

Performance Analysis of a Single Host System with Three Types of Heterogeneous Software

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Abstract: Computer systems are usually studied with intention to the evaluation of their reliability measures in terms of availability. This studies the availability and profit of a single system operating with the help of three heterogeneous software. Markov Model of the system is derived through the system state transition table and differential difference equations which are further used to evaluate the system availability, mean time to system failure and profit. Based on assumed numerical values given to system parameters, graphical illustrations are given to highlight important results.

Key words: Availability, mean time to failure, profit, host, software, Markov Model

INTRODUCTION

Maintaining a high or required level of reliability and availability is especially essential in information industry, communication systems, power plants, etc. One of the most critical competitive factors in the computer systems market is the reliability of the system, given that the software failure may stop entire system. Applications of computer systems are pictured in banking organization, education organization, military, space projects and medical systems are intolerant of failures as the economy and lives may depend on the reliable operation of the computer systems. Computer systems exhibit two types of failures-hardware and software. Provision of standby unit is vital towards achieving high reliability. System reliability is improved through a standby unit support which is capable of performing similar function with the operational unit but with different degree and desirability. Redundancy is a technique used to improve system reliability and availability. It consists of techniques for increasing system effectiveness through reducing failure and maintenance cost minimization. Several techniques have been suggested by the researchers, designers and engineers for performance improvement of the systems. The unit wise redundancy technique has been considered as one of these in the development of stochastic models for computer systems. Welke *et al.* (1995) have discussed reliability modelling of a hardware/software system. The technique of unit wise redundancy in cold standby mode has also been used in computer systems. Malik and Anand (2010), Kumar *et al.* (2015) and Malik (2013) analyzed different computer system models with unit wise

cold standby redundancy and different repair policies. But it is also proved that component wise redundancy is better than unit wise redundancy, so, far as reliability is concerned. Malik and Munday (2014) developed a stochastic model for a computer system with hardware component in cold standby redundancy. Anand and Malik (2012) studied a cold standby computer system by giving priority to hardware repair activities over software replacement. Malik and Sureria (2012) and Kumar *et al.* (2015) analyzed computers systems with cold standby redundancy under different failures and repair policies. Kumar and Malik (2012, 2014) have discussed modelling of a computer system with priority to preventive maintenance over software replacement and priority to hardware repair over replacement, respectively. Kumar *et al.* (2015) have analyzed the performance of a computer system with fault detection of hardware.

Existing literatures ignores the reliability, availability, mean time to system and profit modelling of computer systems incorporating different software with similar task on a single host and the impact such heterogeneous software on computer system performance. Example of such heterogeneous software on computer system can be seen in operating systems (Windows 7, 8, 10, Windows XP, Vista, Ubuntu), application packages (Latex and MS Word), Mathematical software (Mathematica, MATLAB, Maple), etc. Some of this software can be install on a single host. This heterogeneous software will assist in reducing operating costs and the risk of a catastrophic breakdown, extending the availability and working time, increasing the revenue generated for a system.

The problem considered in this study different from the resesarch of discussed researchers above. In this study, a single host with three cold standby heterogeneous software is considered and derived its corresponding mathematical models. The focus of our analysis is primarily to capture the effect of both failure and repair rates on the measures of system effectiveness like availability and profit.

MATERIALS AND METHODS

Description and states of the system: The system is composed of a hardware and three types of software. At initial state, the system is working with type 1 active software while type 2 and 3 software are on standby. Each of the software fails independent of the state of the other and has an exponential failure distribution with parameters λ_{s_i} ($i = 1, 2, 3$). Whenever an active software fails, it is immediately repaired with the service rate equal to μ_{s_i} . Repair time distribution of each software is assumed to be exponentially distributed and the next standby software continue as an active software. It is assumed that switching from standby to operation is perfect and instantaneous. System hardware failed with parameter λ_H and service rate with parameter μ_H where both hardware failure and service time are assumed to be exponential. System failure results from the failure of hardware or both types of software.

