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Concerning the Issue of Object Classification by Form

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Abstract: During the solution of applied problems related to the detection and classification of objects creating a real scene as the source of information the images are used often. On the images the scene objects are represented by their projections on a plane perpendicular to the observation trend. Therefore, during the classification of objects an important telltale sign is the form of their projections which are the subsets (figures) on a plane. In contrast to the visual classification when the comparison of plane figures by form may be performed almost at a subconscious level, an automatic classification requires a formal definition of this term. The development of an object projection according to a real scene image called segmentation is quite a challenging issue. The projection obtained from the segmentation usually differs from previously prepared projection with which the comparison occurs. This means that the method of projection comparison by form must be resistant to the errors arising during segmentation. The research consists of two parts. In this first part, the formalization of a plane figure form as the probabilities of chord length distributions, cut out by the figure from a random line. The independence of the form on shifts and turns is proved. It is shown that the comparison of figures by form is reduced to the test of two sample homogeneity hypothesis. In the second part of the proposed definition of the form is used for the classification of vehicles. The initial information for solving this problem are the images taken from aircrafts.

Key words: Flat figure form, random lines, chord length distribution, the hypothesis of homogeneity, image taken

INTRODUCTION

During the visual interpretation of images the form of objects presented on the stage is an important telltale sign. However, its use in automated deciphering encounters serious difficulties associated with the lack of convenient formalization. Therefore, despite a relatively large number of publications (Zhang and Lu, 2004; Furman, 2003; Mestetsky, 2009; Yukhno *et al*., 2001), dedicated to the presentation of different approaches to the definition (formalization) of form, the research in this area, still remain relevant ones.

In the research (Fofanov and Zhelezov, 2007), the formalization of the form definition for a limited convex related subset of the real plane R^2 s the distribution of probabilities P_B a chord length which is cut out by the subset B from a randomly selected straight line of the plane. The pluralities are set by closed rectifiable curves C that describe the equations of the following form $F(x,y) = 0$. It is assumed that curves do not have multiple points and that the function F is continuous one. Next, the subsets B will be called figures and the curves C will be called their boundaries. The first part of this research is devoted to the proof of the invariance of the proposed form definition in (Fofanov and Zhelezov, 2007) concerning shifts and turns. When you solve the issues for comparing the shape of two figures, it is proposed to use the methods of mathematical statistics. First, they get the samples from the chord lengths for comparable figures. Then the samples are used to test the hypothesis of uniformity. Here, we propose a generalization form for figures that are not convex ones.

In the second part of the research, the proposed definition of the form is used for the classification of vehicles.

FORMALIZATION OF FORM

Regardless of the approach in use, the form definition must satisfy a number of obvious requirements. In particular, the figures obtained from its source by parallel transfer (shift) or its rotation relative to a selected point shall have the same shape. These transformations form as we know, a group of movements to R^2.

Let C is the boundary of a convex shape B and L(C) is its length. We fix some point O on C as a reference one and set a positive direction. This allows you to set between the curve C and the points of the segment [0, L(C)] for a straight line a certain mutual correspondence. The point $t \in [0, L(C)]$ is associated with

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the point M = (x,y) on C, located at a distance t from O. That is there are the representations f and g on [0, L(C)] such as that x = f(t) and y = g(t). Let's extend these representations for the remainder of the line by the means of the equation f (I + L(C)) = f(t) and g (t+L(C)) = g (t), i.e., let's assume that f and g are periodical ones. Since, we are talking about the limited figures, there are numeric constants X_1 and X_2, Y_1 and Y_2 such that $X_1 \le f(t) < X_2$, $Y_1 \le g(t) < Y_2$.

