

Estimation of Erosion Rate at Meanders of Karoon River Within South of Ahvaz to Darkhovein Using Results of Mathematical Model and Experimental Equations

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Abstract: Karoon river as biggest Iran's river has always subjected to lateral changes due to its specific conditions such as big floodwaters and its soil texture. In this research it is tried to calculate erosion rate of meanders of Karoon river within South of Ahvaz to Darkhovein using results of Mike 11 model and also application of an analytical model. Annual erosion rate of Karoon river at its meanders was calculated by incorporating of results of mathematical model, geotechnics, river geometrical attributes (river's width and curve radius of arcs), Einstein-Brown bed load equation and analytical model. In order to assess accuracy of calculations river's shore displacement has been obtained using existing maps which its comparison with calculated results shows that mathematical model, analytical model, bed load equation and other considered parameters for calculation of erosion rate were reasonable and appropriate and simple analytical model of erosion rate could provide relatively proper values of river's shore erosion rate at studied range.

Key words: Mike 11 model, erosion rate, Karoon, bed load, shear stress

INTRODUCTION

Based on implemented studies and visits at Karoon river, different ranges of this river has different abrasion severity and type. In some ranges, especially upstream ranges, surficial abrasion due to surficial waves and flow's hydrodynamic force is effective factor in abrasion of walls and in some ranges, especially medial ranges, depth and abrasions are main factor in destruction of walls. Importance and criticality rate of an abrasive range means danger scope that it imposes to the region.

Base on made assessments, all parts of Karoon river within Ahvaz to Darkhovein with almost 125 km length are subjected to severe and relatively sever abrasion. About 69 km out of this range are under acute abrasion conditions. Considering very high cost for stabilizing of river shores, sufficient knowledge of situation and abrasion rate at abrasive ranges could considerably help planning for construction and developing activities at river sides including pumping stations, road, rural developing plans, comprehensive city plans, infrastructural facilities, oil, gas and electricity transportation lines. In a way that no danger be imposed on these facilities due to river abrasion. For this reason, in this research it is tried to calculate annual abrasion rate at Karoon river's meanders within South of Ahvaz (from downside of Ahvaz city to Darkhovein) where

abrasion is mostly edge abrasion (fork abrasion and trenches loss) using results of MIKE 11 one dimensional mathematical model and assisting of a simple analytical model for abrasion rate. This range has 29 meanders which all of them has abrasion problem (Mike, 2007).

Literature review: The response of surface processes to climatic forcing is fundamental for understanding the impacts of climate change on landscape evolution (Ma *et al.*, 2016). In previous researches, erosion rate in different locations and with various geology were studied (Scherler *et al.*, 2015; Abrahami *et al.*, 2016; Duan, 2005; Margreth *et al.*, 2016; Khaiat, 1390). Scherler *et al.* (2015) employed a cosmogenic nuclide²¹ Ne analysis to quantify the average erosion rate in the Shapotou area, at the North Eastern margin of the Tibetan Plateau, based on the steady-state erosion model. Habibi and his colleagues in 1381 developed a mathematical model which was able to simulate abrasion and sediment formation in multi branch water passages (Khaiat, 1390). This model is designed as a semi two dimensional model and is able to determine stress distribution at vertical direction on flow and also abrasion and sediment rate distribution at direction of river cross. In prepared model for simulation of flow at unsteady mode, Cent and Nant equations was solved using numerical scheme of finite differences and Prizeman pattern and equations system by bilateral swiping

method. Duan (2005) assessed abrasion speed of sidewalls at surficial destruction mechanism with a cohesive soil. To calculate this speed used surficial abrasion concept of sidewalls. Meaning that surficial abrasion of sidewalls will happen when transportation of materials from sidewalls surface is higher than particles deposition. He provided equation of sidewall surficial abrasion rate with assuming of homogeneity and evenness of sidewalls particles and considering of three imposed forces on sidewall surface-adhesion force, particle floating and lifting forces.