RESULTS AND DISCUSSION

Formulation of the models: In order to analyze the system availability of the system we define $P_i(t)$ to be the probability that the system at $t \geq 0$ is in state S_i also let $P(t)$ be the row vector of these probabilities at time t . The initial condition for this problem is:

$$P(0) = [p_0(0), p_1(0), p_2(0), \dots, p_6(0)] = [1, 0, 0, 0, 0, 0, 0]$$

We obtain the following differential difference equations from Table 1:

Table 1: Transition table of the system

Variables	S_0	S_1	S_2	S_3	S_4	S_5	S_6
S_0	0	λ_H	λ_{s1}	0	0	0	0
S_1	μ_H	0	0	0	0	0	0
S_2	μ_{s1}	0	0	λ_{s2}	0	λ_H	0
S_3	0	0	μ_{s2}	0	λ_{s3}	0	0
S_4	0	0	0	μ_{s3}	0	0	0
S_5	0	0	μ_H	0	0	0	0
S_6	0	0	0	μ_H	0	0	0

$$\left. \begin{aligned} \frac{d}{dt} p_0(t) &= -(\lambda_{s1} + \lambda_H) p_0(t) + \mu_H p_1(t) + \mu_{s1} p_2(t) \\ \frac{d}{dt} p_1(t) &= -\mu_H p_1(t) + \lambda_H p_0(t) \\ \frac{d}{dt} p_2(t) &= -(\mu_{s1} + \lambda_{s2} + \lambda_H) p_2(t) + \lambda_{s1} p_0(t) + \mu_{s2} p_3(t) + \mu_H p_5(t) \\ \frac{d}{dt} p_3(t) &= -(\mu_{s2} + \lambda_H + \lambda_{s3}) p_3(t) + \lambda_{s2} p_2(t) + \mu_{s3} p_4(t) + \mu_H p_6(t) \\ \frac{d}{dt} p_4(t) &= -\mu_{s3} p_4(t) + \lambda_{s3} p_3(t) \\ \frac{d}{dt} p_5(t) &= -\mu_H p_5(t) + \lambda_H p_2(t) \\ \frac{d}{dt} p_6(t) &= -\mu_H p_6(t) + \lambda_H p_3(t) \end{aligned} \right\} \quad (1)$$

The above differential equations are written in the form $P' = MP$.

Where:

$$M = \begin{pmatrix} -(\lambda_{s1} + \lambda_H) & \mu_H & \mu_{s1} & 0 & 0 & 0 & 0 \\ \lambda_H & -\mu_H & 0 & 0 & 0 & 0 & 0 \\ \lambda_{s1} & 0 & -(\mu_{s1} + \lambda_{s2} + \lambda_H) & \mu_{s2} & 0 & \mu_H & 0 \\ 0 & 0 & \lambda_{s2} & -(\mu_{s2} + \lambda_{s3} + \lambda_H) & \mu_{s3} & 0 & \mu_H \\ 0 & 0 & 0 & \lambda_{s3} & -\mu_{s3} & 0 & 0 \\ 0 & 0 & \lambda_H & 0 & 0 & -\mu_H & 0 \\ 0 & 0 & 0 & \lambda_H & 0 & 0 & -\mu_H \end{pmatrix} \quad (2)$$

The time dependent analytic solution is difficult to obtain. Using the approach adopted in Wang and Kuo (2000), Wang and Ke (2003), Wang and Pearn (2003) and Wang *et al.* (2005), the MTTF is obtained by taking the transpose matrix of M and delete the rows and columns for the absorbing state and designation the new matrix by Q. The expected time to reach an absorbing state is evaluated from:

$$E = [T_{P(0) \rightarrow P(\text{absorbing})}] = P(0)(-Q^{-1})[1, 1, 1]^T \quad (3)$$