Then, let's assume that f and g are both Borel ones. Therefore, it is desirable to indicate the class of figures for which this assumption is satisfied. First of all, we refer to the well-known result (Tutubalin, 1972), according to which all continuous functions are the Borel ones. In the case, when the set B is a polygon, one may prove the continuity of f and g functions directly. In fact, let t_1 and t_2 are the point from [0, L(C)], corresponding to the vertices $M_1 = (x_1, y_1)$ and M_2 of the polygon, which are connected by a line $M_2 = (x_1, y_2)$ segment, set by the equation ax+by+c = 0. Without the limitation of generality, we assume that $t_1<t_2$ and that $x = f(t_1)<f(t_2) = x_2$. If M = (x, y) is a point on the selected segment of the boundary C corresponding to the point $t \in \{t_1, t_2)$, then at $a \neq 0$ and $b \neq 0$ it is easy to show that:

$$x = f(t) = \frac{x_2 - x_1}{t_2 - t_1}(t - t_1) + x_1,$$
$$y = g(t) = -\frac{a}{b}f(t) - \frac{c}{b}$$

That is f and g are continuous functions. If a = 0, $b \neq 0$, then:

$$x = f(t) = \frac{x_2 - x_1}{t_2 - t_1}(t - t_1) + x_1,$$
$$y = g(t) = -\frac{c}{b}$$

At $a \neq 0$ and b = 0 in a similar way with the previous case, we obtain the following:

$$x = f(t) = -\frac{c}{a},\ y = g(t) = \frac{y_2 - y_1}{t_2 - t_1}(t - t_1) + y_1$$

That is the displays f and g taking the point $t \in [t_1, t_2]$ to the point M = (x,y) of the segment, connecting the vertices $M_1 = (x_1, y_1)$ and $M_2 = (x_1, y_2)$ of the boundary are continuous ones. It is known that the representation p: R→R of the following form:

$$p(t) = \begin{cases} 1/L(C), & t \in [0, L(C)] \\ 0, & t \in R \setminus [0, L(C)] \end{cases}$$

is the density of a random value ξ uniformly distributed on [0, L(C)], determined by the equation ξ(t) = t. We will call it further a random point.

Each pair of random points on t_1, t_2 corresponds to the pair of points $M_1 = (f(t_1), g(t_1))$, $M_2 = (f(1_2), g(t_2))$ on the curve C. By assumption, f and g are Borel functions, so the coordinates of the points M_1 and M_2 are random values. Each pair M_1, M_2 of points on C explicitly defines a particular line on R^2. Obviously, there is a certain mutual correspondence between the lines crossing the figure B and the pairs of points on the curve C. Similar with the random dots, such lines will be called randomly straight ones.

The intersection of a convex figure B with a random straight line is a segment. Then, it will be called a chord, a cutout figure B from a random line. Since the chord $d(M_1, M_2)$ length has the form:

$$d(M_1, M_2) = \sqrt{(x_1 - x_2)^2 - (y_1 - y_2)^2}$$

It is a random variable at Borel f and g. Its distribution of probabilities P_B is offered to consider as a form of a convex figure B. Two convex figures B and A will be considered the same by form if the same distributions of probabilities P_B and P_A coincide for the lengths of chords, cut out by these figures from random lines. Let's show that the probability distribution of the chord lengths is independent of the coordinate system selection at the boundary C.

Theorem 1: The distribution of the chord length, cut out figure from a random direct line does not depend on the choice of coordinates on C.

Proof: If you select another calculation origin $\hat{O}$ on the curve C, each pair t_1, t_2 of random points from [0, L(C))] will define two pairs of points on the curve C. The pair M_1, M_2 in the coordinate system with the origin at O and the pair $\hat{M}_1, \hat{M}_2$ in the coordinate system with the origin $\hat{O}$. The chord $d(M_1, M_2)$ and $d(\hat{M}_1, \hat{M}_2)$ lengths will in general, different. However, the distribution of the chord lengths will be the same in both cases.

Let f is the Borel representation of the type $[0, L(C)] \rightarrow [X_1, X_2]$, corresponding to the reference point O, t uniformly distributed on [0, L(C)] is a random variable. Therefore, f(t) is also a random variable and its density is the following one (Tutubalin, 1972):

$$p_f(x) = p(f^{-1}(x))\,|(f^{-1})'(x)| = \frac{p(f^{-1}(x))}{|f'(x)|}$$

Since P is a uniform distribution density, we obtain:

$$p_f(x) = \begin{cases} \frac{1}{L(C)\,|\,f'(x)\,|}, a \le f^{-1}(x) < a + L(C), \\ 0,\ \text{inothercases} \end{cases}$$

Note that the inequality $0 \le f^{-1}(x) \le L(C)$ on the right part of the last equality is equivalent to $X_1 \le f(t) < X_2$.