In (von Blanckenburg *et al.*, 2012) it was shown that the Earth surface is constantly receiving cosmogenic meteoric beryllium-10 from the atmosphere. At the same time, the stable isotope beryllium, 9 is released from rocks by weathering. The resulting $^{10}\text{Be}/^9\text{Be}$ isotope ratio depends on denudation and weathering rate. A steady-state weathering zone mass balance quantifies this system in sediment and river water (Karamesouti *et al.*, 2016) aimed at evaluating the use of the Revised Universal Soil Loss Equation (RUSLE) and the Pan-European Soil Erosion Risk Assessment (PESERA) models in predicting the changes in spatial variability of soil erosion following a wildfire event. They used a site in Greece on which a wildfire occurred in the Summer of 2007 as a case study. Their results provided a better understanding of the effect of fire on soil erosion in the Mediterranean and else where as well as the ability of those widely used models as efficient tools to be used for this purpose.

Introduction of software and governing equations: MIKE 11 Software is prepared by Denmark Hydraulic Institute (DHI) and is able to one dimensional simulation of flow, sediment displacement and water quality in a unsteady way in rivers and irrigation networks. This software uses finite difference method to one dimensional solve of governing equations of flow, sediment displacement and water quality.

MIKE 11 hydrodynamic module implicitly solves continuous equations of movement in unsteady mode using Abbott-Ionescu numerical method (finite differences method) and do calculations of unsteady flow in lakes and vessels. River flow governing one dimensional equations includes continuity equations and momentum (Cent and Nant equations) which are used in hydrodynamic model calculations (Mike, 2007).

Continuity equations:

$$\frac{\partial Q}{\partial X} + \frac{\partial A}{\partial t} = q \tag{1}$$

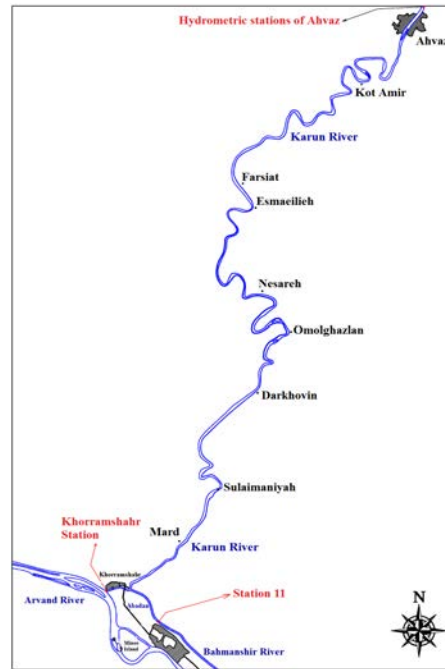


Fig. 1: Used range in mathematical and research model

Momentum equation:

$$\frac{\partial Q}{\partial t} + \frac{\partial(\beta \frac{Q^2}{A})}{\partial x} + qA \frac{\partial h}{\partial x} + \frac{gQ|Q|}{C^2 AR^*} = 0 \tag{2}$$

Introduction of plan's range and used statistics and information in research: Studied scope in this research was a range of Karoon river in downside of Ahvaz City which is started from Krishan do village and continues up to Darkhovein city and average longitudinal slope of river bed in this range is around 0.000052.

Length of studied range is around 120 km but knowing that flow in plan's range is below critical level and noting that control section of below critical flow is in downside so to consider this issue, mathematical model of Karoon river was prepared from Ahvaz hydrometrical station (upside of 7th bridge) to Arvand river mouth (Khorramshahr level) and for Bahmanshir river up to 11 station level at upside with estimated length of 213 km (205 km for Karoon and 8.35 km for Bahmanshir) Fig. 1. Also 186 transversal section were used for modeling of studied range.

This range has 29 meanders which all of them are under almost severe abrasion because of flow conditions at river's arches and formation of secondary flow and also existence of sand lenses inside soil texture and soil weakness. These meanders are commonly locations where

