Research Journal of Applied Sciences 10 (2): 40-48, 2015
ISSN: 1815-932X

Mechanical-Mathematical Model of Mantle Diapirism

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Abstract: The analysis of geophysical and geological data on tectonic processes occurring in the peripheral layers of the Earth, under the influence of local heated mantle material elevations from the lower mantle is conducted. The assumption that there is a difference between the densities of substances originating from the lower mantle and substances of the overlying asthenospheric layer is used. Here, this process is considered as a process of hydrodynamic instability. Physical model of high-viscosity liquids at low Reynolds numbers is used to describe the process. Based on analysis data in the literature on the mantle and salt diapirism, a mechanical-mathematical model of mantle material elevation process through the density difference between the overlying and underlying layers is proposed. An analytical solution of a mathematical problem resulting from application of mechanical-mathematical modeling of mantle diapirism is obtained. Comparison of the results of the analytical solution shows good agreement with the data obtained from the experiments and observations of the salt dome.

Key words: Mechanical-mathematical model, mantle diapirism, observations, mantle diapirism, high-viscosity

INTRODUCTION

In the study of the Earth one important task is to determine the dependence of the processes and phenomena observed at the Earth's surface on the processes occurring in the deep interior of the Earth. This task is important and its solution is relevant when studying the structure and development of the Earth's crust.

According to the geophysical and geological studies, there is a weak (asthenospheric) layer, bounded above by the solid lithosphere and below by a solid mesosphere (Hain, 2003; Belousov, 1991; Dobretsov *et al*., 2001; Yerzhanov, 1964; Walcott, 1970; Ranalli, 1993; Bills *et al*., 1994; De Bremacher, 1977). And, here arises the problem of how to take into account the effect of the lower regions of the Earth to the movements and processes in the lithosphere and asthenosphere layer and how this affects the earth's surface.

The limited amount of information about the physical properties of substances of the underlying mantle and the processes taking place in it, forces us to make some assumptions.

In recent studies (Kropotkin, 1996; Puscharovskiy and Melanholina, 1992; Harper, 1978; Manglik *et al*., 1995; Lopez, 1991; Tychkov *et al*., 1999; Nakado and Takeda, 1995), the researchers mostly suggest the existence of local elevations of heated light materials from the lower mantle which is the source of as the no spheric substance movement both in the vertical direction and its horizontal spread. This approach is also used to explain the origin of the so-called "hot spots" (Lopez, 1991; Tychkov *et al*., 1999). It was observed that in addition to the mid-ocean ridges, island arcs and active continental margins, intense magmatism is also seen in some areas within the plates, a typical example of this Hawaii (Kropotkin, 1996; Puscharovskiy and Melanholina, 1992; Harper, 1978; Manglik *et al*., 1995; Lopez, 1991; Tychkov *et al*., 1999; Nakado and Takeda, 1995). Such anomalous areas (hot spots) in the mid-ocean ridges are present in Iceland and the Azores. According to the theory of plate tectonics "hot spots" are associated with streams emanating from the hot depths of the Earth's mantle, probably due to high intensity of magmatic material stream and tecto no sphere permeability.

From the above, one can assume that there exist local uplift and subsidence of subasthenospheric base. As

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noted above, here the task of determining the analytical form of these uplifts and subsidence is formulated, i.e., it is required to determine the form of the function describing the change of the subasthenospheric base.

So, the following problem is put forth here: how to determine the effect of active processes in the underlying mantle on the asthenosphere and lithosphere on the process of terrestrial structure formation? In other words, it is necessary to define the boundary conditions at the boundary of the asthenosphere and mesosphere, ieon the subasthenospheric base.

In the literature, about the Earth different hypotheses and assumptions about the mechanisms of interaction between as the no spheric layer with the underlying mantle are considered (Hain, 2003; Belousov, 1991; Dobretsov *et al*., 2001; Yerzhanov, 1964). In most cases, the tectonic activity is associated with the elevation of strongly heated molten mantle material and this is seen as the main cause of many tectonic processes. As noted above, mantle substances are deposited into the asthenosphere through restricted areas of sub as the no spheric base. Depending on the intensity of the processes occurring in the underlying mantle and tecto no sphere permeability, various options of the interaction mechanism of asthenosphere with the underlying layer are possible. These include the mechanism of mantle diapirism, advection, the appearance of the plume and others.

Therefore for the solution of this problem, you can consider various boundary conditions on the sub as the no spheric basis, arising from the assumptions and hypotheses about the mechanisms of these processes.