The explicit expression for the MTTF for the system is obtained from:

$$MTTF = \frac{\mu_{s1}(\mu_{s2} + \lambda_H + \lambda_{s3}) + \lambda_H(\mu_{s2} + \lambda_H + \lambda_{s3}) + \lambda_{s1}(\mu_{s2} + \lambda_H + \lambda_{s3}) + \lambda_{s2}\lambda_{s3}}{(\mu_{s2} + \lambda_{s2} + \lambda_{s3})(\lambda_H^2 + \lambda_H\lambda_{s1}) + \lambda_H^2(\mu_{s1} + \lambda_{s1}) + \mu_{s1}\lambda_H(\lambda_{s2} + \lambda_{s3}) + \lambda_{s2}\lambda_{s3}(\lambda_H + \lambda_{s1})}$$

where, $P(0) = [1, 0, 0]$ and:

$$Q = \begin{pmatrix} -(\lambda_H + \lambda_{s1}) & \lambda_{s1} & 0 \\ \mu_{s1} & -(\mu_{s1} + \lambda_H + \lambda_{s2}) & \lambda_{s2} \\ 0 & \mu_{s2} & -(\mu_{s2} + \lambda_H + \lambda_{s3}) \end{pmatrix}$$

Equation 2 is expressed explicitly in the form:

$$\begin{pmatrix} \dot{p}_0(t) \\ \dot{p}_1(t) \\ \dot{p}_2(t) \\ \dot{p}_3(t) \\ \dot{p}_4(t) \\ \dot{p}_5(t) \\ \dot{p}_6(t) \end{pmatrix} = \begin{pmatrix} -(\lambda_{s1}+\mu_{H1}) & \mu_{H1} & \mu_{S1} & 0 & 0 & 0 & 0 \\ \lambda_{H1} & -\mu_{H1} & 0 & 0 & 0 & 0 & 0 \\ \lambda_{s1} & 0 & -(\mu_{s1}+\lambda_{s2}+\lambda_{H1}) & \mu_{S2} & 0 & \mu_{H1} & 0 \\ 0 & 0 & \lambda_{s2} & -(\mu_{s2}+\lambda_{s3}+\lambda_{H1}) & \mu_{S3} & 0 & \mu_{H1} \\ 0 & 0 & 0 & \lambda_{s3} & -\mu_{S3} & 0 & 0 \\ 0 & 0 & \lambda_{H1} & 0 & 0 & -\mu_{H1} & 0 \\ 0 & 0 & 0 & \lambda_{H1} & 0 & 0 & -\mu_{H1} \end{pmatrix} \begin{pmatrix} p_0(t) \\ p_1(t) \\ p_2(t) \\ p_3(t) \\ p_4(t) \\ p_5(t) \\ p_6(t) \end{pmatrix}$$

Let T denote the time-to-failure of the system. The steady-state availability (the proportion of time the system is in a functioning condition or equivalently, the sum of the probabilities of operational states) and busy period probabilities due to repair of type 1-3 and hardware are given by:

$$\left. \begin{aligned} A_T(\infty) &= p_0(\infty) + p_2(\infty) + p_3(\infty) \\ B_{P1}(\infty) &= p_2(\infty) \\ B_{P2}(\infty) &= p_3(\infty) \\ B_{P3}(\infty) &= p_4(\infty) \\ B_{P4}(\infty) &= p_1(\infty) + p_5(\infty) + p_6(\infty) \end{aligned} \right\} \quad (4)$$

In the steady state, the derivatives of the state probabilities become zero and therefore, Eq. 2 become MP = 0 which is in matrix form:

$$\begin{pmatrix} -(\lambda_{s1}+\lambda_{H1}) & \mu_{H1} & \mu_{S1} & 0 & 0 & 0 & 0 \\ \lambda_{H1} & -\mu_{H1} & 0 & 0 & 0 & 0 & 0 \\ \lambda_{s1} & 0 & -(\mu_{s1}+\lambda_{s2}+\lambda_{H1}) & \mu_{S2} & 0 & \mu_{H1} & 0 \\ 0 & 0 & \lambda_{s2} & -(\mu_{s2}+\lambda_{s3}+\lambda_{H1}) & \mu_{S3} & 0 & \mu_{H1} \\ 0 & 0 & 0 & \lambda_{s3} & -\mu_{S3} & 0 & 0 \\ 0 & 0 & \lambda_{H1} & 0 & 0 & -\mu_{H1} & 0 \\ 0 & 0 & 0 & \lambda_{H1} & 0 & 0 & -\mu_{H1} \end{pmatrix} \begin{pmatrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \\ p_5(\infty) \\ p_6(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Subject to following normalizing conditions:

$$p_0(\infty) + p_1(\infty) + p_2(\infty) + p_3(\infty) + p_4(\infty) + p_5(\infty) + p_6(\infty) = 1 \quad (6)$$