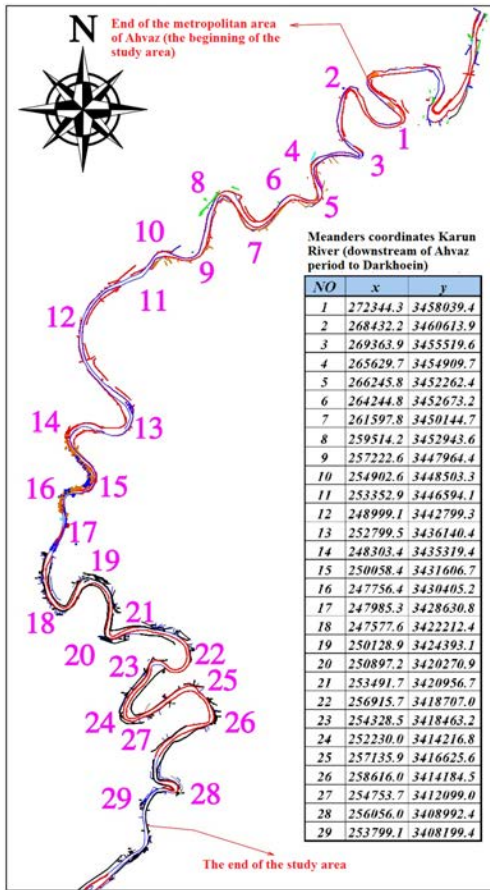


Fig. 2: Plan's range and arches location and their coordinates

villages and some facilities such as pumping stations, towns and agricultural lands are located beside of them. This issue leads that most of the years in addition to problems of flooding by river, an issue called walls abrasion and destruction of lands and facilities cause problem for people and authorities. Plan scope and also layout and coordinates of UTM of arches of research plan range are shown in Fig. 2. Also images of two river's arches are shown in Fig. 3. As it is seen in figures, abrasion rates were severe and mostly caused destruction of facilities and agricultural lands.

Mathematical model of Karoon river in under research scope is taken considering general characteristics of sections and is calibrated based on mentioned methods and manning coefficients of each section are determined. So, 0.025 is used as calibrated coefficient in recent studies. In these studies, level-flow rate curve of Farsiat station is used for calibration.

With made studies and noting this issue that average floodwater in studied range has been between 2500-3100 m³/s in all years except recent years (due to



Fig. 3: Karoon river erosion in area of Malihan and Suichi villages (8 and 23 meanders)

drought) and also noting that most of the abrasions in this range has occurred during floodwater, so for study and mathematical model inputs, floodwater hydrograph with 2 years return period was selected as boundary condition of upside of studied range (Fig. 4) and modeling was implemented for calculation of maximum water level values considering introduced flow rate and its results are analyzed.

Moreover, considering that flow conditions at downside of plan range is tidal and river also is divided into two branch so water level changes at measured time period in two stations of Khorramshahr level measuring for Karoon river and level measuring station of 11 station for Bahmanshir river were applied into mathematical model as downside boundary condition (Fig. 5 and 6).

After indicated issues for mathematical model and its required information, it is needed to indicate some points regarding analytical model which is used in this research for calculation of abrasion rate as following.

Simple model of estimation of abrasion rate of outer river shores which is used in this research for the first time is used to estimate long term displacement rate of meander's ranges at a restoration planning (Richardson, 2002). Mentioned model is prepared in a way that directly uses results of one dimensional hydraulic models such as Hec-Ras and Mike 11.

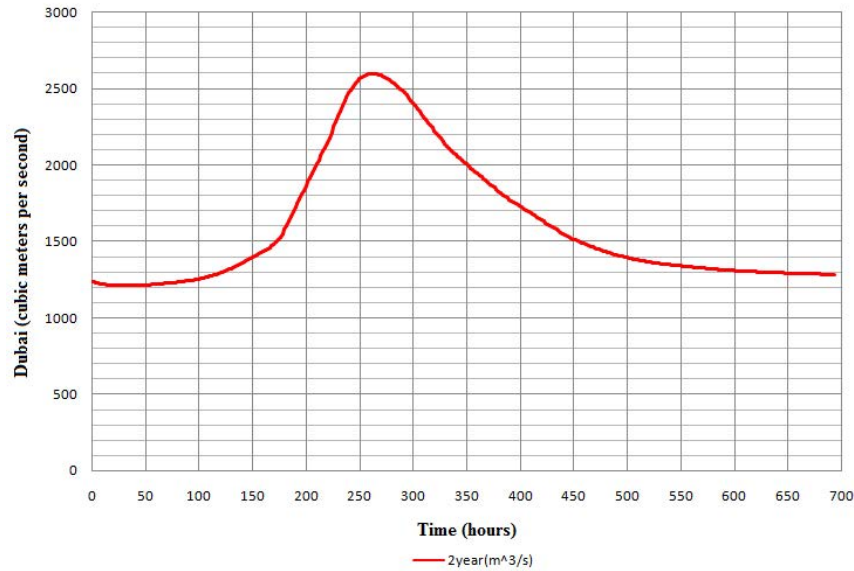


Fig. 4: Karoon river’s floodwater hydrograph in Ahvaz station with 2 years return period

