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Chaotic Properties of the Modified Henon Map

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Abstract: In this study, a dynamical system of modified Henon map on two dimension with the form $MH_{a,b}\binom{x}{y} = \binom{1+ax+\cos 2\pi y}{by}$ is studied. We find some general properties and we show some chaotic properties of it. The proposed study prove that the modified Henon map has positive Lypaunov exponent and sensitivity dependence to initial condition. Fpr applying the suggested scheme, Mat lab programs are usedto draw the sensitivity of modified Henon map and compute the lyapunov exponent.

Key words: Modified the henon map, fixed point, attracting-expanding area, Lyapunov exponent, sensitive dependence on initial conditions

INTRODUCTION

There are several definitionsfor chaos were proposed. When the system is sensitive to initial condition on its domain or has positive Lyapanov exponent at each point in its domain then this system will be chaotic (Denny, 1992). Chaotic behavior of lows dimensional map and flows has been generally considered and described (Sprott and Chaos, 2003). Previously, the French space expert-mathematician Michel Henon was scanned for simple two-dimensional squeezing extraordinary properties of more complication system the result was family of the form:

$$H_{a,b}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1-ax^2+y \\ bx \end{pmatrix}$$

where, a,b are parameter and real number (Denny, 1992). This is a nonlinear two dimensional map which can also be written as a two-step recurrence relation:

$$X_{n+1} = 1 - ax^2_n + by_{n-1}$$

The parameter b is a measure of the rate of area contractionand the Henon map is the most general two-dimensional quadratic map with the property that the contraction is Independent of x and y. For b = 0 , the Henon map reduces to the quadratic map which. Follows period doubling route to chaos. Bounded solutions exist for the Henon map over a range of a and b values. Henon map had two fixed points. Which can be either attracting, saddle or repelling points depending on the choice of parameters (a, b).

Henon map had two fixed points. Which can be either attracting, saddle or repelling points depending on the choice of parameters (a, b) (Shameri, 2012). In this research, we introduce a new map in two dimension, we will call it the modified Henon map as:

$$MH_{a,b}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} 1+ax+\cos 2\pi y \\ by \end{bmatrix}$$

Preliminaries: Let I: $R^n \to R^n$ be a map, we say I is C^∞ if its P-th partial derivatives exist and continuous for all P∈Z and it is called diffeomorphism if it is one-to-one onto C^∞ and its inverse is C^∞ let W be subset of R^2 and μ be any element in R^2 concider G:W→R^2 be a map. Furthermore assume that the first partials on R^2 by DG :

$$(u_0) = \begin{pmatrix} \frac{\partial f_1}{\partial x}(u_0) & \frac{\partial g_1}{\partial y}(u_0) \\ \frac{\partial f_2}{\partial x}(u_0) & \frac{\partial g_2}{\partial y}(u_0) \end{pmatrix}$$

For all u_0= ∈R^2 the determinate of DG (u_0)is called Jacobian of G at u_0 and denoted by JG (u_0) = detD G (u_0). So Gis said to be area expanding at u_0 if $|\det D\,G(u_0)|>1$ is said to be area contracting at u_0 if $|\det D\,G(u_0)|<1$. Let B be n×n matrix the real numberλ is called Eigen value of B . The point $(^p/_q)$ is called fixed point ifit G $(^p/_q)$= $(^p/_q)$ is repelling fixed point if $\lambda_1$and λ_2 >1 in absolutevalue and it is an attracting fixed pointif λ_1 and λ_2<1in absolute value B ∈ GL (2,Z) with det (B) = ±1 is called hyperbolic matrixif $|\lambda_1 \neq 1$ wher λ_1 are the eigenvalue (Denny, 1992).

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MATERIALS AND METHODS

General properties of modified Henon map: In this study, we find the fixed point and study the general properties ofmodified Henon map (one to one, onto, C∞ and invertible) which make it diffeomorphism and find the value of a, b which $MH_{a,b}$ has area contracting or expanding.

Proposition (3.1): If a≠1 and b≠1 then modified Henon $MH_{a,b}$ maphas unique fixed point.

