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Analysis of Three Frequency Undulator Intensity and Gain Due to off Axis Contribution in Free Electron Laser

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Abstract: In this study, researchers study the three frequency undulator radiation and free electron laser gain for higher harmonics with the inclusion of off axis contribution. When electron enters the field off axis it causes additional oscillations due to which intensity and gain reduction take place. To enhance the intensity and gain, researchers introduce a new scheme, i.e., the three frequency undulator scheme.

Key words: Free electron laser, undulator, scheme, gain, axis

INTRODUCTION

In a Free Electron Laser (FEL) (Colson and Freedman, 1983; Colson, 1981, 1977; Madey, 1971; Benson and Madey, 1989; Brau, 1990; Colson *et al*., 1985; Gupta and Mishra, 2002; Dattoli *et al*., 1992, 1993, 1995; Dattoli and Bucci, 2000; Mishra *et al*., 2003; Chouhan and Mishra, 2002), a relativistic beam of electrons passes through a periodic transverse magnetic field, to produce coherent radiation. The main advantage of the FEL is that it is tunable. In standard FEL electron passes through on-axis the undulator field.

When electron enters on axis the interaction region of undulators of a free electron laser device, it sees an on-axis field and executes small oscillations around the axis. However, this on-axis field does not satisfy Maxwell's equation. Hence, the off-axis fields are calculated from the Maxwell equations. With the presence of these off-axis fields, the electron executes additional oscillations known as betatron oscillations. These extra oscillations drive away electrons from the resonance and FEL gain drops. Both off-axis and angular injection of the electron induces betatron oscillations. When it enters the field off axis causes additional field components of the undulator field. And this extra additional oscillation causes the degradation in the intensity and gain free electron laser.

The two-frequency and two-harmonic undulator (Dattoli *et al*., 2002; Yang and Ding, 1998; Dattoli and Voykov, 1993; Asakawa *et al*., 1992; Ciocci *et al*., 2001, 1993; Iracane and Bamas, 1991), Optical Klystron Undulator (Schmitt and Elliott, 1987; Elleaume, 1986; Boscolo and Colson, 1985; Wang *et al*., 1990) are some examples which have attracted wide interests in this context. In this study, researchers analyze the case of a three frequency (Niculescu *et al*., 2008) undulator scheme to increase the intensity and gain of a free electron laser in the presence of betatron oscillations.

MATERIALS AND METHODS

Undulator radiation: Researchers assume the electron moves on axis in a three frequency undulator scheme whose on-axis field given by:

$$\vec{B} = \left[0,\ B_0 a_1 \left\{\sin(k_u z) + \frac{a_2}{a_1}\sin(2k_u z) + \frac{a_3}{a_1}\sin(3k_u z)\right\}\hat{y},\ 0\right] \quad (1)$$

Where:
k_u = $2\pi/\lambda_u$, λ_u is the undulator wavelength
B_0 = Peak field strength

The trajectory of the electron is determined through the Lorentz equation. This gives:

$$x(t) = -\left[\frac{Kc}{\gamma\omega_u}\sin(\omega_u t) + \frac{Kc\delta_1}{2\gamma\omega_u}\sin(2\omega_u t) + \frac{Kc\delta_2}{3\gamma\omega_u}\sin(3\omega_u t)\right] \quad (2)$$

Where:
K = $eB_0a_1/m_0c\omega_u$ and K defines the undulator parameter
δ_1 = $(a_2/2a_1)$
δ_2 = $(a_3/3a_1)\omega_u = k_uc$

The z-motion is:

