

## Modeling Product Variants Based on the Number of Initial Components and Assembled Modules

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**Abstract:** Developing product variants with respect to satisfying diverse customer needs with reasonable costs in terms of mass customization has been recognized as a new paradigm for today's manufacturing. In mass customization, products are usually made of several modules while each module is composed from several number of variants. High number of product variants is achieved through a combination of assembled modules. In this study, we present a methodological framework for creating all possible product variants based on unlimited number of initial components. Generation of all possible product architectures is aimed at investigation of the influence of product variety on assembly processes, structural complexity.

**Key words:** Product variants, supply chain, modular configuration, mass customization, customer

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### INTRODUCTION

Competition in the global business environment is accelerating in different areas due to the combined impact of globalization and technology and rapidly changing customer needs. Mass customization is one of the main business strategies focused on design and development of products that can be individually tailored to customer needs. It allows OEMs to provide customers with products that bring complete satisfaction and enjoyment at near mass production cost (Pine, 1992). The key to mass customization is to provide products with high variety. In mass customized production, the products are made by flexible assembly processes consisting of several different modules. Usually such an assembly process is connected with a continuously moving conveyor(s). At the last station, an operator assembles components or modules into the final product. Thus, the number of all possible components determines the assembly systems' complexity. The intent in this study is to determine all possible product variants based on the number of initial components. Then, the number of all possible product variants could reflect a product-based structural complexity of an assembly system.

### LITERATURE REVIEW

Mass customization caused that the variety of products increased drastically in recent years. In order to satisfy the needs of the market, manufacturers are

motivated and made to produce in high varieties at competitive prices, respectively (Sloane, 1973). According to Svensson and Barfod (2002), "the cost of customization have been reduced so dramatically that many customers have a actual choice between a mass-produced product and a customized version". Today's production systems must be able to handle such a variety and cope with the expected quality and quantity. Manufacturing systems evolved from "simple" moving assembly lines to mixed-model assembly systems which are considered to be the major enablers to handle the increased variety (Zhu *et al.*, 2008). They are used in various industries in two main versions as Assembly Supply Chains (ASC):

- Modular assembly supply chain (Sayed and Lohse, 2013; Fredriksson, 2002; Modrak *et al.*, 2012)
- Non-modular assembly supply chain (Wang and Ceglarek, 2005; Dini and Santochi, 1992; Modrak and Marton, 2013)

However, there is one particular challenge in the application of mixed-model assembly systems. It is complexity caused by product variety of mass customization. It has been a huge challenge for number of scientists to describe, apply and optimize systems based on complexity of ASCs. Therefore, researchers in different areas offer a number of complexity measures that differ in their approach to dealing with specific complications and theoretical methods used. The most frequent approach is

based on Shannon information theory by which entropy is defined as a measure of uncertainty (Shannon, 1948). Zhu *et al.* (2008) applied this theoretical assumptions to definition of complexity model aimed for mixed-model assembly systems. Thier indicator so called Operator Choice Complexity is developed to evaluate the complexity at each station and for the entire assembly line. Deshmukh *at al.* (1998) proposed another entropy measure combining part types and their ratios in a manufacturing system. ElMaraghy *et al.* (2005) defined a code-based structural complexity index to detect the amount of information in the manufacturing systems as well as another complexity indicators based on the probability of asuccessful assembly of a product in a manufacturing process. Another approach was presented by Suh (2005) which analysed the complexity in the context of product design in achieving functional or design requirements. It is evident that different assembly variations in mixed model assembly lines have profound impact on complexity and on performance of the system. Therefore, it is important to reveal and understand the linkage behind the complexity, product variety and assembly variation in mixed-model assembly lines (Modrak, 2009). Knowing the complexity of designed system depending on the number of assembly variations and system configuration, it can be very essential already in the early stages of a production or product design to find the best balanced design for a new product or whole production.

**INITIAL ASSUMPTIONS**

The definition of the term “supply chain” introduced by Modrak (2009) says that supply chain is a network of the organizations involved through upstream and downstream linkages in the different processes and activities that produce value in the form of products and

services delivered to the ultimate consumer. Beamon and Chen (2001) added that each functional level of supply chain network is represented by numerous facilities that along with the structure of the material and information flows contribute to the complexity of the chain.

In order to specify a suitable generic assembly supply chain model, it is useful to present existing classification of supply chain structures. For this purpose, researchers can use existing classifications (Modrak, 2009; Beamon and Chen, 2001; Wang *et al.*, 2010) compiled into Fig. 1.