Proof: By the definition of fixed point, we get:

$$MH_{a,b}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} 1+ax+\cos 2\pi y \\ by \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

Then by = y since b ≠ 1 then y = 0 since 1+ax+cos2π(0) = x then bx(a-1) = -2⇒x = -2/(a-1). Such that α≠1 then:

$$\begin{bmatrix} 2/1\text{-}a \\ 0 \end{bmatrix}$$

is the fixed point. Let $[{}^{x}/_{y}] = [{}^{n}/_{s}]$ Also by the definition of fixed point

$$MH_{a,b}\begin{pmatrix} n \\ s \end{pmatrix} = \begin{bmatrix} 1+an+\cos 2\pi s \\ b_s \end{bmatrix} = \begin{bmatrix} n \\ s \end{bmatrix}$$

Since bs = s and b ≠ 1. Then, s = 0, Also since 1+an+cos2π(0) = x and a ≠0 then n= -2/(a-1). But this contradiction So:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} n \\ s \end{bmatrix}$$

Such that if a ≠ 0 then:

$$\begin{bmatrix} 2/1\text{-}a \\ 0 \end{bmatrix}$$

is the unique fixed point.

Proposition (3.2): If a≠1, b = 1 then $MH_{a,b}$ has infinite fixed point.

Proof: By definition of fixed point:

$$\begin{bmatrix} 1+x+\cos 2\pi y \\ by \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

Since, b = 1 then y =y, 1+ax-cos2πy = x ⇒x = -1-cos 2πy/a-1.Then, $Mh_{a,b}$ has infinite fixed point:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \dfrac{-1-\cos 2\pi y}{a-1} \\ y \end{bmatrix}$$

Proposition (3.3): If a = 1, b≠ 1then $MH_{a,b}$ has no fixed point.

Proof: By definition of fixed point:

$$\begin{bmatrix} 1+x+\cos 2\pi y \\ by \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

Since b≠ 1 then by = y ⇒ y= 0, x-x = 1+cos2πby then $H_{a,b}$ has no fixed point.

Proposition (3.4): The Jacobian of the modified Henon map $MH_{a,b}$ is ab

Proof:

$$DH_{a,b}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \dfrac{\partial f_1}{\partial x} & \dfrac{\partial f_1}{\partial y} \\ \dfrac{\partial f_2}{\partial x} & \dfrac{\partial f_2}{\partial y} \end{pmatrix} = \begin{bmatrix} a & -2\pi \sin 2\pi y \\ 0 & b \end{bmatrix}$$

Then; j = det $DH_{a,b}(V_0)$ = ab

Proposition (3.5): Let $MH_{a,b}$ be modified Henon map

- If |a|<1 and |b|<1 then $MH_{a,b}$ is area contracting map and|j|<1
- If |a|>1 and |b|>1 then is area expanding map |j|>1

Proof: if |a|<1 and |b|then|J| = |ba| = |b||a|<1. Therefore, by definition area contracting the Jacobian of modified Henon map<1.

Similarity proof (2): By definition area expanding the Jacobian of modified Henon map >1.

Proposition (3.6): The modified Henon map $MH_{a,b}$ is area contracting if

- |b|>1, b≠ 0 and |a|<1/|b|
- |a|>1, a ≠ 0 and |b| <1/|a|

Proof: If |b|>1, b ≠ 0 and |a|<1/|b|then |j| = |b||a|⇒|J|<|b|.1/|b|

$|b|<1$ So the Jacobian of modified Henon map<1 so from definition area contracting.

Similarity proof (2): By definition area contracting the Jacobian of modified Henon map<1.

Proposition (3.7): The modified Henon mapis $MH_{a,b}$ area expanding if

- $|a|>1$, $a\neq 0$ and $|b|>1/|a|$
- $|b|>1$, $b\neq 0$ and $|a|>1/|b|$

Similarity proof (proposition (3.6)).

Proposition (3.8): The eigenvalue of modified Henon map $MH_{a,b}\begin{pmatrix}x\\y\end{pmatrix}$ are a, b.