Let $\hat{O}$ is another reference point on the curve C, to which the point $a \in [0, L(C)[$ corresponds within the first coordinate system. If $\hat{f}: [0, L(C)[\ [X_1, X_2]$ is the display, corresponding the reference point $\hat{O}$, then for every point we get $\hat{f}(t) = f(a+t)$. From this equation, it follows that $\hat{f}^{-1}(x) = f^{-1}(x) - a$. Therefore,

$$p_{\hat{f}}(x) = p(\hat{f}^{-1}(x))\,|\,(\hat{f}^{-1})'(x)\,| = = \frac{p(\hat{f}^{-1}(x))}{|\,\hat{f}'(x)\,|}$$

Taking into consideration the type $\hat{f}$, we obtain:

$$p_{\hat{f}}(x) = \begin{cases} \frac{1}{L(C)\,|\,f'(x)\,|}, a \le f^{-1}(x) < a + L(C) \\ 0,\ \text{inothercases} \end{cases}$$

The inequality $a \le f^{-1}(x) < a + L(C)$ of the equation right side is equivalent to $X_1 \le f(t) < X^2$. Thus $P_f(x)$.

Let's consider a new coordinate system which only differs from the original one by direction. In this case for every point $t \in [0, L(C)]$ we get that $\hat{f}(t) = f(L(C) - t)$. From this equation, it follows that $\hat{f}^{-1}(x) = L(C) - f^{-1}(x)$ and its derivative $\hat{f}^{-1}(x) = -f^{-1}(x)$. Thus:

$$p_{\hat{f}}(x) = \begin{cases} \frac{1}{L(C)\,|\,f'(x)\,|}, 0 \le L(C) - f^{-1}(x) < L(C), \\ 0,\ \text{inothercases} \end{cases}$$

Therefore, $p_{\hat{f}}(x) = p_f(x)$, QED. In the same way, we prove the independence from the chosen display calculation origin of g type $[0, L(C)] \rightarrow [Y_1, Y_2]$ that associates each random point $t \in [0, L(C)]$ with a second coordinate $Y = g(t)$ of the point $M_2(x, y)$ on the curve C.

FORM PROPERTIES

Let's show that the shape of the figure is not changed under a parallel displacement. Let $\tau = (\tau_1, \tau_2) \in R^2$ is a fixed vector, B is a subset on R^2 and is a set obtained by parallel transition of B on the vector τ, i.e.:

$$B_\tau = \{(x,y) \in R^2: x = b_1 + \tau, y = b_2 + \tau, (b_1, b_2) \in B\}$$

In this case, the equation of the set boundary B_τ has the following form:

$$F(x - \tau_1, y - \tau_2) = 0$$

If the boundary C of the set B is rectifiable, then the boundary C_τ of the set B_τ is rectifiable too. In fact, let $M_1 = (x_1, y_1)$ and $M_2 = (x_2, y_2)$ are the vertices of the polygon inscribed in the border C. With the parallel transfer of B on the vector $\tau = (\tau_1, \tau_2)$ they will be transited in points $M_1 = (x_1+\tau_1, y_1+\tau_2)$ and $M_2 = (x_2+\tau_1, y_2+\tau_2)$ borders C_τ. It is evident that $d(M_1, M_2) = d(M_1, M_2)$. This means that every polygon inscribed in C, corresponds to the polygon inscribed in C_τ with the same perimeter. If C is rectifiable, then at the reduction of the distance between the series vertices of the polygon perimeters built on the boundaries C and C_τ converge to the same limit which is the length C and C_τ. Consequently if B is the figure, then B_τ is also a figure and vice versa. The following assertion is fair for the figures B and B_τ.