One of the mechanisms of mantle substance elevation and their penetration into the asthenosphere suggests that mantle substances accumulate in the bottom of the asthenosphere, spreading horizontally, thus forming a new layer of high-viscosity liquids. In this case, movements in asthenospheric layer should be modeled as the movement of double-layered liquid with the density of the underlying layer less than the density of the overlying layer. Obviously, the problem of hydrodynamic instability in double-layered high-viscosity liquids arises. There may in turn, be possible following options: when the supply of new mantle material from the lower layer continues and when such a “make-up” stops.

In the case, when the amount of accumulated mass of mantle material is significant or when the “makeup” with new arrivals of this substance from the lower mantle continues, elevation of the lower layer substances and dropping of heavier upper layer substances occur. As a resultof this process, the top of the boundary between these layers may reach the upper boundary of the as the no sphere, i.e., the base of the lithosphere which can lead to major faults, rift zones in the body of the lithosphere, accompanied by events such as volcanism, magmatism, seismic activity and etc.

Another possible option is when the intensity of these processes is insufficient or tecto no sphere permeability by the rising mantle material is insignificant, then at some level elevation of the boundary between layers may stop. Incomplete convection occurs. Such a phenomenon in geotectonics is called “advection”. With such a mechanism different processes that affect the movement in the asthenosphere and lithosphere are also possible.

In the case, when a limited amount of mantle material gets into the asthenosphere and the process of receiving them stops, then there may occur appearance of the so-called plume. Plume is a single solid mass of mantle material, bounded on all sides by asthenospheric substances. Because of the difference in density between the plume and the environment this body moves up, creating certain conditions for the emergence of different specific processes. Plume is detected by geophysical methods as in tectonically active regions, so under relatively stable continental plates. The study of the plume became relevant due to the study of tectonically active regions such as rift zones, in particular, the Baikal rift zone (Dobretsov *et al*., 2001; Tychkov *et al*., 1999).

The above-described mechanisms of mantle material penetration into the asthenosphere are possible in cases where they are not mixed with the material of the asthenosphere.

If mantle substances flowing into the asthenosphere from the lower mantle are mixed with the material of the asthenosphere, the process will have a completely different look. Then the problem of asthenospheric motions will be formulated another way. In this case, the motion will be considered in the viscous layer when in some local area on its lower boundary the rate of elevation or subsidence (or flow rate) is given. And, there are various options: either this is a continuous process when for a long period of geologic time there is “metabolism” between the asthenosphere and the underlying mantle (convective mechanism) or this process is associated with short-term “release” of mantle material into the asthenosphere (“pulsing” mechanism).

In contrast to traditional views of the convective motions in the mantle (D.P. McKenzie, J.M. Roberts, N.O. Weiss, Nakada Masao, A. Grigoriev, T.L. Tolkunova, etc.), Erzhanov Zh.S proposed to consider the model which assumes the existence beneath the asthenosphere of “sinks” and “sources” of mantle materials. This mechanism of mantle material flows would allow the description of interesting phenomena such as moving apart (spreading) and subduction of lithospheric plates. Interest in this issue is due to the ever-increasing expression of interest in the investigation of ocean margins (Yu.M. Puscharovsky, E.N. Melankholina, Peyve A.V., S.V. Ruzhentsev, etc.) and mid-ocean ridges. The solution to such problems would also be useful to describe lito spheric plate tectonics which would make it possible to assess many of the allegations of the continental drift hypothesis.

A brief overview of possible mechanisms of tectonic movements under the influence of endogenous processes, estimated in various geological hypotheses is conducted. It showed the wide range of problems arising during the solution of this problem, especially problems of continuum mechanics.

Later in this study will be considered the issues of formulation of the boundary conditions on problems of the interaction of asthenosphere with the underlying mantle for some options of the above mechanisms.

PHYSICAL ESSENCE OF THE HYDRODYNAMIC IN STABILITY MECHANISM

Now the question is: how does the process of mutual penetration of liquid layers with higher dynamic viscosity coefficients take place and when the density of the lower layer is less than the density of the top layer? Below is a description of the process occurring at the boundary of these two layers of high-viscosity liquids.

Let's consider a Cartesian coordinate system xoz (Fig. 1) and at some initial time in the neighborhood of a point (for definiteness with x = 0) at the boundary between the layers of liquid an equilibrium violation happened. This means that a function ξ (x, t) that describes this boundary, obtains an increment in the neighborhood of the point x = 0, i.e., with $\Delta x>0$ the rate of change of the function is $\Delta\xi = \xi(\Delta x, t)-\xi(0, t)$. Then the pressure difference created in the lower layer $\Delta p = p_1(\Delta x, t)-p_1(0, t)$.

