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Stochastic Modeling of the Process of Solid Bodies Fracture

A.V. Koroliov, A.A. Koroliov, A.F. Balayev, B.M. Iznairov, O.V. Zakharov,
A.N. Vasin, O.P. Reshetnikova, S.A. Savran, A.S. Yakovishin,
K.S. Neygebauer and Ye V. Mukhina
Yuri Gagarin State Technical University, 410054 Saratov, Russia

Abstract: The defects of solid body materials occur in a random manner and are unevenly distributed within their volume that's why it should be supposed that fracture of solid bodies has a probabilistic nature. This is evidenced by the example of the brittle fracture theory according to which fracture occurs under the influence of microcracks propagation inside a material. There was offered a mathematical model of solid bodies fracture based on probabilistic approach. There were analyzed the conditions of cracks development with account of random character of their dimensions and distribution within the volume of a material. A condition of occurrence of a critical state of a solid body resulting in its fracture was determined. It was established that the nature of solid bodies fracture depends on expenditure of energy applied at time of fracture. Moreover, this relation has a random nature since internal energy of saturation of solid body material is a random variable. Creation of the said Probabilistic Mathematical Model of solid bodies fracture will allow to forecast strength characteristics of various products more accurately as well as to improve their reliability without necessity to ensure unreasonable assurance coefficient.

Key words: Stochastic modeling, brittle fracture, microcracks, elastic strain energy, fracture probability

INTRODUCTION

There exists a lot of works dedicated to the mechanism of materials fracture (Berezin and Poroshi, 2012; Mamayeva *et al*., 2008; Ostsemin and Utkin, 2009; Vladimirov, 1984). But the most of them consider the fracture process as a deterministic one. Actually exterior load on a material is concentrated in its defects and since the defects within the material volume are distributed unevenly the stresses occurring in the material defects are randomly distributed. Destruction begins from the major defects which in the process of destruction join into one large crack. That's why fracture of materials is a stochastic process and the process of fracture may be forecasted with a certain probability. Otherwise, it would be necessary to use unreasonably high assurance factors at time of strength calculation which results in significant financial and economic losses. A range of researchers (Berezin and Poroshi, 2012; Mamayeva *et al*., 2008) offered a probabilistic approach to analysis of the process of fracture but the dependences offered by them had empirical nature and did not take into account the mechanism of fracture.

Since, the defects of solid body materials occur in a random manner and are unevenly distributed within their volume it should be supposed that failure of solid bodies has a probabilistic nature. Let's demonstrate this by an example of the Griffith's theory of brittle fracture which states that failure occurs under the influence of microcracks propagation within a material. Let's carefully follow the Griffith's calculations but on the condition that there is a number of randomly located microcracks with random dimensions within the material instead of one. The micriocracks have flat round form with the radius l.

DESCRIPTION OF STOCHASTIC MODEL OF SOLID BODIES FRACTURE

According to the Griffith's statements let's analyze a rectangular plate under the action of tension force. Elastic strain energy of the plate is determined as:

$$W = k_\mu \sigma^2 V/(2E) \quad (1)$$

Where:

- σ = Tensile stress inside the plate
- E = Elastic modulus of the plate material
- V = Volume of the plate material
- $k\mu$ = Coefficient which makes allowance for a type of deformation

Corresponding Author: A.V. Koroliov, Yuri Gagarin State Technical University, 410054 Saratov, Russia

For plane stress condition $k\mu = 1$ for plane deformation condition $k\mu = 1-\mu^2$ where μ is the Poisson's ratio. In case of presence of cuts there can be observed local stress relief in the adjacent zones which results in decrease of the elastic strain energy accumulated in the plate by the value equivalent to:

$$W_l = (\pi k_\mu k_\theta \sigma^2 \sum l^3)/E \qquad (2)$$

where, k_θ is a coefficient which makes allowance for an angular position of microcracks with regard to the direction of action of the exterior load (given that equally possible angular position of microcracks $k_\theta = 2/\pi$). As it follows from Eq. 1 and 2 the released elastic energy is equal to:

$$W\text{-}W_l = k_\mu \sigma^2 abt/(2E)\text{-}\pi(\pi k_\mu k_\theta \sigma^2 \sum l^3)/E \qquad (3)$$