Theorem 1: The figure B and the figure B_τ obtained from it with a parallel shift on the vector $\tau = (\tau_1, \tau_2)$ have the same form, i.e., $P_B = P_{B_\tau}$.

Proof: Let t_1 and t_2 are accidental points on [0, L(C)]. From the stated above, it follows that if $M_1 = (x_1, y_1)$ and $M_2 = (x_2, y_2)$ the ends of the chord cut out by figure B, then after a parallel shift, they will go to the ends $M'_1 = (x_1+\tau_1, y_1+\tau_2)$ and $M'_2 = (x_2+\tau_1, y_2+\tau_2)$ of the chorde, cut out by figure B_τ. At that $d(M_1, M_2) = d(M'_1, M'_2)$. Therefore if the representations f and g are the Borel ones, then the lengths of the two chords are random variables with the same distribution, QED.

Let's prove the invariance of the form under turns. Let α is a fixed angle, measured from the horizontal axis anticlockwise B is the subset on R^2, a B_α the result of B turn reative to 0 = (0, 0) on the angle α, i.e.:

$$B_\alpha = \{(x, y) \in R^2: x = b_1 \cos\alpha - b_2 \sin\alpha, y = b_1 \sin\alpha + b_2 \cos\alpha, (b_1, b_2) \in B\}$$

In this case, the equation the subset B_α border becomes the following one:

$$F(x \cos\alpha + y \sin\alpha - x \sin\alpha + y \cos\alpha) = 0$$

If the boundary of the set B is rectifiable, then the boundary of the set B_α is also rectifiable. Indeed; let $M_1 = (x_1, y_1)$ and $M_2 = (x_2+y_2)$ are the vertices of the polygon inscribed in the border C. At turn B on the angle α the will turn to the following points:

$$M'_1 = (x_1 \cos\alpha - y_1 \sin\alpha, x_1 \sin\alpha + y_1 \cos\alpha)$$

and:

$$M'_2 = (x_2 \cos\alpha - y_2 \sin\alpha, x_2 \sin\alpha + y_2 \cos\alpha)$$

C_α borders of the figure B_α. It is easy to verify directly that:

$$d(M'_1, M'_2) = \sqrt{\begin{aligned}&((x_1 - x_2)\cos\alpha - (y_1 - y_2)\sin\alpha)^2 + \\ &((x_1 - x_2)\sin\alpha + (y_1 - y_2)\cos\alpha)^2\end{aligned}}$$
$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = d(M_1, M_2)$$

This means that every polygon inscribed in C, corresponds to the polygon, inscribed in C_α with the same perimeter. If C is rectifiable one, then at the decrease in the distance between the tops of the sequence from polygon perimeters, built on the boundaries C and C_α, converge to the same limit which is the length C and C_α. Thus if B is the figure, then B_α is also the figure and vice versa. The following statement is fair for the figures B and B_α.

Theorem 2: Figure B and the figure B_α obtained from it by turn relative the coordinate origin on the corner α have the same form, i.e., $P_B = P_{B_\alpha}$.

Proof: Let t_1 and t_2 are accidental points on [0, L(C)]. If $M_1 = (x_1, y_1)$ and $M_2 = (x_2, y_2)$ are the ends of the chord cut out by figure B, then after a turn relative to the beginning of coordinates by the angle α, they will turn to the points M'_1 and M'_2 of the boundary C_α of the figure B_α. It is easy to verify directly that the two chords have the same length. Indeed:

$$d(M'_1, M'_2) = \sqrt{\begin{aligned}&((x_1 - x_2)\cos\alpha - (y_1 - y_2)\sin\alpha)^2 + \\ &((x_1 - x_2)\sin\alpha + (y_1 - y_2)\cos\alpha)^2\end{aligned}}$$
$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = d(M_1, M_2)$$

Therefore if the representations f and g are the Borel ones, the lengths of both chords are random variables with the same distribution, QED.