Because of the small thickness of the liquid layer it was assumed that the dimensionless pressure in the lower layer is given by Eq. 1 (Lavrentiev and Shabat, 1972):

$$p_1 = \frac{\rho_1 - \rho_2}{\rho_1} \times \xi_1 - z + \frac{\rho_2}{\rho_1} \times \xi_2 \qquad (1)$$

Where:
ξ_1 = ξ_1 (x, t)
ξ_2 = Yet considered a constant

In this case, the pressure drop is equal to $\Delta p = (\rho_1-\rho_2)\times[\xi_1(\Delta x, t)-\xi^1(0, t)]$. If we assume that the function ξ (x, t) reaches its maximumat x = 0, then with a positive value of the argument increment ($\Delta x>0$) function increment is equal to $\Delta\xi_1 = \xi_1(\Delta x, t)-\xi_1(0, t)$.

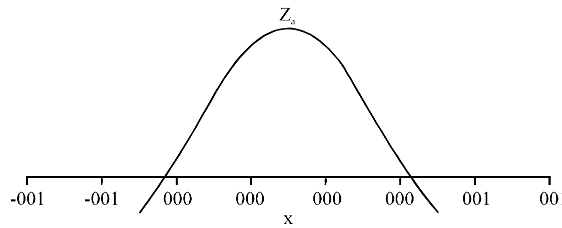

Fig. 1: Graph of the function $Z_a(x, t)$ when n = 1

It follows that with $\rho_1>\rho_2$ the pressure drop $\Delta p<0$ for the area x>0. Because of such a negative pressure differential, movement occurs in the lower layer of the liquid the horizontal velocity of which is directed from the center of the boundary elevation (x = 0) toward its lowered areas, i.e., flow of liquid occurs to restore equilibrium condition. This process is well studied and its mechanism is largely understood. Results of the solution of a similar problem are used to describe the elevation of Earth's crust after deglaciation in particular, post-glacial uplift of Fennoscandia (Walcott, 1970).

Now suppose $\rho_1<\rho_2$, then for the area x>0 pressure drop is $\Delta p>0$. This means that due to the positive pressure drop, movement in the opposite direction occurs, i.e., substances of the lower layer of the nearby area move to the center of boundary elevation (x = 0) and the top layer substances that are above the area of boundary elevation between the layers are moved to the areas of its descent.

In the points of elevation sufficiently remote from the center, the pressure drop is virtually zero, hence, the horizontal velocity of the liquid there must be negligibly small. Therefore, rising bottom layer substances are compensated by substances close to the center of the elevationof areas and there occurs lowering of the boundary $z = \xi_1(x, t)$; these areas will be filled with substances of the upper layer. This process in turn, increases the pressure drop that accelerates the elevation of substances at the elevation center neighborhood points.

Lowering of the more dense materials of the upper layer in the center of the considered region prevents mass transfer from the outermost regions. This ensures the locality of substance elevation of the lower layer. Increasing the pressure drop can cause, under favorable conditions, the appearance of new local substance elevations of the lower layer in the areas situated at a sufficient distance from the considered local boundary elevation between layers.

STATEMENT OF THE PROBLEM OF DETERMINING THE BOUNDARIES BETWEEN VISCUOUS LAYERS

The above shows the physical nature of the studied process occurring at the interface of two layers with different densities. For a quantitative analysis of this process you should try to get the boundary variation law in the form of an analytical formula. For this, mathematical problems resulting from mechanical-mathematical modeling of the process described here, must beformulated and solved.

Obviously, the mathematical problems that will be posed and solved here are related to partial differential equations. To search for partial solutions of these equations as well as for the analysis of the results, initial and boundary conditions should be given. In contrast to conventional problems of mathematical physics, there are some features in the formulation of these problems as well as in the formulation of the conditions for their solutions.

As noted earlier, the process of lower layer substanceelevation andof upper layer substance lowering of the highly viscous liquids is very slow and long. Therefore, there could be considered the problem "without initial conditions" (Tikhonov and Samarskii, 1977). It is assumed that at $t \to -\infty$ the boundary between the layers of the liquid was in the original equilibrium and the slow rise of a small section of the border due to violations of the equilibrium state began. Then the problem should be considered in the time interval $-\infty<t<t_1$ where t_1 the times corresponding for the top of the boundary to reach the upper boundary of the upper layer. After this point, there is a "gap" of the upper layer and a violation of the continuity conditions which was expected in the problem statement.