Following by Wang and Kuo (2000) and Wang *et al.* (2006), we substitute Eq. 6 in the last row of 5 to compute the steady-state probabilities.

$$\begin{pmatrix} -(\lambda_{s1}+\lambda_{H1}) & \mu_{H1} & \mu_{S1} & 0 & 0 & 0 & 0 \\ \lambda_{H1} & -\mu_{H1} & 0 & 0 & 0 & 0 & 0 \\ \lambda_{s1} & 0 & -(\mu_{s1}+\lambda_{s2}+\lambda_{H1}) & \mu_{S2} & 0 & \mu_{H1} & 0 \\ 0 & 0 & \lambda_{s2} & -(\mu_{s2}+\lambda_{s3}+\lambda_{H1}) & \mu_{S3} & 0 & \mu_{H1} \\ 0 & 0 & 0 & \lambda_{s3} & -\mu_{S3} & 0 & 0 \\ 0 & 0 & \lambda_{H1} & 0 & 0 & -\mu_{H1} & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \\ p_5(\infty) \\ p_6(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Solving Eq. 5 using 6, the steady-state probabilities are given below:

$$\left. \begin{aligned} p_0(\infty) &= \frac{\mu_{S1}\mu_{S2}\mu_{S3}}{\mu_{S3}(\lambda_{H1}+\mu_{H1})(\lambda_{s1}\lambda_{s2}+\mu_{S1}\mu_{S2})+\mu_{S2}\mu_{H1}\lambda_{s1}(\lambda_{H1}+\mu_{S3})+\lambda_{s1}\lambda_{s2}\lambda_{s3}(2\mu_{S3}+\mu_{H1})} \\ p_1(\infty) &= \frac{\mu_{S1}\mu_{S2}\mu_{S3}\lambda_{H1}}{\mu_{S3}(\lambda_{H1}+\mu_{H1})(\lambda_{s1}\lambda_{s2}+\mu_{S1}\mu_{S2})+\mu_{S2}\mu_{H1}\lambda_{s1}(\lambda_{H1}+\mu_{S3})+\lambda_{s1}\lambda_{s2}\lambda_{s3}(2\mu_{S3}+\mu_{H1})} \\ p_2(\infty) &= \frac{\mu_{S2}\mu_{H1}\mu_{S3}\lambda_{s1}}{\mu_{S3}(\lambda_{H1}+\mu_{H1})(\lambda_{s1}\lambda_{s2}+\mu_{S1}\mu_{S2})+\mu_{S2}\mu_{H1}\lambda_{s1}(\lambda_{H1}+\mu_{S3})+\lambda_{s1}\lambda_{s2}\lambda_{s3}(2\mu_{S3}+\mu_{H1})} \\ p_3(\infty) &= \frac{\mu_{S2}\mu_{H1}\mu_{S3}\lambda_{s2}}{\mu_{S3}(\lambda_{H1}+\mu_{H1})(\lambda_{s1}\lambda_{s2}+\mu_{S1}\mu_{S2})+\mu_{S2}\mu_{H1}\lambda_{s1}(\lambda_{H1}+\mu_{S3})+\lambda_{s1}\lambda_{s2}\lambda_{s3}(2\mu_{S3}+\mu_{H1})} \\ p_4(\infty) &= \frac{\mu_{H1}\lambda_{s1}\lambda_{s2}\lambda_{s3}}{\mu_{S3}(\lambda_{H1}+\mu_{H1})(\lambda_{s1}\lambda_{s2}+\mu_{S1}\mu_{S2})+\mu_{S2}\mu_{H1}\lambda_{s1}(\lambda_{H1}+\mu_{S3})+\lambda_{s1}\lambda_{s2}\lambda_{s3}(2\mu_{S3}+\mu_{H1})} \\ p_5(\infty) &= \frac{\mu_{S2}\mu_{S3}\lambda_{H1}\lambda_{s1}}{\mu_{S3}(\lambda_{H1}+\mu_{H1})(\lambda_{s1}\lambda_{s2}+\mu_{S1}\mu_{S2})+\mu_{S2}\mu_{H1}\lambda_{s1}(\lambda_{H1}+\mu_{S3})+\lambda_{s1}\lambda_{s2}\lambda_{s3}(2\mu_{S3}+\mu_{H1})} \\ p_6(\infty) &= \frac{\mu_{S3}\lambda_{H1}\lambda_{s2}}{\mu_{S3}(\lambda_{H1}+\mu_{H1})(\lambda_{s1}\lambda_{s2}+\mu_{S1}\mu_{S2})+\mu_{S2}\mu_{H1}\lambda_{s1}(\lambda_{H1}+\mu_{S3})+\lambda_{s1}\lambda_{s2}\lambda_{s3}(2\mu_{S3}+\mu_{H1})} \end{aligned} \right\} \quad (7)$$