Convergent class of structures represent assembly type of supply chains and is one in which each node of the chain has at most one successor but may have any number of predecessors. Convergent supply chains can be further divided into two basic groups; modular SCs and non-modular SCs (Hu *et al.*, 2008). In the modular structure, the intermediate sub-assemblers are understood as assembly modules while the non-modular structure consists only from suppliers (initial nodes) and a final assembler (end node). Moreover, it is suggested here to divide the modular SCs into two specific categories; modular SCs with minimal number of echelons and modular SCs with maximal number of echelons. This categorization is conditioned on the requirement that number of initial nodes is the same for these two altered structures. In the modular configuration, the final producer purchases sub-components from intermediate sub-assemblers instead of doing all the assembly activities himself. Modular assembly is typical for many industries such as automotive, agricultural equipment, aerospace and others. Generating all possible combinations of structures brings enormous difficulties in order to optimize the design and operation of assembly supply chains.

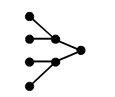
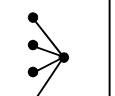
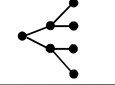
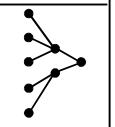
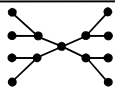
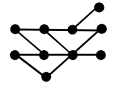
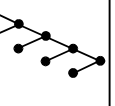
Classification of supply chain structure	Examples	Classification of assembly supply chain structures	Examples
Convergent (assembly) SC		Non-modular assembly SC	
Divergent SC		Modular assembly SC (minimal number of echelon)	
Conjoined SC			
General SC		Modular assembly SC (maximal number of echelon)	

Fig. 1: Supply chain structure classification

Proposed approach is based on the assumption defined by Wang *et al.* (2010) about the existence of: dominant demands on final product among all variants or the demand shares are equal across all variants. For assembly operations where the Dominant Demand Exists (DDE), the convergent, non-modular supply chain structure will be a matter of interest of our study for several reasons. First of all it is clear that the non-modular convergent SC is the least complex even without application of a complexity metric (Fig. 1). Secondly, according to Modrak *et al.* (2002) the optimal assembly SC structure is the one with the smallest number of links.

Therefore, researchers assume the non-modular convergent Assembly Supply Chain Structure to be the most suitable for assembly processes in general and researchers are going to use them for the generation of assembly supply chain configurations which is the matter of this study.

**GENERATING OF PRODUCT VARIANTS**

If researchers want to describe the framework for creation all possible product variants through all possible product configurations based on initial components, we will use the following notation:

- Class of product variants/configurations CL (based on number of basic components)
- Sub-class of product variants/configurations  $P_i$ ,  $i = 1, \dots, a$ , where  $i$  number of initial components
- Sub-configurations of the  $i$ th sub-class  $G_{ji}$ ,  $j = 1, \dots, b$ , where  $j$  number of sub-configurations of the  $i$ th sub-class
- Initial assembly product configuration component  $C_{ji}^z$ , where  $\Sigma$  number of initial components
- Optional assembly product configuration component  $C_{ji}^o$ ,  $o = 0, \dots, d$ , where  $o$  number of optional components
- Basic assembly product configuration component  $C_{ji}^b$ ,  $b = 1, \dots, e$ , where  $b$  number of basic components
- Product variation of the  $i$ th sub-class of the  $j$ th sub-configuration  $v_{ki}$ ,  $k = 1, \dots, f$ , where  $k$  number of product variations  $i$ th sub-class,  $j$ th configuration
- Total number of variations of the  $i$ th sub-class  $\Sigma v_i$
- Number of component sub-configurations of the  $i$ th sub-class for the given number of initial components  $G_i^z$

It is firstly essential to establish the total number of component configurations  $\Sigma G_i^z$  of the product in relation to the definition of possible product variants depending on the number of all possible additional components. Researchers are going to build it on the following assumptions:

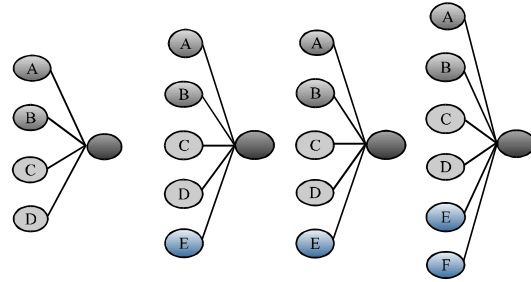


Fig. 2: Component product configurations for sub-class  $P_6$  when we have 4 basic and 2 optional components