Here, it is advisable to turn to the physical nature of the problem. In this task, the motion of the liquid itself and the boundary change between the layers will be free. The impact of all factors other than gravity and viscosity is neglected. Movements are due to density differences of the layers. Then, we can consider the following statement to hold: during the free movement of heavy incompressible highly viscous liquids in the gravitational field under the influence of the density difference of its layers, a change in their boundaries at any point leads to corresponding changes in its other locations.

From this statement, it follows that for the solution of the problem of theboundary change between the layers of the liquid, it is enough to consider its position in only one characteristic point. This means that you can record (measure) values of peak elevation height (for $x = 0$) of local elevation for any point in time. For example for $t = 0$ (the starting time) or $t = 1$ (end of the reporting period). Note that the choice of points in time is conditional.

This approach is useful for practice. In most cases, during the study of the processes occurring in the Earth's crust the data aredefined for the last periods or from situation in real time. Then, the task of determining them in earlier geological periods is placed. This approach will be used further to solve partial problems.

Now, it is necessary to consider the boundary conditions. As has been suggested, a separate local boundary elevation between layers of liquid is considered. Because of the isotropy of the considered layers, position of the boundary layer is symmetric about the vertical z axis. This allows you to limit the consideration of semi-infinite region $0 \le x + \infty$, considering the point $x = 0$ to be the elevation center. Then the function $\xi(x, t)$ that determines the boundary under consideration, satisfies the condition $\xi(-x, t) = \xi(x, t)$. Since, at $x = 0$ the function $\xi(x, t)$ reaches its maximum you can use the following boundary condition here:

$$\frac{\partial \zeta(x,t)}{\partial x} = 0$$

On the right boundary of the interval $[0, \infty]$ the condition that the required function's $\xi(\infty, t) = 0$ equality to zero is given.

It should be noted that the analytical solution of this problem in an infinite point condition does not cause any problems. However, the numerical solution of the problem with this boundary condition causes some difficulty. In many cases, the condition at infinity is replaced by the condition at the end point $x = x_N$ for which and also also for values $x>x_N$, the required function has sufficiently small values or its derivative can be considered = 0.

DETERMINING THE UPPER DOME-SHAPED BOUNDARY OF THE ELEVATING MANTLE FLOW

Suppose, it is assumed that a certain layer of less dense mantle material appeared at the bottom of the asthenospheric layer of the local area. The substances of the two layers do not mix. This raises the problem of determining the boundary between these layers when the density of the lower layer substance is less than the density of the top layer. Attention is drawn to the fact that this boundary is moving.

Problem that is stated in the previous paragraph, about the definition of an analytic function describing the change of subasthenospheric base can be solved in two ways. In one case, you need to consider the problem of hydrodynamic instability when the underlying layer of liquid has a lower density than the density of the overlying layer of liquid. Another way to determine the required function can be an approximation of the available experiment data and observations of such phenomena.

The second method of determining the standard form of the desired function on the basis of data analysis on the phenomenon known as the salt strata elevation process and the formation of salt domes is considered initially. In geology, the process is called salt diapirism (Harbaugh and Bonham-Carter, 1970; Ramberg, 1985). A great deal of research has been devoted to the problem of studying the salt strata and its relevance is undeniable. It is primarily concerned with the exploration and production of oil and gas. In this study, these issues are not directly addressed. However, the research results of these processes performed by different researchers of thick salt tectonics and published in the press are used. The reason for this is the analogy between the process of salt diapirism and mantle diapirism, assumed as one of the causes of tectonic movements in the "lithosphere-asthenosphere" system.

According to popular opinion among geologists about the appearance of the salt domes (Harbaugh and Bonham-Carter, 1970), the elevation mechanism of salt whose density is less than the density of the rock surrounding it is similar to mantle diapirism when molten light mantle substances elevate. There are experimental studies, in particular, Ramberg (1985) which show the shape of the border of rising matter. And Howard (Harbaugh and Bonham-Carter, 1970) shows a graphical view of the boundary using computer simulation of the results of salt strata elevation observations (Fig. 1).

Comparison of the results obtained by Harbaugh and Bonham-Carter (1970) and Ramberg (1985) shows that the graph of the function shown in Fig. 1, agrees well with the results of experiments conducted by Ramberg. However, this chart shows only the general graphical form of the border. It is impossible to determine its analytical formula and to describe the dynamics of the process, i.e., change of the required function at time t using the graph. Despite this, we can accept this function graphas an approximating curve of the Ramberg experimental data and salt dome elevation observations. It should be noted that Howard noted a number of properties of the function that can be used to construct an analytic formula of this graph.

