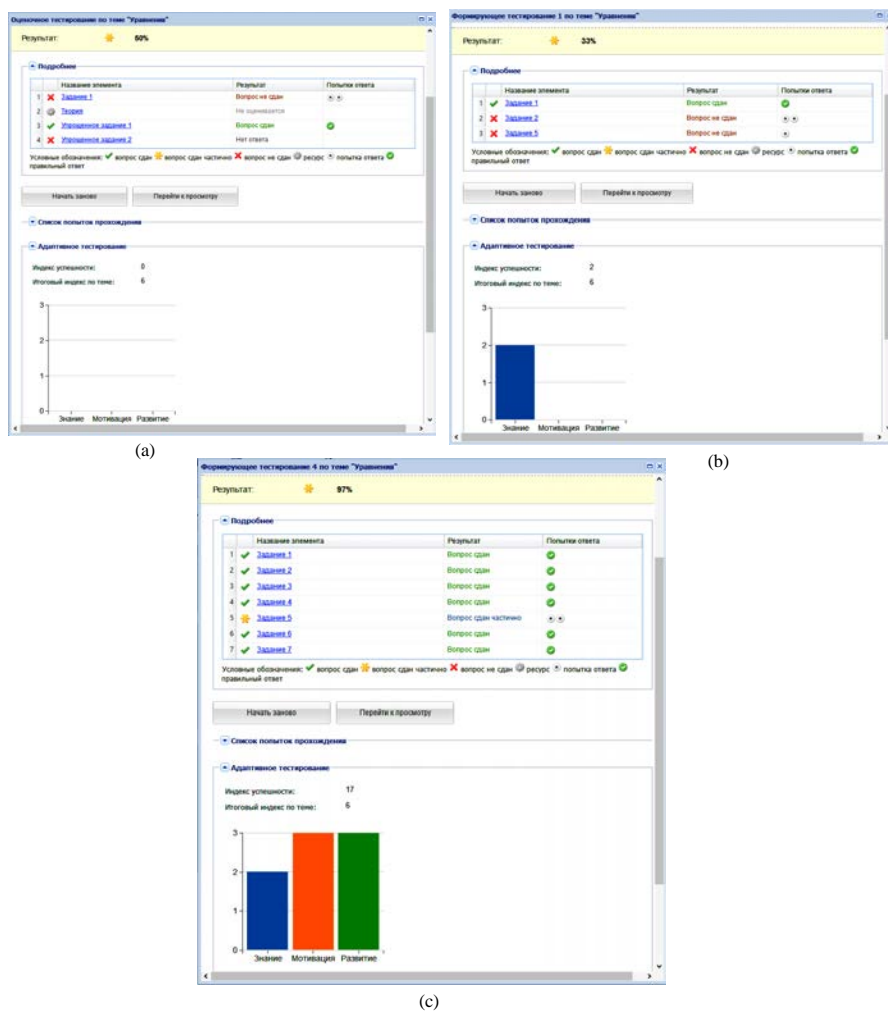


Fig. 13: a-c) Sequentially (left to right), the results of diagnostic, forming tests No. 1 and No. 4 of a 6th grade pupil are presented. At transition to the next topic "Divisibility of Numbers", this pupil showed, on the diagnostic testing, the results and a giftedness profile similar to the results of forming test No. 4

correct it, and the System affords a chance to make this way again, to perform a task of the same type; a chance to advance in the subject matter at a comfortable pace and to reflect on the actions.

Regarding the System's feasibility of using in the educational process, the approbation participants noted the System's universal nature: it can be used at lessons; for assigning interactive homework; in supplementary education (mathematical circles and optional classes).

Thus, the approbation results can be considered positive. The approbation participants recommended using and distributing the System among the general education institutions of the Russian Federation. The vast majority of approbation participants expressed a willingness to use this System in their professional activities in the next academic year.

## CONCLUSION

Despite the importance and efficiency of the developed program-methodical solutions for organization of pupils' adaptive teaching process, pedagogical justification of the educational material presented in the most appropriate educational systems requires additional studies. In particular, the problem of objective selection of latent parameters of training and personal characteristics that should form the basis for the development of various diagnostic and forming procedures remains far from fully solved.

In solving the problem of the pupils' mathematical giftedness diagnostics and development, it is advisable to choose a triad of the following characteristics of an individual pupil (giftedness profile) as a unit of analysis

of such giftedness formation: composition and structure of individual experience, learner's level of training; cognitive activity and in particular, the nature of his/her meaning and goal formation at various stages of teaching and search activity as well as the pupil's level of mathematical intuition, his/her knowledge of heuristic methods of mathematical activity in solving any research problems. The above components can be presented in the pupil's giftedness structure at different levels, and one or another combination of conditions predetermines a possibility of his/her teaching and development.

In order to determine a pupil's giftedness type and level recorded in the corresponding giftedness profile and to create favorable conditions for the development of a particular pupil's "underachieved" components, an adaptive computer-based teaching system design supported by 1C platform has been developed. The project calls for a need of constant account of the real development of pupil's giftedness in order to optimize the choice of direction of such development realized in the process of transition from the "condition of lower-level giftedness" to the "condition of higher-level giftedness" in solving chains of specially selected mathematical problems. In this case, an appropriate educational and developmental content plays a role of "supporting" content-methodological instruments "within" the basic course in mathematics at all stages of the educational process: diagnostic, forming and reflexive. The system's adaptability is ensured by variability of automated transitions between the tasks of respective blocks in a particular module included in the content. At the same time, within each module pupils self-diagnose their current level of mathematical abilities and, according to it, they realize their individual trajectory at mastering an appropriate method of heuristic search.

The current and final diagnosis of the pupil's creative potential in the framework of a particular module is made by implementing a specially developed evaluation system produced in the performance of tasks on a relevant topic. According to the results of solved problems, the educational process management system generates a report displaying the initial and final levels of the diagnosed components of pupil's giftedness, as well as an individual success index that represents an integral estimate of the pupil's performance. Comparison of these indicators leads to the conclusion about effectiveness of the work done.

As a result of approbation of the proposed adaptive automated teaching system developed on the basis of "1C:Education" at schools of several Russian cities (access to the system was organized via the Internet), it has been found out that it allows to objectively identify an individual character of the pupil's mathematical giftedness and the problems affecting the successful study of the subject matter, to create favorable conditions for the development of "underachieved" components of such giftedness and to facilitate to acquiring the pupils' communication and regulatory competence.

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