When you rotate relative to an arbitrary point $\alpha \in R^2$, you must first perform a parallel transition of the figure on the vector -a and then turn the shifted figure relative to the origin of coordinates and then perform the parallel transition of the turned figure to the vector a.

COMPARISON OF FIGURES BY FORM

Thus, the comparison of figures A and B by form from a formal point of view, means the test of the distributions P_A and P_B equality for the chord lengths, cut our by these figures from straight lines selected randomly on a plane. The obtaining for specific figure A of its distribution P_A by theoretical calculations in general may be quite challenging. Therefore, it is advisable to use the methods of mathematical statistics. They allow you to check the equality of distributions P_A and P_B by comparing random samples obtained from these distributions. Note that obtaining of random samples for chord lengths practically does not depend on the kind of figure.

Indeed, let A and B are comparable figures and n_A and n_B is the number of non-dependent random lines intersecting A and B, respectively. Let $l_A = (l_{A.k})_{1\le k\le n_A}$ and $l_B = (l_{B.k})_{1\le k\le n_B}$ are the samples of the chord lengths cut out from these lines by the figures A and B. The assumption about the equality of distributions P_A and P_B in mathematical statistics is called the hypothesis of homogeneity (Cramer, 1948). In order to test it let's divide, the set of real numbers R into s of intervals. Let $m_{A,j}$ and $m_{B,j}$ is the number of sample elements l_A and l_B respectively within the jth interval. It is known that at equality of distributions P_A and P_B the distribution of statistics $X^2_{n_A+n_B}$ of the type:

$$X^2_{n_A+n_B} = n_A n_B \sum_{j=1}^{s} \frac{1}{m_{A,j} + m_{B,j}} \left(\frac{m_{A,j}}{n_A} - \frac{m_{B,j}}{n_B} \right)^2$$

strives at $(n_A+n_B)\to+\infty$ top the distribution χ^2 c(s-1) by the degree of freedom. Therefore, the probability to be in the range of $[t+\infty]$ for statistics $X^2_{n_A+n_B}$ is equal approximately to $P(\chi^2 \ge t)$.

This allows us to reduce the comparison of figures by shape to test the hypotheses of homogeneity. In fact, let's set a small positive number α (for example, 0.01 or 0.005) and solving the equation $P(\chi^2 \ge t_\alpha) = \alpha$ with respect to t_α, let's create the critical region $[t_\alpha+\infty]$. If for the calculated value of the statistics $X^2_{n_A+n_B}$ the inequality $X^2_{n_A+n_B} \ge t_\alpha$ is performed the hypothesis of distributions P_A and P_B equality is rejected. Thus, α has a clear meaningful interpretation. It is equal to the probability of a correct hypothesis rejection.

Finally, we turn to the description of non-convex figure form. The principal difference between a non-convex figure from the convex one is that its intersection with a straight line may consist of several segments which will also be called chords. Consequently, the direct application of the proposed definition of the form for the comparison of non-convex shapes is not suitable.

Therefore, we associate each non-convex figure not one but a pair of random variables. The first random variable will describe the number of chords that the figure cuts out of a straight line. The second one is the sum of all chords lengths cut out from the line. Thus, a non-convex shape of the figure may be seen as a two-dimensional random variable or its distribution. Two non-convex figures should be regarded as the same shapes if their distributions coincide. Obviously, the figures obtained from each other by parallel transition or turn, will have the same shape. If you apply the definition of a non-convex figure to describe convex figures, the random variable that describes the number of cut out chords will always accept (with the probability equal to unity) the value of 1. This means that the definition of the convex shape form is a special case of determining the form for a non-convex figure. Thus for the comparison of two figures by shape, it is quite difficult to obtain a two-dimensional sampling and test the hypothesis of homogeneity for each of them.

SUMMARY

The form definition offered in the research may be used to compare plane figures in order to solve the applied problems.

CONCLUSION

The proposed definition of form for a plane figure is invariant under transitions and rotations. The method of figure comparison by form from a computational point of view is quite simple: it is reduced to the hypothesis of homogeneity testing. It does not require prior training and is independent of any constants, an unfortunate choice of which affects the outcome.

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