Proof: The eigenvalue of:

$$DHM_{a,b}\begin{pmatrix}x\\y\end{pmatrix} = Det^{(DH_{a,b}(v)-\lambda I)}$$

$$\det\begin{bmatrix}a-\lambda & -2\pi\sin 2\pi y\\ 0 & b-\lambda\end{bmatrix}=0\Rightarrow(a-\lambda)(b-\lambda)=0$$

Then; $\lambda_1 = a$, $\lambda_2 = b$.

Proposition (3.9): -Let be modified Henon map and $a\neq 0$, $b\neq 0$ then;

- If $|a|<1$ and $|b|<1$ then the fixed point of $MH_{a,b}$ is attracting fixed point
- If $|a|>1$ and $|b|>1$then the fixed point of $MH_{a,b}$ is repelling fixed point
- If $|a|>1$ and $|b|<1$then the fixed point of $MH_{a,b}$ is saddle fixed map
- If $|a|<1$ and $|b|>1$then the fixed point of $MH_{a,b}$ is saddle fixed map

Proof: By proposition (3.5-3.7) and definition it's satisfying (1- 4).

Proposition (3.10): If $b\neq 0$, $a\neq 0$ then modified Henon map $MH_{a,b}$ is diffeomorpism.

Proof: $MH_{a,b}$ Is one-to-one map.
Let:

$$\begin{pmatrix}x_1\\y_1\end{pmatrix},\begin{pmatrix}x_2\\y_2\end{pmatrix}\in R^2$$

such that:

$$MH_{a,b}\begin{pmatrix}x_1\\y_1\end{pmatrix}=MH_{a,b}\begin{pmatrix}x_2\\y_2\end{pmatrix}$$

Then;

$$\begin{pmatrix}1+ax_1+\cos 2\pi y_1\\ by_1\end{pmatrix}=\begin{pmatrix}1+ax_2+\cos 2\pi y_2\\ by_2\end{pmatrix}$$

So:

$$by_1 = by_2 \Rightarrow y_1 = y_2$$

$$1+ax_1+\cos 2\pi y_1 = 1+ax_2+\cos 2\pi y_2$$

$$ax_1 = ax_2 \Rightarrow x_1 = x_2$$

$MH_{a,b}$ is onto: Let $\begin{pmatrix}u\\v\end{pmatrix}$ any element inR^2 such that $y = v/b$ and:

$$x=\frac{u-1-\cos 2\pi v/b}{a}$$

Then:

$$MH_{a,b}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}1+ax+\cos 2\pi y\\ by\end{pmatrix}$$

Let;

$$\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}u\\v\end{pmatrix}\in R^2$$

$$u=1+ax+\cos 2\pi y\Rightarrow ax=u-1-\cos 2\pi y$$

$$x=\frac{v-1-\cos 2\pi y}{a} \quad (1)$$

$$v=by\Rightarrow y=\frac{v}{b}$$

Replacing Eq. 2 in 1 we get on:

$$x=\frac{v-1-\cos 2\pi v/b}{a} \quad (2)$$

Then there exist:

$$\begin{pmatrix}\dfrac{v-1-\cos 2\pi w/b}{a}\\ \dfrac{w}{b}\end{pmatrix}\in R^2$$

Such that:

$$=\begin{pmatrix} 1+a\left(\dfrac{u-1-\cos 2\pi v/b}{a}+\cos 2\pi v/b\right) \\ b\dfrac{v}{b} \end{pmatrix}=\begin{pmatrix} u \\ v \end{pmatrix}$$

$$MH_{a,b}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} u \\ v \end{pmatrix}$$

Then $MH_{a,b}$ is onto.
$MH_{a,b}$ is C^{∞} since:

$$MH_{a,b}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{bmatrix} 1+ax+\cos 2\pi y \\ by \end{bmatrix}$$