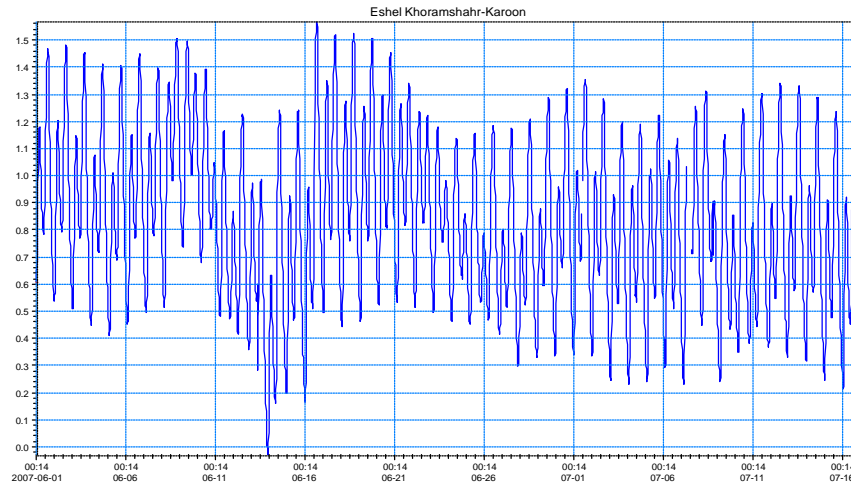


Fig. 5: Fluctuations of taken water level values per time in Khorramshahr station (Karoon river)

It is required to correct output results of above one dimensional models for expression of hydraulic conditions at river arch in order to calculate abrasion rate of river sidewalls. For this intention, a conversion factor (K) is provided for conversion of shear stress at sidewalls to shear stress of whole section and it is assumed that ratio of speed at outer sidewall to average speed of section is equal to ratio of length of arch’s central line to length of outer arch of bending. Hershel at 1897 stated that ratio of shear stress of outer sidewall to average shear stress is directly related to speeds ratio powered by two:

$$K = \frac{T_{toe}}{T_{avg}} = \left(\frac{V_{toe}}{V_{avg}} \right) = \left(\frac{B}{2R} + 1 \right)^2 \quad (3)$$

Where:

- K = Shear stress conversion coefficient
- T_{toe} = Outer wall shear stress
- T_{avg} = Section one dimensional shear stress
- V_{toe} = Flow speed at outer wall side
- V_{avg} = Section one diemsional average speed
- R = Arch’s curve radius
- B = channel width in meter

Calculated shear stress in above correlation could be converted to volume of eroded materials of the bed using a proper model for sediment displacement for local conditions. Also analytical model assumes that difference of volume of displaced sediment by average flow and outer sidewall flow is appeared as outer arch’s

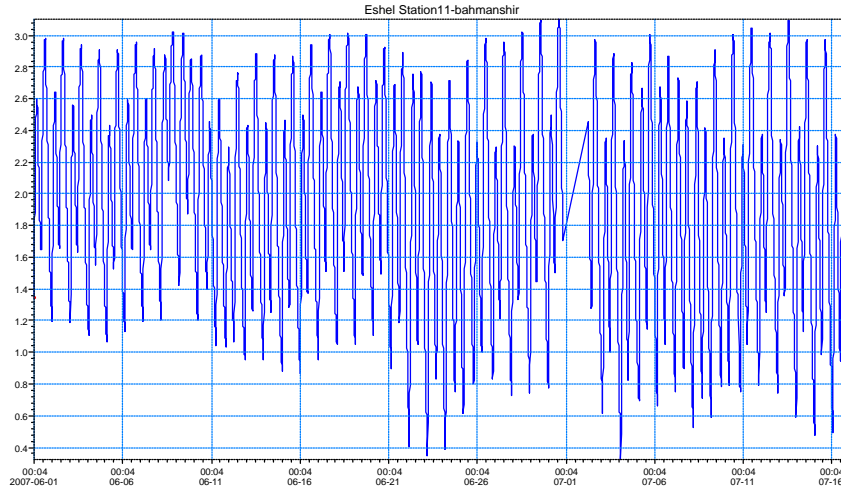


Fig. 6: Fluctuations of taken water level values per time in 11 station

erosion. Considering Fig. 7, volumetric rate of sidewall erosion can be obtained by dividing of erosion volume by shore height:

$$E_r = \frac{E_v}{h} \quad (4)$$

Where:

E_r = Edge erosion rate (m^3/sec)

E_v = Erosion volume with unit of m^3/sec per channel length unit

h = River side height (above of the thalweg of outer wall) in meter

In this research, considering river conditions and made assesments, Eingshtein-Brown bed load displacement (Eq. 5-8) has been used for hydrolic attributes of daily flows for the sake of dtermining of volume of daily eroded materials:

$$\varphi = \frac{q_{bv}}{\gamma_s k [g (\frac{\gamma_s}{\gamma} - 1) d_{50}^3]^{0.5}} \quad (5)$$

Where:

q_{bv} = Sediment rate per channel width unit

γ_s = Soil density (kg/m^3) γ kg/m^3

d_{50} = Particles average size (m)

If $d_{50} > 1$ mm then $k = 0.79$ and $d_{50} < 1$ mm then:

$$k = \left[\frac{2}{3} + \frac{36v^2}{gd_{50}^3 (\frac{\gamma_s}{\gamma} - 1)} \right]^{0.5} - \left[\frac{36v^2}{gd_{50}^3 (\frac{\gamma_s}{\gamma} - 1)} \right]^{0.5} \quad (6)$$

Where:

v = Kinematic viscosity (m^2/sec)

g = Gravity acceleration

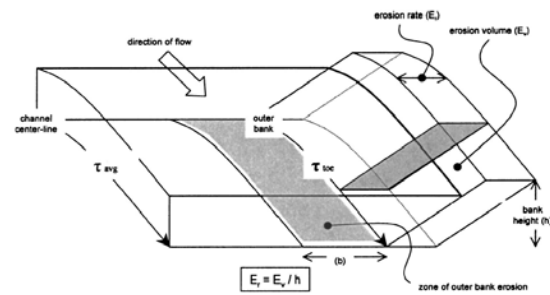


Fig. 7: Relation between eroded materials volume in outer wall and its sidewall erosion rate

$$\frac{1}{\Psi} = \frac{\tau}{(\gamma_s - \gamma)d} \quad (7)$$

Where:

$1/\Psi$ = Shields parameter

τ = Shear stress (kg/m^2)

d = particles average size (m)

$$\text{If } \frac{1}{\Psi} > 0.09 \text{ then } \varphi = \left(\frac{1}{\Psi}\right)^3 \quad (8)$$

After calculations using above correlations, erosion volume is calculated by correlation:

$$E_v = (q_{bv:toe} - q_{bv:avg}) \cdot b \quad (9)$$

In above correlation, b is eroded width of outer edge which is assumed equal with the edge height.

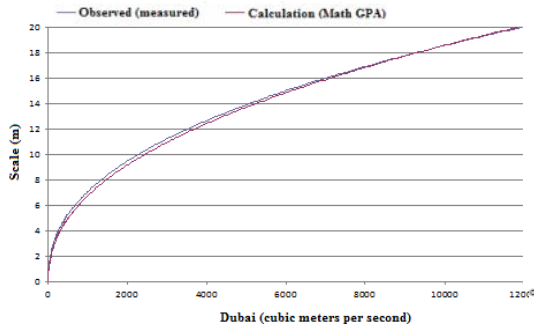


Fig. 8: Curve of calculated and measured level flow rate by model in farsiat station for verification

Table 1: Calculation of values of statistical correlations (error and compatibility) for mathematical model verification

Statistical correlation type	Error (%)	a	R ²
Value	6.5	1.11	0.967

MATERIALS AND METHODS

Verification of mathematical model: For comparison of results and assessment of compatibility between model results and measured values, statistical correlations of compatibility factor (R^2), difference relative to 45° line (a) and absolute Average of Error (ERROR) are used which their values are shown in Table 1 for verification of mathematical model. Considering made statistical calculations, model simulated water level height values with relatively high precision. Also curve related to results of measured and calculated (mathematical model results) level flow rate curve of Farsiat station (as medium station for model verification) during verification is shown in Fig. 8.