The expressions for the steady-state availability and busy period probabilities due to repair of 1-3 types and hardware given in Eq. 4 are:

$$\left. \begin{aligned} A_T(\infty) &= \frac{\mu_{S1}\mu_{S2}\mu_{H1}\mu_{S3}+\lambda_{s1}\mu_{S2}\mu_{H1}\mu_{S3}+\mu_{H1}\mu_{S3}\lambda_{s1}\lambda_{s2}}{\mu_{S3}(\lambda_{H1}+\mu_{H1})(\lambda_{s1}\lambda_{s2}+\mu_{S1}\mu_{S2})+\mu_{S2}\mu_{H1}\lambda_{s1}(\lambda_{H1}+\mu_{S3})+\lambda_{s1}\lambda_{s2}\lambda_{s3}(2\mu_{S3}+\mu_{H1})} \\ B_{P1}(\infty) &= \frac{\lambda_{s1}\mu_{S2}\mu_{H1}\mu_{S3}}{\mu_{S3}(\lambda_{H1}+\mu_{H1})(\lambda_{s1}\lambda_{s2}+\mu_{S1}\mu_{S2})+\mu_{S2}\mu_{H1}\lambda_{s1}(\lambda_{H1}+\mu_{S3})+\lambda_{s1}\lambda_{s2}\lambda_{s3}(2\mu_{S3}+\mu_{H1})} \\ B_{P2}(\infty) &= \frac{\mu_{H1}\mu_{S3}\lambda_{s1}\lambda_{s2}}{\mu_{S3}(\lambda_{H1}+\mu_{H1})(\lambda_{s1}\lambda_{s2}+\mu_{S1}\mu_{S2})+\mu_{S2}\mu_{H1}\lambda_{s1}(\lambda_{H1}+\mu_{S3})+\lambda_{s1}\lambda_{s2}\lambda_{s3}(2\mu_{S3}+\mu_{H1})} \\ B_{P3}(\infty) &= \frac{\mu_{H1}\lambda_{s1}\lambda_{s2}\lambda_{s3}}{\mu_{S3}(\lambda_{H1}+\mu_{H1})(\lambda_{s1}\lambda_{s2}+\mu_{S1}\mu_{S2})+\mu_{S2}\mu_{H1}\lambda_{s1}(\lambda_{H1}+\mu_{S3})+\lambda_{s1}\lambda_{s2}\lambda_{s3}(2\mu_{S3}+\mu_{H1})} \\ B_{P4}(\infty) &= \frac{\mu_{S1}\mu_{S2}\mu_{S3}\lambda_{H1}+\lambda_{H1}\lambda_{s1}\mu_{S2}\mu_{S3}+\mu_{S3}\lambda_{H1}\lambda_{s1}\lambda_{s2}}{\mu_{S3}(\lambda_{H1}+\mu_{H1})(\lambda_{s1}\lambda_{s2}+\mu_{S1}\mu_{S2})+\mu_{S2}\mu_{H1}\lambda_{s1}(\lambda_{H1}+\mu_{S3})+\lambda_{s1}\lambda_{s2}\lambda_{s3}(2\mu_{S3}+\mu_{H1})} \end{aligned} \right\} \quad (8)$$