Then All partial derivatives continuous and exist such

$$\frac{\partial f_1}{\partial x_1}=a,\frac{\partial^2 f_1}{\partial x^2}=0,\ldots\ldots\ldots\ldots\frac{\partial^n f_1}{\partial x^n}=0 \quad n\in N$$

$$\frac{\partial f_1}{\partial y_1}=-2\pi\sin 2\pi y,\ \frac{\partial^2 f_1}{\partial y^2}=4\pi^2\cos 2\pi y \quad \forall n\in N$$

$$\frac{\partial f_2}{\partial x}=0\ldots\ldots\ldots\ldots\frac{\partial^n f_2}{\partial x^n}=0 \qquad n\in N$$

$$\frac{\partial f_2}{\partial y}=b,\frac{\partial^2 f_2}{\partial y^2}=0\ldots\ldots\ldots\frac{\partial^n f_2}{\partial y^n}=0 \qquad \forall n\in N, n\geq 2$$

$MH_{a,b}$ has an inverse:

$$MH^{-1}_{a,b}\begin{pmatrix} u \\ v \end{pmatrix}=\begin{bmatrix} \dfrac{u-1-\cos(2\pi v/b)}{a} \\ \dfrac{v}{b} \end{bmatrix}$$

Such that let:

$$MH^{-1}_{a,b}\circ MH_{a,b}\begin{pmatrix} u \\ v \end{pmatrix}=H_{a,b}\circ H^{-1}_{a,b}=\begin{pmatrix} u \\ v \end{pmatrix}=\begin{pmatrix} u \\ v \end{pmatrix}$$

Then:

$$MH^{-1}_{a,b}\circ MH_{a,b}=MH^{-1}_{a,b}\begin{pmatrix} 1+au+\cos(2\pi u) \\ bv \end{pmatrix}$$

$$=\begin{pmatrix} 1+au+\cos(2\pi bv/b)- \\ \dfrac{1-\cos(2\pi v/b)}{a} \\ \dfrac{bv}{b} \end{pmatrix}=\begin{pmatrix} u \\ v \end{pmatrix}$$

And:

$$MH_{a,b}\circ MH^{-1}_{a,b}\begin{pmatrix} u \\ v \end{pmatrix}=MH_{a,b}\begin{pmatrix} \dfrac{u-1-\cos(2\pi v/b)}{a} \\ \dfrac{v}{b} \end{pmatrix}$$

Then $MH_{a,b}$ has an inverse and it is invertible.

Remark:

- If a = 0, b = 0 then is not onto
- If a≠ 0, b = 0then is not onto
- If a = 0, b≠ 0 and then is not onto

Remark: If a = 0 then $MH_{a,b}$ not one to one, so it is not diffeomorphism.

Proposition (3.11): $DMH_{a,b}\begin{bmatrix} x \\ y \end{bmatrix}$ is a Hyperbolic matrix If $|a|\neq 1$, $|b|\neq 1$ iff $|ab|=1$.

Proof: Let $DMH_{a,b}\begin{bmatrix} x \\ y \end{bmatrix}$ be a hyperbolic matrix then by definition:

$$DMH_{a,b}\begin{bmatrix} x \\ y \end{bmatrix}\in GL(2,R)$$

Then:

$$\det\left(DMH_{a,b}\begin{bmatrix} x \\ y \end{bmatrix}\right)=ba\pm 1$$

Hence $|ba| = 1$
⇐) Let $|ba| = 1$ then:

$$\det\left(DMH_{a,b}\begin{bmatrix} x \\ y \end{bmatrix}\right)=ba\pm 1$$

$$DMH_{a,b}\begin{bmatrix} x \\ y \end{bmatrix}\in GL(2,R)$$

and by the relation between roots and coefficients $|ba| = |\pm 1| = 1$ so, if $|b|\neq 1$ and $|a|\neq 1$then $|a| = -1/|b|$such that $|b|\neq 0$and by proposition (3.3) $\lambda_1 = a$, $\lambda_2 = b$ are two real number and since: $|\lambda_1| = |a|\neq 1$ and $|\lambda_2| = |b|\neq 1$ and since R is totally order set so either $|a|>1$ or $|a|>1$ or $|b|<1$ if $|a|>1$ then$|b| = 1/|a|$ and if $|b|<1$then $|a| = 1/|b|>1$