RESULTS AND DISCUSSION

After implementation of primary actions and calculations including preparation of river layout, transversal sections, floodwater hydrograph, roughness factor, level and. And introduction and application of them, mathematical model was conducted which mathematical model results for river meanders for studied range (29 meanders) including flow rate, speed, water level values and shear stress (required parameters in analytical model) are as columns of 3-6 of Table 2. In addition to hydraulic parameters, it is required to derive some primary information of studied range meanders such as river width and river curve radius for implementation of some calculations such as determining of fork shear stress using correlation 3 from existing maps. This was done for studied range using 1:2500 topography maps of Karoon river which its results is indicated in columns of 7 and 8 of Table 2.

To conduct calculations, it is required to have some soil geotechnical parameters related to river trenches and also some water specifications. Required parameters in this regard (based on taken samples) and also water parameters are indicated in Table 3.

After deriving and clarification of required parameters from available information and studies, then after, calculations of erosion rate for arches of studied range will be done using correlations of 5-8. Calculations' trend is in a way that initially shields parameter is calculated with consideration of information, then Φ is determined and then q_{bv} is calculated for two conditions of $q_{bv(ave)}$ (average sediment discharge at river section) and $q_{bv(toe)}$ (sediment discharge at river trenches fork).

For calculation of $q_{bv(toe)}$, τ_{toe} (fork shear stress) should be determined initially which this issue is determined via application of parameters of width and river curve radius for arches using correlation 3. With inserting τ_{toe} in shields parameter ($1/\Psi$), this parameter is determined then Φ and then after $q_{bv(toe)}$ are determined. For $q_{bv(ave)}$ also this process is repeated which of course τ_{ave} is used.

After calculation of primary required values, values of E_v and E_r are calculated using correlations of 4, 9. It is notable that unit of calculated E_r is m/s which its value should be converted to m/year (annual erosion value). Calculated values in this respect are indicated in Table 4 where last column is annual erosion value at location of arches of Ahvaz South to Darkhovein (m).

As indicated in Table 4, annual erosion value for Karoon river shore at Ahvaz South is sensible which is necessary to take required measures for organizing and stabilizing of river shore in erosive ranges especially meanders and also keeping of appropriate distance for construction of facilities. Because in marginal area of Karoon river in Ahvaz South, there are lots of rich agricultural lands in addition of existence of facilities (harbor, pumping station, irrigation and draining networks, electricity transfer lines,...) and villages which definitely their destruction will lead to endanger production of agricultural products in location in addition to damage people assets.

Assessment of evidences shows reasonability of made calculations for erosion value of arches. However, to ensure about research outcomes, it was decided to assess river erosion value in studied range by two periods of river layout map. For this intention, river layout map in intended range was prepared at two scales of 1:25000 and 1:2500 which are related to 1372 and 1382, respectively 10 years period duration (Fig. 9) and value of difference of river trenches in arches location (river lateral displacement) in two layouts during this 10 years was

Table 2: Calculated results of karoon river's mathematical model and values of width and river's curve radius in studied range's meanders

Kilometer (m)	WS elevation (m)	Q total (m ³ /sec)	Velocity channelle (m/sec)	Shear stress (N/m ²)	(B) River width (m)	Curve Radius (R) of river (m)
29235	11.921	2583.32	1.21	2.16	240	650
34473	11.617	2580.59	0.99	2.11	240	640
40413	11.314	2577.41	0.84	2.18	270	785
44051	11.170	2575.37	0.83	1.90	280	903
46930	10.909	2573.56	1.39	2.29	210	876
48745	10.819	2572.49	1.19	2.02	295	860
51610	10.681	2570.66	1.11	2.16	340	1115
55854	10.488	2567.56	1.14	2.19	240	664
62136	10.156	2563.86	1.21	2.49	225	1127
63903	10.089	2562.80	1.04	2.29	350	1470
66042	9.968	2561.46	1.02	2.31	310	2440
73235	9.477	2557.19	1.35	3.68	185	4157
81792	8.949	2552.94	1.44	2.39	193	1259
86161	8.689	2550.85	1.49	2.26	190	1074
90105	8.508	2548.47	1.21	2.62	130	1090
92291	8.419	2547.09	1.07	2.52	148	800
94338	8.298	2545.87	1.20	2.56	175	2967
102945	7.804	2540.08	1.23	2.14	190	579
105833	7.674	253800.00	1.12	2.82	240	1232
110481	7.501	2535.30	0.59	2.18	190	518
113932	7.251	2533.86	1.37	3.46	250	2747
117983	6.968	2532.29	1.30	2.99	195	1314
120345	6.846	2531.51	1.02	1.89	283	577
125761	6.558	2529.41	0.97	2.18	203	495
131587	6.190	2526.63	1.15	2.35	280	990
134114	6.053	2525.37	1.36	2.13	145	646
138093	5.895	2519.33	0.80	0.92	220	3699
142375	5.742	2507.15	1.20	1.99	160	575
145369	5.684	2505.45	1.07	0.82	203	2332