Let $z = Z_a(x, t)$ denote the required function. Time t will be considered a parameter for now and the dependence of the required function on this parameter will be is unknown for now. Because a dependence on t can not be set using the Howard and Ramberg graphical data.

It should be noted that we are considering some single (local) elevation of lower layersubstance. Because of the strong viscosity of the layer, it is assumed that another elevation is placed a sufficient distance from the first.

This raises the problem of determining an analytical formula that describes the border between the rising liquid and its environment shown in Fig. 1 where the z is vertical, x is horizontal axis, t-time.

Based on the analysis conducted by Harbaugh and Bonham-Carter (1970) and Ramberg (1985), it is first necessary to formulate the conditions that must satisfy the required function $z = Z_a(x, t)$, i.e., formulate its basic properties. They are:

- The function $z = Z_a(x, t)$ must have a point of local maximum (minimum). This means that the material of the lower layer rises up through some relatively small "channels". Without loss of generality we can assume that the local maximum (minimum) is attained at $x = 0$ and can be called the center of elevation (lowering)
- The function $z = Z_a(x, t)$ must be defined and continuous everywhere $x \in (-\infty +\infty)$
- The function $z = Z_a(x, t)$ is even and symmetrical with respect to axis z, i.e., condition $Z_a(-x, t) = Z_a(x, t)$ holds
- At points far enough away from the center of elevation ($x = 0$), the function $Z_a(x, t)$ has very small values, i.e. as $x \to +\infty$ and $x \to -\infty$ limit $\lim Z_a(x, t) = 0$
- From the conservation of mass (volume) of the substances follows the condition of the equality to zero of the following integrals:

$$\int_{-\infty}^{+\infty} Z_a(x,t)\,dx = \int_0^{+\infty} Z_a(x,t)\,dx = \int_{-\infty}^{0} Z_a(x,t)\,dx = 0$$

- Areas of elevation and subsidence are equal, i.e.:

$$\left|\int_{-x_0}^{+x_0} Z_a(x,t)\,dx\right| = 2\left|\int_0^{+\infty} Z_a(x,t)\,dx\right|$$

where, $x = \pm x_0$- the points of intersection of the graph of a function with the horizontal axis, i.e., $Z_a(\pm x_0, t) = 0$. All the above mentioned properties of the function are related to the kinematics of the problem of a slow elevation of light substances up and heavy down.

General view of the required function $Z_a(x, t)$ which satisfies all the above conditions can be written as follows:

$$Z_a(x, t) = \psi(t) \times \exp(-\varphi(t) \times x^2) \times [1 - b(t) \times x^{2n}] \quad (2)$$

Where:

$\psi(t)$, $\varphi(t)$, $b(t)$ = Unknown function of time for now

n = An integer (n = 1, 2, 3,...) and also $\varphi(t)>0$

Function (Eq. 1) is valid for local elevation. If we consider a local subsidence, the sign in front of it will be negative.

From the condition of the integrals' equality to zero (condition 5), you can get the relationship between the functions $\varphi(t)$, $b(t)$ and parameterm n:

$$b(t) = \frac{[2\,\varphi(t)]^n}{1\times 3\times 5\ldots\times(2\,n-1)} \tag{3}$$

Then Eq. 2 may have a different view:

$$Z_a(x,t) = \psi(t)\times \exp[-\varphi(t)\times x^2]\times \{1-\frac{[2\times\varphi(t)\times x^2]^n}{1\times 3\times 5\cdots\times(2\times n-1)}\} \tag{4}$$

and the required function will depend on two unknown functions $\psi(t)$, $\varphi(t)$ as well as on the parameter n.

Function $\psi(t)$ determines the law of motion of the elevation center (at x = 0) and the elevation amplitude of the top of itsgraph. If we consider the elevation, then $\psi(t)>0$, conversely, if subsidence, then $\psi(t)<0$.

From Eq. 1 it follows that the null value of the function $Z_a(x, t)$ is reached at two points $-x_0$ and $+x_0$ symmetrically located with respect to the ordinate:

$$10 - x_0 = \frac{[1\times 3\times 5\cdots\cdot(2n-1)]^n}{\sqrt{2\times\varphi(t)}} \tag{5}$$

This shows that the points $x = \pm x_0$ are "mobile", i.e., they depend on the value of the function $\varphi(t)$. The lower the value of $\varphi(t)$, the greater the value of x_0 and vice versa. This means that the function $\varphi(t)$ describes the "width" of the function $Z_a(x, t)$ graph in the horizontal direction.

The above means that there is a relationship between functions $\psi(t)$ and $\varphi(t)$ that must be determined from the dynamic conditions of the problem of hydrodynamic instability of high-viscosity liquids, when the lower layer has a lower density than the density of the upper layer.