RESULTS AND DISCUSSION

Sensitive dependence on initial condition of modified Henon map $MH_{a,b}$: The $K:X\to X$ is said to be sensitive dependence on initial conditions if there exist $\eta>0$ such that for any $p_0\in X$ and any open set $W\subset X$ containing p_0 there exists $q_0\in W$ and$m\in z^+$ such that $d(K^m(p_0), K^m(p_0))>\eta$

That is $\exists\eta>0, \forall p, \forall\delta>0, \exists q\epsilon B_\delta(p), \exists m: d(f^m(p_0), f^m(q_0))\geq\eta$ (Elaydi, 2000). Despite the fact that there is no widespread concurrence on definition of chaos is for the most part concurred that a chaotic dynamical system should exhibit sensitive dependence on initial conditionsas chaotic. Iftichar *et al.* (2013). Let $P = (P_1, P_2\ P_3 \dots\dots\dots, P_n)$ and $q = (q_1, q_2\ q_3 \dots\dots\dots, q_n)\ R^n$ we write if and only if therer exist ({1, ..., n}) such that p.

Proposition (4.1): If $|b|>1$ or $|a|>1$ then $Mh_{a,b}$ has sensitive dependence on initial condition

Proof: Let:

$$X=\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}\in R^2$$

Since:

$$MH_{a,b}(x)=\begin{bmatrix} 1+ax_1+\cos 2\pi y_1 \\ by_1 \end{bmatrix}$$

Case1: If $|x|\leq 1$ by hypothesis and by definition

$$MH_{a,b}(x)<\begin{bmatrix} 1+ax_1 \\ by_1 \end{bmatrix}$$

And:

$$MH^2_{a,b}(x)<\begin{bmatrix} 1+a^2x_1 \\ b^2y_1 \end{bmatrix}$$

That is:

$$MH^n_{a,b}(x)<\begin{bmatrix} 1+a^nx_1 \\ b^ny_1 \end{bmatrix}$$

Thus if $|b|>1$, $n\rightarrow\infty$

$$MH^n_{a,b}(x)\rightarrow\infty$$

Let:

$$y=\begin{bmatrix} x_2 \\ y_2 \end{bmatrix}\in R^2$$

Such that $d(x, y)<\delta$

$$d(MH_{a,b}(x).MH_{a,b}(y))=\sqrt{(1+ax)^2+(by)^2}$$

$$d(MH^2_{a,b}(x).MH^2_{a,b}(y))=\sqrt{(1+a(1+ax))^2+(b(by))^2}$$

$$d(MH^n_{a,b}(x).MH^n_{a,b}(y))=\sqrt{(1+a(1+ax))^{2n}+(b(by))^{2n}}$$

If $|ab|>1$ and:

$$d(MH^n_{a,b}(x).MH^n_{a,b}(y))\rightarrow\infty$$

Hence $MH_{a,b}$ has sensitive dependence on initial condition.

Case2: If $|x|>1$ then the iterates of modified Henon map are diverge .thus it has sensitive dependence on initial condition (Fig. 1). Then we study the sensitive dependent on initial condition of map by varying the point (as follow (i = 1, 2) control parameters (a, b) by using(matlab).

The lyapunov exponents of modified Henon Map $Mh_{a,b}$: Let $F: R^n\rightarrow R^n$ be continuous differential map .The map will have n Lyapunov exponents, say

$$L_1^{\pm}(y,v_1), L_2^{\pm}(yv_2), L_3^{\pm}(y,v_3)\dots L_1^{\pm}(y,v_n),$$

For a minimum Lyapunov exponent that is:

$$L^{\pm}(y,v)\ (\max\{L_1^{\pm}(y,v_1), L_2^{\pm}(yv_2), L_3^{\pm}(y,v_3)\dots L_1^{\pm}(y,v_n)\}$$