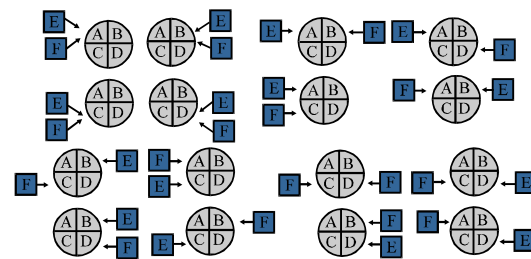


Fig. 3: Product variants of two optional components for  $P_6$ ; A, B, C, D: initial components; E, F: optional components

- Let the product group with a number of initial components be denoted by sub-class of product variants
- Each such class consists of at least one basic assembly komponent

In cases when the number of basic components ( $C_{ji}^b$ ) equal four and number of optional components ( $C_{ji}^o$ ) equals two, researchers get four possible component configurations as in Fig. 2. From Fig. 2, it is apparent that we will take into account the sub-configuration with minimum number of optional components (without optional components) as well:

- In order to generate a number of product component configurations in all possible classes of product variants, the following equation can be applied

$$G_{mi} = (2)^{C_{mi}^o} \tag{1}$$

- Each sub-configuration can be assigned by a number of product variations. For instance, for sub-configurations depicted in Fig. 3 with two optional components, there are 16 product variations

$C_{j_6}^{\Sigma}$	$G_{j_6}$	$G_{j_6}^{\Sigma}$	$V_{i_6}$	$\Sigma V_{i_6}$
4		$\left\{ \left[ \frac{2!}{(2-0)!} \right] / 0! \right\} = 1$		1
5		$\left\{ \left[ \frac{2!}{(2-1)!} \right] / 1! \right\} = 2$		4
5				4
6		$\left\{ \left[ \frac{2!}{(2-2)!} \right] / 2! \right\} = 1$		16
		$\Sigma G_{j_6}^{\Sigma} = 4$		$\Sigma v_i = 25$

Fig. 4: Fragment of product assembly variation procedure for CL = 4 and sub-class  $P_4$

- The number of all possible configurations and product variants for a given sub-class is determined in Fig. 4
- Based on Fig. 1, it is apparent that for each sub-class  $P_i$ , it is necessary to specify the values of  $v_i$  for individual product component configurations ( $G_{j_i}^{\Sigma}$ ). These numbers of configurations can be determined using the following equation for sub-class  $P_6$ :

$$\Sigma G_{j_6}^{\Sigma} = \left\{ \left[ \frac{2!}{(2-0)!} \right] / 0! \right\} + \left\{ \left[ \frac{2!}{(2-1)!} \right] / 1! \right\} + \left\{ \left[ \frac{2!}{(2-2)!} \right] / 2! \right\}$$

$$\Sigma G_{j_6}^{\Sigma} = 1 + 2 + 1 = 4 \tag{2}$$

Similarly, researchers can formulate equations for all possible individual sub-classes of product component configurations. For, example for the sub-class  $P_7$ , it will be:

$$\Sigma G_{j_7}^{\Sigma} = \left\{ \left[ \frac{3!}{(3-0)!} \right] / 0! \right\} + \left\{ \left[ \frac{3!}{(3-1)!} \right] / 1! \right\} + \left\{ \left[ \frac{3!}{(3-2)!} \right] / 2! \right\} + \left\{ \left[ \frac{3!}{(3-3)!} \right] / 3! \right\}$$

$$\Sigma G_{j_7}^{\Sigma} = 1 + 3 + 3 + 1 = 8 \tag{3}$$

Individual addends represent the values of individual component configurations  $v_i$  in the given order. Summing all the addends, the total number of product variations  $\Sigma v_i$  is obtained. A summary procedure for the  $\Sigma v_i$  determination of each sub-class  $P_i$  of CL = 4 is depicted in Fig. 5.

Definition of the total number of variants of any sub-class  $P_i$  and class CL depending on the number of base and optional components can then be expressed as follows:

$$v_i = \sum \left( \left\{ \left[ \frac{C_{j_i}^o!}{(C_{j_i}^o - C_{j_i}^g)!} \right] / C_{j_i}^g \right\} C_{j_i}^{b C_{j_i}^g} \right) \tag{4}$$

Where:

- $C_{j_i}^o$  = No. of optional components,  $o = 0, \dots, d$
- $C_{j_i}^b$  = No. of basic components,  $b = 1, \dots, e$
- $C_{j_i}^g$  = No. of components towards  $C_{j_i}^o$ ,  $g = 0, \dots, C_{j_i}^o$

Based on the obtained values of product variations that practically represent complexity level of assembly process it is possible to express a relation between these two variables. This relation is shown graphically in Fig. 6 for selected classes of product variants and for specific numbers of base components. Finally, in this study researchers summarize results of the research in to Table 1. In this Table 1, it is possible to identify number of