SOME PROPERTIES OF THE FUNCTION $Z_a(x, t)$ FOR THE CASE n = 1

Let the parameter n = 1. In this case, the first function $Z_a(x, t)$ of the family (Eq. 4) will have the form:

$$Z_a(x,t) = \psi(t)\times \exp[-\varphi(t)\times x^2]\times [1-2\times\varphi(t)\times x^2] \tag{6}$$

The first partial derivative of this function with respect to time t is defined as:

$$\frac{\partial Z_a}{\partial t} = \exp[-\varphi(t0\times x^2)]\times\{[\psi'(t)-\psi(t)\times\varphi'(t)\times x^2]\times [1-2\times\varphi(t)\times x^2]-2\times\psi(t)\cdot\varphi'(t)\times x^2\} \tag{7}$$

Now, you need to find it first and second derivatives with respect to x:

$$\frac{\partial Z_a}{\partial x} = 2\times\psi(t)\times\varphi(t)\times\exp[-\varphi(t)\times x^2]\times [2\times\varphi(t)\times x^2 - 3\times x];$$

We can show that this function $Z_a(x, t)$ satisfies the following properties.

Property 1: At the point $x_1 = 0$ maximum of the function $Z_z(x, t)$ is reached. Here, its maximum value is equal to max $Z_a(0, t) = \psi(t)$.

Indeed, the condition of the maximum at this point aresatisfied. In fact, at this point, its first derivative with respect to x is zero (a necessary condition) and the second derivative is negative if $\psi(t)>0$ (a sufficient condition).

Property 2: Similarly, we can show that at the points $x_{2,3} = \pm\sqrt{1.5\times\varphi(t)}$ the minimum of the function min $Z_a(x, t) = 0.431\times\psi(t)$ is reached.

Property 3: Function $Z_a(x, t)$ is zero at two points:

$$x_{4,5} = \pm\frac{1}{\sqrt{2\times\varphi(t)}} \approx \pm\frac{0.71}{\sqrt{\varphi(t)}}$$

Property 4: Function $Z_{x,t}$ has four inflection points:

$$x_{6,7} = \pm\sqrt{\frac{3+\sqrt{6}}{2\times\varphi(t)}} \approx \pm\frac{1.65}{\sqrt{\varphi(t)}};$$

$$x_{8,9} = \pm\sqrt{\frac{3-\sqrt{6}}{2\times\varphi(t)}} \approx \pm\frac{0.5246}{\sqrt{\varphi(t)}}$$

Thus, the graph of the function is characterized by nine points. Coordinates of these points are dependent on the time t and their values change simultaneously with time (Fig. 1).

ANALYTICAL SOLUTION OF THE PROBLEM

Based on the analysis of Harbaugh and Bonham-Carter (1970) and Ramberg (1985) research results, a form of the function $Z_a(x, t)$ which describes closely enough the boundary between rising (light) substances and heavier upper substances has been obtained. However, the results of these studies can not take into account the dynamics of the process, so the variable t in the function $Z_a(x, t)$ played the role of some parameter. Meanwhile, the dependence of the function on time is not explicitly defined. Consequently, the function $Z_a(x, t)$ is defined, depending on the unknown functions $\varphi(t)$ and $\psi(t)$. These functions can determine the dynamics of the process at hand but the published information does not reflect the nature of these functions. To determine the unknown functions $\varphi(t)$ and $\psi(t)$, it is required to formulate and solve the problem of hydrodynamic stability in double-layered high-viscosity liquids when the density of the lower layer is less than the density of the top layer.

Let's consider two layers of high-viscosity liquids for which the Reynolds numbers will be small. It is assumed that the density ρ_1 of the lower layer is less than the density ρ_2 of the top layer. The upper surface of the top layer is considered free and the bottom (base) of the lower layer is considered a stationary surface. On the boundary surface between the layers the conditions of continuity and equality of speed hold.