Where $v = (v_1,v_2,\dots,v_n)$. Where all y in R^n in direction V the Lyapunov exponent was defined of a map F at y by $L^{\pm}(y,v)= \mathrm{Lim}\top(n\rightarrow\infty)1/n\ \ln\|DF_y^n v\|$ whenever the limit exists. where $v = (v_1, v2,\dots,v_n)$" (Sturman *et al.* 2006). The usual test for chaos is calculation of the largest Lyapunov exponent (Bin and Zhang, 2006). A positive largest Lyapunov exponent indicates chaos. When one has access to the equations generating the chaos, and which measure the rates of separation from the current orbitpoint along m orthogonal directions .The Lyapunov exponentis greater than zero. A quantitative measure of the sensitive dependence on the initial conditions is the Lyapunov exponentit's the average rate of divergence (or convergence) of two neighboring trajectories in the phase space.

Proposition (5.1): If either then thehas positive Lyapunov exponents.

Proof: -If $|a|<1$ and $|b|>1$by proposition $|\lambda_1| = |a|$ if $|a|<1$ since

$$L_1\left(\begin{pmatrix} x \\ y \end{pmatrix}, v_1\right)=\lim_{n\rightarrow\Leftarrow}\frac{1}{n}\ln\left\|DMH_{a,b}\begin{pmatrix} x \\ y \end{pmatrix}, v_1\right\|<0$$

But if $|b|>1$ then:

$$L_2\left(\begin{pmatrix} x \\ y \end{pmatrix}, v_2\right)=\lim_{n\rightarrow\Leftarrow}\frac{1}{n}\ln\left\|DMH_{a,b}\begin{pmatrix} x \\ y \end{pmatrix}, v_2\right\|>0$$

So, the Lyapunov exponent.

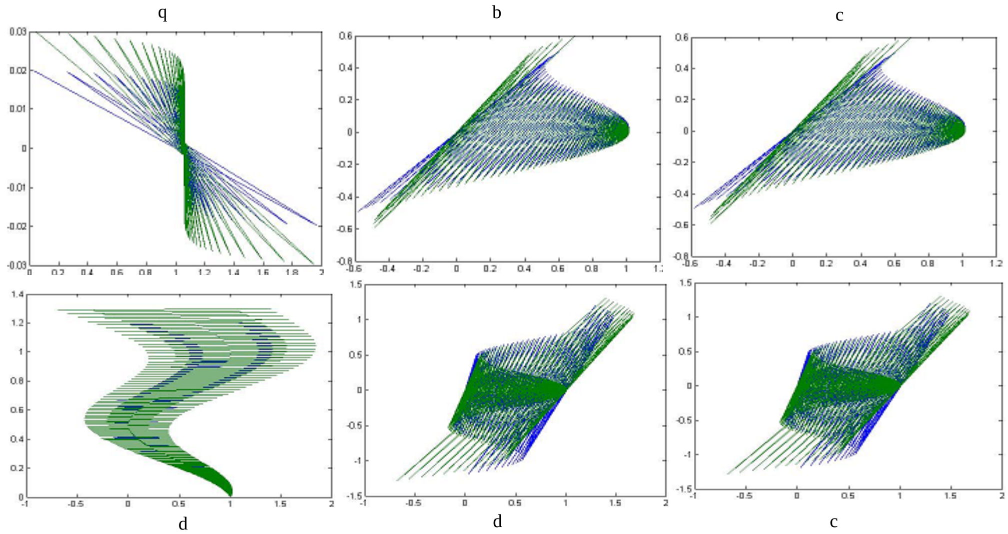

Fig. 1: Modified Henon map is not sensitive dependence on initial condition

Table 1: Negative lyapunov exponent

Variable	(x , y)	a	b	L_1	L_2
A	(0.04,0.03)	-0.88	-0.79	-0.1278333715	-0.2357223335
B	(0.5,0.4)	-0.98	-0.99	-0.0202027073	-0.0100503359
C	(0.1,0.2)	-0.70	-0.99	-0.3566749439	-0.0100503359
D	(1.3,1.2)	-0.99	0.97	-0.1278333715	-0.0304592075
E	(1.4,1.3)	-0.98	-0.99	-0.0202027073	-0.0100503359
F	(0.7 ,0.6)	0.77	0.96	-0.2613647641	-0.0408219945