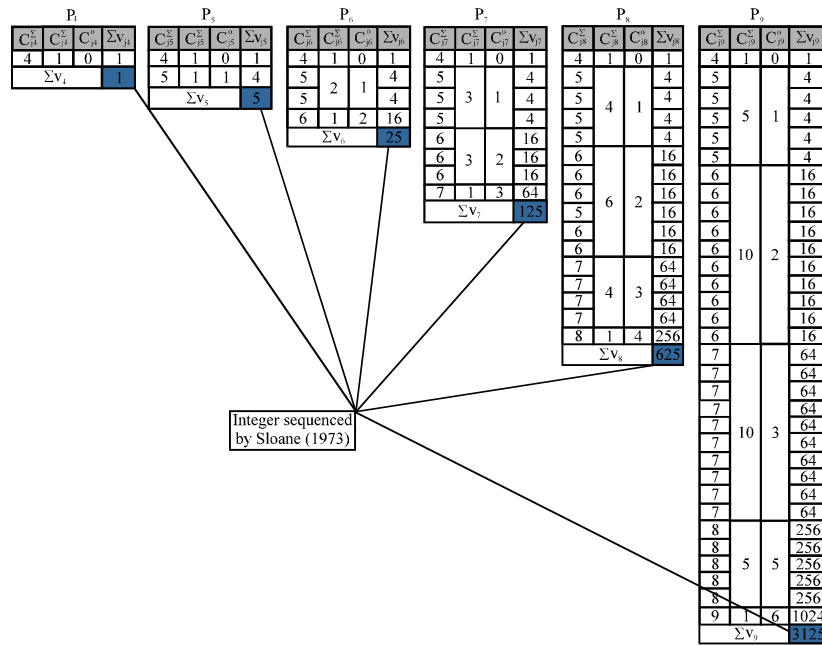


Fig. 5: Generation of total assembly product variants for P<sub>4</sub> to P<sub>9</sub> of CL = 4 resulting by integer sequence A000351

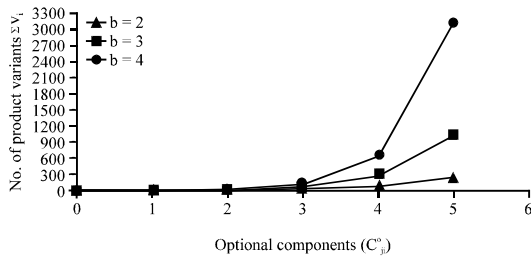


Fig. 6: Relation between the numbers of optional components and number of product variants

**Table 1: Selected numbers of product configurations and variants**

CL#	Sub-class	Nr. of basic components	Nr. optional components	Nr. of product configuration	Nr. of product variants
CL#1	P <sub>2</sub>	1	1	1	1
	P <sub>3</sub>	1	2	3	3
	P <sub>n</sub>	1	n	-	-
CL#2	P <sub>2</sub>	2	0	1	1
	P <sub>3</sub>	2	1	2	3
	P <sub>4</sub>	2	2	4	9
	P <sub>n</sub>	2	n	-	-
CL#3	P <sub>3</sub>	3	0	1	1
	P <sub>4</sub>	3	1	2	4
	P <sub>5</sub>	3	2	4	16
	P <sub>6</sub>	3	3	8	64
	P <sub>7</sub>	3	4	16	256
	P <sub>8</sub>	3	5	32	1024
P <sub>n</sub>	3	n	-	-	
CL# <sub>x</sub>	-	-	-	-	-

configurations and variants for desired case depending on the number of basic and optional components.

### CONCLUSION

The topic on assembly systems complexity has become widely used in research communities. Complexity aspects of assembly systems account for most of the complexity of the product itself. Assembly systems complexity issues can be further divided into structural complexity (such as subassembly cells, final assembly cell and their relations) and assembly sequence complexity (Svensson and Barfod, 2002; Fujimoto *et al.*, 2003).

As it was outlined in introduction section of this study, the focus was to explore so called product complexity. Product complexity refers to the increasing number of product options and encompasses those aspects that relate directly to each component of the product. The main contribution of this study can be summarized as follows:

- In order to specify all possible component-based configurations of assembly models suitable for mass customization manufacturing concepts, it was recommended to use the non-modular ASC structure. The reason for that was minimal configuration complexity level of such structure compared to other possible configurations containing the same number of initial nodes

- A new modeling framework for creation of all possible product variants based on unlimited number of optional components has been developed. The combinatorial determination of such product variants follows the integer sequence A000244. The all possible product variants then directly reflect complexity level of assembly process

The proposed approach to determine all possible product variants based on product variety options could be adopted in decision-making process when product variety level is specified.

### ACKNOWLEDGEMENTS

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