Table 3: Required information related to soil texture and water specification

Parameter name	Values
D ₅₀ (mm)	0.0895
G _s	2.676
γ (kg/m ³)	2671.18
γ (kg/m ³)	998.2
ν (m ² /sec)	1.003×10 ⁻⁶

Table 4: Erosion volume values and calculated erosion rate in arches

Kilometer (m)	Bank				Kilometer (m)	Bank height			Er m/year
	Ev (m ³ /sec, m)	height (h/m)	Er (m/s)	Er (m/years)		Er (m ³ /s,m)	(h/m)	Er (m/s)	
29235	1.14E-06	7.13E-05	2.25	16	92291	7.22E-07	17	4.25E-08	1.34
34473	1.09E-06	4.95E-08	1.56	17	94338	2.06E-07	11	1.87E-08	059.00
40413	1.06E-06	5.88E-08	1.85	18	102945	9.37E-07	15	6.25E-08	1.97
44051	6.07E-07	6.75E-08	2.13	19	105833	1.08E-06	14	7.71E-08	2.43
46930	7.47E-07	4.67E-08	1.47	20	110481	1.17E-06	16	7.29E-08	2.30
48745	8.42E-07	7.02E-08	2.21	21	113932	8.17E-07	12	6.81E-08	2.15
51610	8.69E-07	7.90E-08	2.49	22	117983	9.24E-07	11	8.40E-08	2.65
55854	1.15E-06	7.20E-08	2.27	23	120345	1.19E-06	16	7.42E-08	2.34
62136	7.67E-07	4.26E-08	1.34	24	125761	1.37E-06	21	6.53E-08	2.06
63903	7.46E-07	6.21E-08	1.96	25	311587	1.02E-06	13	7.83E-08	2.47
66042	3.55E-07	3.55E-08	1.12	26	134114	5.55E-07	15	3.70E-08	1.17
73235	4.53E-07	3.49E-08	1.10	27	138093	9.64E-09	10	9.64E-10	0.03
81792	4.90E-07	4.09E-08	1.29	28	142375	6.02E-07	18	3.35E-08	1.06
86161	4.90E-07	3.50E-08	1.10	29	145369	1.04E-08	13	7.96E-10	0.03
90105	4.82E-07	4.01E.08	1.27						

obtained as average and by dividing of obtained average figure in each arch by 10, annual river lateral displacement (erosion) was obtained. Calculate and observed erosion rate and their difference is provided in Table 5.

Also to compare calculated and observed data, using indicated correlations in model verification topic,

comparison error value of calculated and observed results was determined that are indicated in Table 6. With assessment of Table 6 and obtained error value, it can be concluded that calculated values of annual erosion rate for arches of Ahvaz South to Darkhovein are reasonable and appropriate and it is possible to use them with a little

Table 5: Comparison of calculated and observed erosion rate in arches location

Kilometer (m)	Erosion rate calculation (m/year)	Average rate of erosion (m/year)	Erosion rate differential (m/year)	Kilometer (m)	Erosion rate calculation (m/year)	Average rate of erosion (m/year)	Erosion rate calculation (m/year)
29235	2.249	2.000	0.249	92291	1.339	1.917	-0.577
34473	1.562	2.556	-0.994	94338	0.590	0.542	0.049
40413	1.853	4.958	-3.105	102945	1.970	1.108	0.861
44051	2.128	1.528	0.600	105833	2.430	1.729	0.701
46930	1.473	1.833	-0.360	110481	2.298	0.844	1.454
48745	2.213	1.028	1.185	113932	2.146	2.438	-0.291
51610	2.491	2.375	0.116	117983	2.648	3.250	-0.602
55854	2.270	2.313	-0.042	120345	2.339	2.500	-0.161
62136	1.343	1.528	-0.185	125761	2.059	1.639	0.420
63903	1.959	1.167	0.793	131587	2.468	2.389	0.079
66042	1.120	1.417	-0.297	134114	1.166	0.917	0.249
73235	1.099	1.458	-0.359	138093	0.030	1.483	-1.453
81792	1.289	1.292	-0.003	142375	1.055	0.833	0.222
86161	1.104	1.383	-0.279	145369	0.025	0.333	-0.308
90105	1.265	0.847	0.418				