Table 2: Positon in true

Variable	(x , y)	a	b	L_1	L_2
1	(1.2, 1.7)	-1.0028	1.00010	0.0027960873	0.0000999950
2	(1.3, 1.2)	-0.9900	1.00019	-0.0100503359	0.0001899820
3	(0.4,0.2)	-1.0088	1.00500	0.0087615057	0.0049875415
4	(0.06,0.05)	1.0030	-0.99000	0.0029955090	-0.0100503359
5	(1.1, 1.2)	-1.0099	1.00040	0.0098513161	0.0003949200
6	(1.3,1.2)	-1.0016	1.00009	0.0015987214	0.0000899960

Table 3: Arbitrary point

Variable	A	b	L_1	L_2
1	1.0000	1.000	0	0
2	-1.0000	-1.000	0	0
3	-1.0000	-1.006	0	0.005982071
4	-1.5000	-1.000	0.4054651081	0
5	1.0022	1.000	0.0021975835	0
6	1.0000	1.450	0	0.3715

$$L^{\pm}(x,v) = \max\{L_1^{\pm}(x,v_1), L_2^{\pm}(x,v_2)\}$$

That is $MH_{a,b}$ has positive Lyapunov exponent.

Proposition (5.2) (Michael and Garrett, 2002): If $L^{\pm}(x, v) = x>0$ for some vector, then there is a sequence $\{m_i\}$,$I^{\to}\infty$ Such that for every :

$$\delta>0 \; \| dDMH_{a,b}\begin{pmatrix} x \\ y \end{pmatrix} v \| \geq e^{(x-\delta)}m_i \; \| v \|$$

This implies that, for a fixed; there is a point $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \in R^2$ such that:

$$d(DMH_{(a,b)}(m_i)((x@y), DMH_{(a,b)}(m_i)$$
$$(x_2@y_2)) \geq 1/2e^{((x-\delta)m_i))|(d(x@y))|}$$

This does not imply sensitive dependence on initial condition. We use the mat lab program to compute Lyapunov exponent in several value of parameters a and b. Table 1 show that the points I n Fig. 1 have negative Lyapunov Exponent. Table 2 show that the Proposition (5.1) is true In Table 3 we choose arbitrarily point (0, 0), this table show us that the Lyapunov Exponent equal zero If |b| = 1 and |a| =1, that is, this point is a bifurcation point when |b| = 1and |a|=1.

CONCLUSION

In this research,we have presented a two-dimensional dynamical system. The mathematical properties of the modified Henon maps

- If $a \neq 1$ and $b \neq 1$ the $MH_{a,b}$ has unique fixed point, 2
- If a = 1 has $b \neq 1$then $MH_{a,b}$ no fixed point2)
- The eigenvalues of the

$$MH_{a,b}\begin{pmatrix} x \\ y \end{pmatrix}$$

are: $\lambda_1 = a$, $\lambda_2 = b$

- The area contracting and expanding of $MH_{a,b}$
- If $|a|<1$ and $|b|<1$ then $MH_{a,b}$ is an area contracting map
- If $|a|<1$ and $|b|<1$ then $MH_{a,b}$ is an area expanding map
- If $|a|>1$, $a \neq 0$ and $|b|<1/|a|$ or $|b|> 1$, $b \neq 0$ and $|a<1/|b|$ then $MH_{a,b}$ is an area contracting map
- If $|a|>1$, $a \neq 0$ and $|b|>1/|a|$ or $|b|> 1$, $b \neq 0$ and $|a|>1/|b|$
- If $|b|>1$, $b \neq 0$ and $|a|^2>1/|b|$and then $MH_{a,b}$ then M is an area expanding map

The modified Henon map are close and they have sensitive dependence on initial condition, they have positive Lyapunov exponents

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