Table 2: States of the system

States	Descriptions
S ₀	Initial state, hardware and type 1 software are working, type 2 and 3 software are on standby. The system is working
S ₁	Hardware has failed and is under repair, type 1 software is suspended, type 2 and 3 software are on standby. The system is inoperative
S ₂	Hardware and type 2 software are working, type 1 software has failed and is under repair, type 3 software is on standby. The system is working
S ₃	Hardware and type 3 software are working, type 1 and 2 software have failed and are under repair. The system is working
S ₄	Hardware is suspended, type 1-3 have failed are under repair. The system is inoperative
S ₅	Hardware and type 1 software have failed and are under repair, type 2 software is suspended, type 3 software is on standby. The system is inoperative
S ₆	Hardware, type 1 and 2 software have failed and are under repair, type 3 software is on standby. The system is inoperative

Let C₀-C₄ be the revenue generated when the system is in working state and no income when in failed state, cost per unit time for which the repairman is busy due to type 1 software failure, cost per unit time for which the repairman is busy due to type 2 software failure, cost per unit time for which the repairman is busy due to type 3 software failure and cost per unit time for which the repairman is busy due to hardware failure, respectively. The expected total profit per unit time incurred to the system in the steady-state is: Profit = Total revenue generated-Cost incurred by the repair man due to preventive maintenance-cost incurred when repairing the failed units.

The hardware and software are subjected to corrective maintenance due to failures as can be observed in states 1-6 in Table 2. The profit incurred to the system model in steady state can be obtained as:

$$P_F(\infty) = C_0A_V(\infty) - C_1B_{P1}(\infty) - C_2B_{P2}(\infty) - C_3B_{P3}(\infty) - C_4B_{P4}(\infty) \tag{9}$$

Numerical examples: Numerical examples are presented to demonstrate the impact of repair and failure rates on steady-state availability and net profit of the system based on given values of the parameters. For the purpose of numerical example, the following sets of parameter values are used: λ_H = 0.2, λ_{S1} = 0.2, λ_{S2} = 0.3, λ_{S3} = 0.3, μ_H = 0.8, μ_{S1} = 0.7, μ_{S2} = 0.2, μ_{S3} = 0.2, C₀ = 2,500,000, C₁ = 1000, C₂ = 1250, C₃ = 1350 and C₄ = 2500. MATLAB package was used to program the simulations in this study. The results are presented below.

Numerical results of availability and profit with respect to hardware failure rate λ_H are depicted in Fig. 1-3, respectively. Simulations in these fig. show that availability and profit decreases as λ_H increases for different values of hardware repair rate μ_H. It is clear from these fig that system availability and profit display decreasing pattern with respect to λ_H. Thus, the system availability and profit are more sensitive to λ_{S1}. This sensitivity analysis implies that major maintenance should be invoked to the hardware to minimize the failure of the system in order to improve and maximize the system

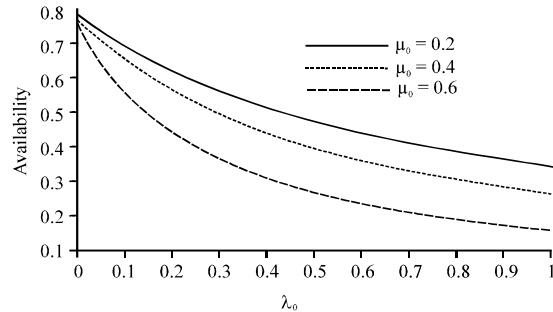


Fig. 1: Availability against hardware failure rate λ_H for different values of μ_H (0.2, 0.4, 0.6)