According to Griffith released elastic energy flows to the top of a cut where stress concentration occurred and there it is used for destruction, i.e., creation of a new surface of a body. The effort for creation of new surfaces is equal to:

$$U = 2\pi k_\theta \gamma_s \sum l^2 \qquad (4)$$

where, γ_s is an energy of a free surface of a material. Providing that the tip edges of the plate are motionless (no effort on the part of external forces) and that the cuts length increased by the small value of Δl then considering (Eq. 2) the value of released deformation energy will be equal to:

$$-\Delta W = -[W(l+\Delta l)-W(l)] = \pi k_\mu k_\theta \sigma^2 (\sum (l+\Delta l)^3 - \sum l^3)/E \approx (3\pi k_\mu k_\theta \sigma^2 \sum l^2 \Delta l)/E \qquad (5)$$

With respect to Eq. 4, the value of energy consumed for increase of the cut area will be equal to:

$$\Delta U = 2k_\theta \pi \gamma_s (\sum (l+\Delta l)^2 - \sum l^2) = 4\pi k_\theta \gamma_s \sum \Delta l \cdot l \qquad (6)$$

Here, two situations are possible according to Griffith:

- $\Delta W > \Delta U$: the quantity of released energy will be more than enough for material fracture at the cut top and the cut will begin to spontaneously lengthen up to division of the plate into two parts. In this case kinetic energy will be generated more and more intensively and the cut length will grow more and more quickly
- $\Delta W < \Delta U$: quantity of released energy will not be enough for the cut increase and the cut will remain fixed

Therefore, the following formula will be a condition for occurrence of the critical state when a cut will spontaneously lengthen: $-\Delta W = \Delta U$.

Let's determine the value of critical stress σ which may give rise to the beginning of microcracks development by equaling Eq. 5 and 6:

$$\sigma = (4\gamma_s E \sum l \cdot \Delta l)^{1/2} / (3 \cdot k_\mu \sum l^2 \cdot \Delta l)^{1/2} \qquad (7)$$

If the value of crack increase is proportional to its length then:

$$\sigma = (4\gamma_s E \sum l^2)^{1/2} / (3 \cdot k_\mu \sum l^3)^{1/2} \qquad (8)$$

As we can see Eq. 8 is similar to the equation obtained by Griffith for one crack. However, as contrasted with the dependency offered by Griffith this equation contains the amounts which depend on the number and distribution of microcracks in the volume of a material. And since the value l is a random one then the value of stress which initiates destruction of a materials will be also random.

If the law of distribution of the values l is known then it is possible to determine the nature and parameters of distribution of the values σ on the basis of Eq. 8. For example often technological break of materials is performed by use of creation of a stress concentrator serving as a microcrack. It's not difficult to establish that in this case given that the value l is distributed according to the normal probability law with the mathematical expectation l_0 and the variability s_l^2 the density of probability ofσ value distribution will have the following form:

$$f(\sigma) = (\lambda \cdot \sigma_0^2/\sigma^3) e^{-u_n(\mu\sigma_0^2/\sigma^2 - 1)} \qquad (9)$$

where, $u_n = l_0/(2^{1/2} s_l)$ and σ_0 is a mathematical expectation for the value σ equal to:

$$\sigma_0 = \eta (\gamma_s E)^{1/2} / (k_\mu \cdot l_0)^{1/2} \qquad (10)$$

Mean square deviation of the value σ is equal to $s_\sigma = \varsigma \sigma_0$. Dimensionless factors λ, μ, η and ς depend of the value u_n and are determined on the basis of Table 1.

As it can be seen from Table 1 the growth of the value of the dimensionless parameter u_n of distribution of random values of the microcrack l is accompanied by significant increase of the value of factor ς which

Table 1: Koroliov A.V. Stochastic modeling of the process of solid bodies failure

u_n	λ	μ	η	ς
2.0	1.98	0.88	1.06	0.10
2.4	2.49	0.92	1.04	0.11
2.8	2.99	0.94	1.03	0.13
3.2	3.46	0.96	1.02	0.16
3.6	3.93	0.97	1.02	0.21
4.0	4.40	0.98	1.01	0.30

characterizes the mean square deviation s_σ. The value of factor η and therefore, the value of mathematical expectation σ_0 almost does not depend on the value of u_n.

By analogy it is also possible to find the parameters of distribution of the value σ for other laws of distribution of the value l.

The Griffith's theory and all of the solutions obtained on its basis are true for brittle solids. This theory may be also applied for elastic and plastic bodies given that adjustments offered by Vladimirov (1984) $r_l >> r_0$ are used.

CONCLUSION

Therefore, the nature of fracture of solid bodies depends on the expenditure of energy applied at time of destruction. Moreover, this relation has a random nature since internal energy of saturation of solid body material is a random variable.

Creation of the probabilistic mathematical model of solid bodies fracture will allow to forecast strength characteristics of various products more accurately as well as to improve their reliability without necessity to ensure unreasonable assurance coefficient.

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