Table 6: Calculation of statistical correlations values for comparing observed and calculated results

Error value calculation method	A	RMSE	Error (%)
Error value of comparison of observed and calculated erosion rate	1.07	0.84	33.09

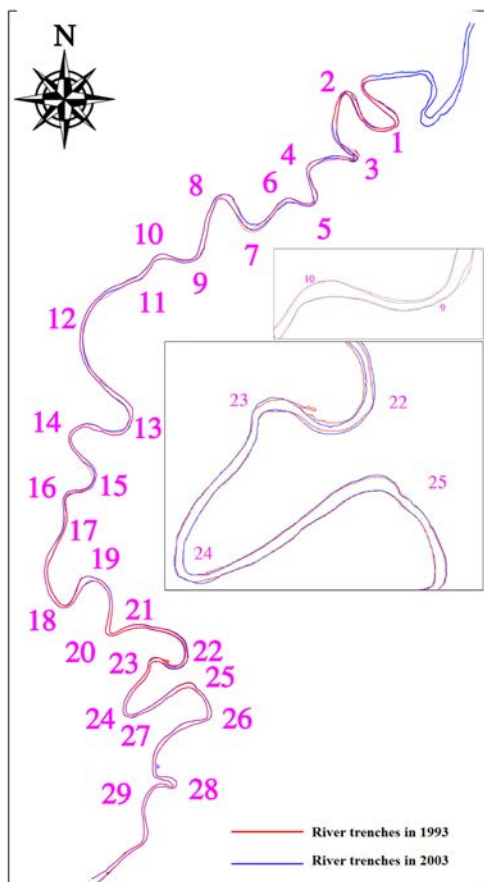


Fig. 9: Changes (displacement) of trenches of Karoon river in plan's range (comparison of 1:25000 and 1:2500 maps with 10 years' time difference)

bit overestimating in river engineering plans (organizing and stabilizing of shore), infrastructure facilities (road, electricity, water, oil and gas transfer lines,...) and other plans.

Also this issue shows that prepared mathematical model for studied range and its input values including floodwater hydrograph, statistics of leveling, roughness coefficients, sea tidal effect and rest of the parameters were completely right and proper and mathematical model is well calibrated and also Einstein-Brown bed load transfer equation (sediment discharge) is selected properly.

CONCLUSION

By made assessments and noting that average flood in studied range have been between 2500-3100 m³/sec in all years except recent years (due to drought) and moreover most of the erosions in this range has been occurred during flood times, so flood hydrograph with 2 years return period was selected as boundary condition of upside of studied range to conduct study of mathematical model inputs.

Made statistical calculations (values of a, R² and error) for verification of mathematical model shows relatively high accuracy of model in simulation of water level height values.

Annual erosion value of Karoon river shores in Ahvaz South is 1.65 m as average that is sensible and is necessary to take required measures for organizing and stabilizing of river shore in erosive rages especially meanders.

Obtained annual erosion rate from observed results is in average 1.71 m in plan's range that by its comparing with calculated erosion rate and also assessment of statistical correlations.

It can be concluded that calculated values of annual erosion rate for arches of Ahvaz South to Darkhovein are reasonable and appropriate and it is possible to use them with a little bit overestimating in river engineering plans (organizing and stabilizing of shore), infrastructure facilities (road, electricity, water, oil and gas transfer lines,...) and other plans.

Reasonability of calculated results shows that prepared mathematical model for studied range and its input values including floodwater hydrograph, statistics of leveling, roughness coefficients, sea tidal effect and rest of the parameters were completely right and proper and mathematical model is well calibrated.

Reasonability of calculated results also shows almost appropriate selection of bed load transfer equation (sediment discharge) which is Einstein-Brown equation.

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