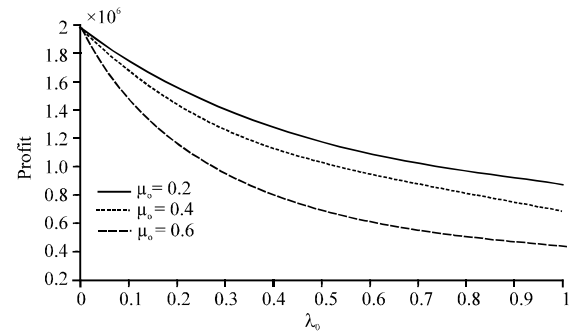


Fig. 2: Profit against hardware failure rate λ_H for different values of μ_H (0.2, 0.4, 0.6)

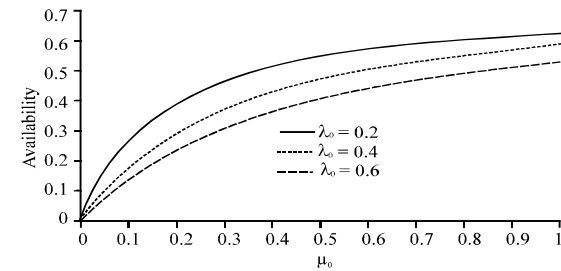


Fig. 3: Availability against hardware repair rate λ_H for different values of λ_H (0.2, 0.4, 0.6)

availability, production output as well as the profit. On the other hand, numerical results of availability and profit with respect to hardware repair rate μ_H are depicted in

Table 3: Variation of availability with respect to failure and repair rates

Variables	$A_T^{(\infty)}$ λ_{S1}	$A_T^{(\infty)}$ λ_{S2}	$A_T^{(\infty)}$ λ_{S3}	$A_T^{(\infty)}$ μ_{S1}	$A_T^{(\infty)}$ μ_{S2}	$A_T^{(\infty)}$ μ_{S3}
0	0.8000	0.8000	0.8000	0.4651	0.3636	0.0000
0.1	0.6726	0.7143	0.7273	0.5000	0.5405	0.5000
0.2	0.6154	0.6567	0.6667	0.5283	0.6154	0.6154
0.3	0.5829	0.6154	0.6154	0.5517	0.6567	0.6667
0.4	0.5620	0.5843	0.5714	0.5714	0.6829	0.6957
0.5	0.5474	0.5600	0.5333	0.5882	0.7010	0.7143
0.6	0.5366	0.5405	0.5000	0.6027	0.7143	0.7273
0.7	0.5283	0.5246	0.4706	0.6154	0.7244	0.7368
0.8	0.5217	0.5113	0.4444	0.6265	0.7324	0.7442
0.9	0.5164	0.5000	0.4211	0.6364	0.7389	0.7500

Table 4: Variation of profit with respect to software failure and repair rates

Variables	$P_F^{(\infty)}$ λ_{S1}	$P_F^{(\infty)}$ λ_{S2}	$P_F^{(\infty)}$ λ_{S3}	$P_F^{(\infty)}$ μ_{S1}	$P_F^{(\infty)}$ μ_{S2}	$P_F^{(\infty)}$ μ_{S3}
0	1.9995	1.9993	1.9991	1.1614	0.9077	-0.0014
0.1	1.6806	1.7849	1.8173	1.2487	1.3502	1.2489
0.2	1.5375	1.6409	1.6657	1.3195	1.5375	1.5375
0.3	1.4562	1.5375	1.5375	1.3782	1.6409	1.6657
0.4	1.4038	1.4596	1.4276	1.4275	1.7064	1.7382
0.5	1.3672	1.3989	1.3323	1.4695	1.7517	1.7848
0.6	1.3403	1.3502	1.2489	1.5058	1.7849	1.8173
0.7	1.3195	1.3103	1.1754	1.5375	1.8102	1.8412
0.8	1.3031	1.2770	1.1100	1.5653	1.8302	1.8595
0.9	1.2898	1.2488	1.0515	1.5900	1.8464	1.8741