For these layers recognition of the "shallow water" is assumed when the amplitudes of local elevation and subsidence of layer boundaries are comparable to their average thickness (vertical dimension) and they are small compared to the horizontal dimensions. Any violation of the equilibrium state at the boundary between the layers is the cause of motion in the layers. The movement in these layers is due to the difference of densities of layers. In Kuralbayev (2005), a system of differential equations for the boundary surfaces is obtained: $z = \xi_1(x, t)$ the boundary between the layers; $z = \xi_2(x, t)$ the free surface of the upper layer. It represents a system of two parabolic-type partial differential equations of second order:

$$\frac{\partial \xi_1}{\partial t} = -a_1^2 \times \frac{\partial^2 \xi_1}{\partial x^2} + a_3^2 \frac{\partial^2 \xi_2}{\partial x^2}, \quad \frac{\partial \xi_2}{\partial t} = -\frac{a_1^2}{2} \times (3h_2 - h_1) \times \frac{\partial^2 \xi_1}{\partial x} + a_2^2 \times \frac{\partial^2 \xi_2}{\partial x^2} \tag{9}$$

Here, we use the following notation of the constants:

$$a_1^2 = \frac{ER}{3} \times \frac{\rho_2 - \rho_1}{\rho_1} \times h_1^3, \quad a_2^2 = \frac{ER}{3} \times \frac{\rho_2}{\rho_1} \times [h_2^3 + (h_2 - h_1)^3 \times (\frac{\eta_1}{\eta_2} - 1)], \quad a_3^2 = \frac{ER}{6} \times \frac{\rho_2}{\rho_1} \times (3h_2 - h_1) \times h_1^2 \tag{10}$$

Where:

h_1	=	The dimensionless initial capacity (thickness) of the lower layer
h_2-h_1	=	Thickness of the upper layer
η_1, η_2	=	Dynamic coefficients of viscosity of the lower and upper layers, respectively
ER	=	$\rho_1 g H^3/\eta_1 UL$ a dimensionless parameter (Erzhanov's number)
U, H, L	=	Adopted characteristic values, speed, thickness and the horizontal size, respectively
g	=	Acceleration of gravity

In this research, an analytical solution of this system of equations which has the following form:

$$\xi_1(x,t) = \frac{A}{\sqrt{(B^2 - 4 \times a^2 \times t)^3}} \times \exp \left(-\frac{x^2}{B^2 - 4 \times a^2 \times t} \right) \times \left(1 - \frac{2 \times x^2}{B^2 - 4 \times a^2 \times t} \right) \tag{11}$$

In the given Eq. 11, a^2 constant parameter that depends on the properties of the considered layers of high-viscosity liquids. It is defined by the following (Kuralbayev, 2005):

$$a^2 = \frac{a_1^2 - a_2^{2^2} + \sqrt{(a_1^2 + a_2^2)^2 - 2a_1^2 a_3^2 (3h_2 - h_1)}}{2} \tag{12}$$

This solution is written in general form; it depends on the unknown constants and integration of A and B. It is easily seen that function (Eq. 10) satisfies all the properties of the function $Z_a(x, t)$ described in the preceding paragraph. Therefore, the following equation: $Z_a(x, t) = \xi_1(x, t)$ can be written within the accuracy of a constant value. Satisfaction of this equation allows to determine the functions $\varphi(t)$ and $\psi(t)$:

$$\varphi(t) = \frac{1}{B^2 - 4 \times a^2 \times t}, \quad \psi(t) = \frac{A}{\sqrt{(B^2 - 4 \times a^2 \times t)^3}} \tag{13}$$

Thus, the solution to the problem of determining the form of the function describing the elevation of light mantle substances under the influence of density differences arising from the high temperatures in the lower mantle has been obtained.

The analysis of the solution to the problem leads to the following conclusions: With an increase in the relative density difference of considered layers ρ_2-ρ_1/ρ_1 elevating bottom layer substances will actively influence the movement of the upper layer and contribute to elevation of the free surface of the upper layer. Depending on the values of this magnitude, the amplitude of the free surface elevation comprised from about 3-10% of the amplitude of the boundary surface between the layers.

By increasing the ratio of the dynamic viscosity of the lower layer to the dynamic coefficient of the top layer η_1/η_2 (at a low viscosity of the top layer), the substances of the upper layer have time to spread in the horizontal direction and the free surface of the layer will change in significantly.

The resulting function $Z_a(x, t) = \xi_1(x, t)$ defines the behavior of local sub as the no spheric border elevations as a result of mantle material elevation from the deep interior of the Earth in the "initial period" of mantle diapirism.

CONCLUSION

In this study, the solution has been performed to the problem related to determination of the upper boundary of the ascending mantle flow changing process under the influence of density differences which led to the hydrodynamic instability in the asthenosphere layer of the Earth. From the analysis of the information available in the geological and geophysical literature (Hain, 2003; Belousov, 1991; Dobretsov *et al*., 2001; Yerzhanov and Rock, 1964; Walcott, 1970b; Ranalli, 1993; Bills *et al*., 1994; De Bremacher, 1977; Kropotkin, 1996; Puscharovskiy and Melanholina, 1992; Harper, 1978; Manglik *et al*., 1995; Lopez, 1991; Tychkov *et al*., 1999; Nakado and Takeda, 1995; Nalpas and Brem, 1993; Harbaugh and Bonham-Carter, 1970; Ramberg, 1985), the basic properties of the function $Z_a(x, t)$ that describes the process in question have been established. Analytical solution of a mathematical problem, obtained by mechanical-mathematical modeling, allowed us to obtain a function whose properties completely correspond to the physical nature of the process of hydrodynamic instability in double-layered high-viscosity liquids. Function obtained from the analytical solution of this problemis in fairly good agreement with the results published by Harbaugh and Bonham-Carter (1970) and Ramberg (1985).

The practical value of the function obtained here is that it describes quite well the kinematics of the process of emergence and evolution of salt domes or mantle diapirism. The use of such a function may be useful in the study of the dynamics of such processes. Function can be used to describe both the individual local dome and to describe the ascending mantle flow beneath rifts and mid-ocean ridges. The possibility of determining the function through some characteristic points facilitates the measurement of modern movements of the earth's crust in the areas of elevation or subsidence of the earth's surface. The proposed function is useful for approximating the results of observations or experimental data in the study of the hydrodynamic instability phenomenon. It can also be used to explain the salt dome elevation mechanism.

REFERENCES

Belousov, V.V., 1991. Tectonosphere of the Earth: Interaction of Upper Mantle and Crust. Nedra, Russia, Pages: 428.

Bills, B.G., D.R. Gurrey and G.A. Marshall, 1994. Viscosity estimates for the crust and upper mantle from patterns of lacustire shoreline deformation in the Eastern Great Basin. J. Geophys. Res., 99: 46-58.

De Bremacher, J.C., 1977. Is the oceanic lithosphere elastic or viscous. J. Geophys. Res., 82: 234-245.

Dobretsov, N.L., A.G. Kirdyashkin and A.A. Kirdyashkin, 2001. Deep Geodynamics. GEO Publ., Novosibirsk, Russian, Pages: 409.

Hain, V.E., 2003. Main Problems of the Modern Geology. World Scientific Publishing, Russia, Pages: 348.

Harbaugh, J.W. and G. Bonham-Carter, 1970. Computer Simulation in Geology. John Wiley and Sons, New York, USA., Pages: 319.

Harper, J.F., 1978. Asthenosphere flow and plate motion. Geophys. Roy Astron. Soc., 5: 123-134.

Kropotkin, P.N., 1996. Tectonic stressesin the earth'scrust. Geotectonics, 2: 9-42.

Kuralbayev, Z.K., 2005. Model-based research of the effect of the uplift of mantle material on tectonosphere. Novosibirsk Technical University Scientific Herald, Moscow, Russia, pp: 37-49.

Lavrentiev, M.A. and B.V. Shabat, 1972. Problems of Hydrodynamics and their Mathematical Models. Nauka, Moscow, Russia, Pages: 416.

Lopez, D.E., 1991. Mantle plumes. Tectonophysics, 187: 373-384.

Manglik, A., A.O. Gliko and R.N. Singh, 1995. Movement of the lithosphere-asthenosphere interface in response to erosion of thickened continental lithosphere: A moving boundary approach. Geophys. J. Int., 122: 479-488.

Nakada, M. and Y. Takeda, 1995. Roles of mantle diapir and ductile lower crust on island-arc tectonics. Tectonophysics, 246: 147-161.

Nalpas, T. and J.P. Brem, 1993. Salt flowered diapirism related to extension at crystal scale. Tectonophysics, 228: 3-4.

Puscharovskiy, Y.M. and E.N. Melanholina, 1992. Tectonic Development of the Earth. World Scientific Publishing, Russia, Pages: 263.

Ramberg, H., 1985. Gravity and Deformation of the Crust. Nedra, Russia, Pages: 399.

Ranalli, G., 1993. Viscosity of the asthenosphere. Nature, 361: 211-211.

Tikhonov, A.N. and A.A. Samarskii, 1977. Equations of Mathematical Physics. Nauka, Moscow, Russia, Pages: 736.

Tychkov, S.A., A.N. Vasilevskiy and E.V. Rychkova, 1999. Evolution of the plume beneatch continental lithosphere with sharp thickness variations. Geol. Geophys., 40: 1182-1196.

Walcott, R.J., 1970. Flexural rigidity, thickness and viscosity of the lithosphere. J. Geophys. Res., 75: 3941-3954.

Yerzhanov, Z.S., 1964. Rock Flowage Theory and its Applications. World Scientific Publishing, Russia, Pages: 200.