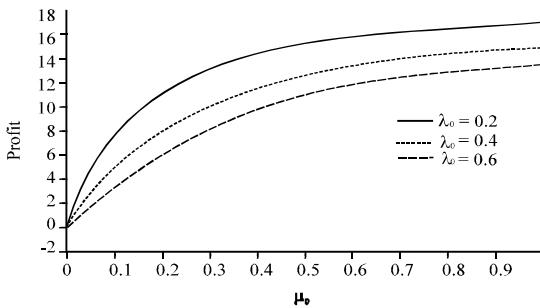


Fig. 4: Profit against hardware repair rate μ_H or different values of λ_H (0.2, 0.4, 0.6)

Fig. 3 and 4, respectively. In these fig, the availability and profit increases as λ_H for different values of hardware failure rate. The gaps between the curves in the figures widen as μ_H increases. This sensitivity analysis implies that preventive and major maintenance to the hardware should be invoked to improve and maximize the system availability, production output as well as generated profit. Table 3-5 provides information how the availability, profit and mean time to system failure of the system changes with respect to software failure and repair rates are fixed at different values. When the failure, repair rates, generated revenue and cost of repairs are fixed at values, $\lambda_{S1} = 0.2$, $\lambda_{S2} = 0.3$, $\lambda_{S3} = 0.3$, $\mu_H = 0.8$, $\mu_{S1} = 0.7$, $\mu_{S2} = 0.2$, $\mu_{S3} = 0.2$, the availability, profit and mean time to system failure of the system tend to decrease. Similar observation can be seen with respect to λ_{Si} ($i = 1, 2, 3$). On the other hand when the availability, profit and mean time to system

Table 5: Variation of MTTF with respect to software failure and repair rates

Variables	MTTF λ_{S1}	MTTF λ_{S2}	MTTF λ_{S3}	MTTF μ_{S1}	MTTF μ_{S2}
0	5.0000	5.0000	5.0000	4.2241	4.4706
0.1	4.7568	4.8276	4.7945	4.3077	4.5313
0.2	4.5794	4.6907	4.6667	4.3750	4.5794
0.3	4.4444	4.5794	4.5794	4.4304	4.6186
0.4	4.3382	4.4872	4.5161	4.4767	4.6512
0.5	4.2525	4.4094	4.4681	4.5161	4.6786
0.6	4.1818	4.3431	4.4304	4.5500	4.7020
0.7	4.1226	4.2857	4.4000	4.5794	4.7222
0.8	4.0722	4.2357	4.3750	4.6053	4.7399
0.9	4.0288	4.1916	4.3541	4.6281	4.7554

failure of the system tend to increase. Similar observation can be seen with respect to μ_{Si} . It is evident from Table 3-5 that improving the availability and mean time to system of the system is amounting to higher productivity as well as generated profit, it is essential that the hardware and each software type should run failure free for long duration with full capacity and efficiency. This can only be achieved through adequate inspection and preventive maintenance to enable the system remain operative with the maximum efficiency for the maximum duration to ensure their reliable operation.

CONCLUSION

In this study, we studied the single host system with three types of dissimilar software in cold standby. Explicit expression for the mean time to system failure, steady-state availability, busy period probabilities due type 1-3 software, hardware repair and profit are derived. The numerical simulations presented in Fig. 1-4 and Table 3-5 provide a description of the effect of the failure rate and repair rate on steady-state availability, mean time to failure and profit.

SUGGESTIONS

On the basis of the numerical results obtained for particular cases, it is suggested that the system availability, mean time to failure and profit of the system can be improved significantly by:

- Adding more software and host in cold standby
- Increasing the repair rate
- Reducing the failure rate of the system by hot or cold duplication method
- Adequate inspection and preventive maintenance

The system can further be developed into system with multiple hosts with heterogeneous software in solving reliability and availability problems. Reliability engineers and system designers are faced with the challenges of competition and market globalization on maintenance

system to improve efficiency and reduce operational costs, this study will serve as a guide in relation to efficiency, reduction of failure and operational costs, increase in production output and revenue mobilized.

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