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$$z(t) = \beta^* ct - \frac{cK^2}{8\gamma^2\omega_u}\sin(2\omega_u t) - \frac{cK^2\delta_1^2}{16\gamma^2\omega_u}\sin(4\omega_u t) - \frac{cK^2\delta_2^2}{24\gamma^2\omega_u}\sin(6\omega_u t) - \frac{cK^2\delta_2}{4\gamma^2\omega_u}\sin(2\omega_u t) - \frac{cK^2\delta_1}{6\gamma^2\omega_u}\sin(3\omega_u t) - \frac{cK^2\delta_2}{8\gamma^2\omega_u}\sin(4\omega_u t) - \frac{cK^2\delta_1\delta_2}{10\gamma^2\omega_u}\sin(5\omega_u t) - \frac{cK^2\delta_1(1+\delta_2)}{2\gamma^2\omega_u}\sin(\omega_u t) \quad (3)$$

Where:

$$\beta^* = 1 - \frac{1}{2\gamma^2}\left[1 + \frac{K^2}{2} + \frac{K^2\delta_1^2}{2} + \frac{K^2\delta_2^2}{2}\right]$$

The representation of the field in Eq. 1 is valid if the electron's path remains near to the undulator axis where there are no betatron oscillations. The dependence of the field on the transverse coordinates is found from the Maxwell equations. At the lowest order in transverse coordinates, the field components are (Dattoli *et al*., 1992, 1995):

$$\begin{aligned} B_x &= \frac{B_0}{2} xy\left[k_u{}^2\delta\{a_1\sin(k_u z)\} + k_h{}^2\sigma\{a_2\sin(k_h z)\}\right] \\ B_y &= B_0\left[1 + \frac{k_u{}^2}{4}\{\delta x^2 + (2-\delta)y^2\}\right]a_1\sin(k_u z) + B_0\left[1 + \frac{k_h{}^2}{4}\{\sigma x^2 + (2-\sigma)y^2\}\right]a_2\sin(k_h z) \\ B_z &= B_0 y\left[a_1 k_u\cos(k_u z) + a_2 k_h\cos(k_h z)\right] \end{aligned} \quad (4)$$

where, $\delta = 2\alpha^2/k_u^2$ and $\sigma = 2\alpha^2/k_h^2$. The equation of motion can now be derived using the field in Eq. 4. Researchers assume that the motion can be decomposed as:

$$x = x_0 + x_1$$

Where:

- x_0 = The reference trajectory due to the field
- x_1 = The additional motion around the reference trajectory due to the extra terms in Eq. 4 depending on the transverse coordinate

Keeping the contributions at the first order only in x_1 and averaging over one undulator period, researchers get the following differential equation ruling the additional motion:

$$\frac{d^2x_1}{dt^2} + \Omega_\beta^2 x_1 = 0 \quad (5)$$

Where:

$$\Omega_\beta^2 = \frac{K_1^2c^2k_u^2}{4\gamma^2}\left[\delta + h^2\sigma\right]$$

The solution of Eq. 5 is given by:

$$x_1(t) = x_0\cos(\Omega_\beta t) \quad (6)$$

where, x_0 represent the off-axis position from the undulator axis and $Y_0 = 0$. The z-motion is:

$$z(t) = \beta^{**} ct - \frac{cK^2}{8\gamma^2\omega_u}\sin(2\omega_u t) - \frac{cK^2\delta_1^2}{16\gamma^2\omega_u}\sin(4\omega_u t) - \frac{cK^2\delta_2^2}{24\gamma^2\omega_u}\sin(6\omega_u t) - \frac{cK^2\delta_2}{4\gamma^2\omega_u}\sin(2\omega_u t) - \frac{cK^2\delta_1}{6\gamma^2\omega_u}\sin(3\omega_u t) - \frac{cK^2\delta_2}{8\gamma^2\omega_u}\sin(4\omega_u t) - \frac{cK^2\delta_1\delta_2}{10\gamma^2\omega_u}\sin(5\omega_u t) - \frac{cK^2\delta_1(1+\delta_2)}{2\gamma^2\omega_u}\sin(\omega_u t) + \frac{x_1^2\Omega_{\beta_x}}{8c^2}\sin(2\Omega_{\beta_x} t) \pm \frac{Kx_1\Omega_{\beta_x}}{2\gamma c(\omega_u \pm \Omega_{\beta_x})}\cos(\omega_u \pm \Omega_{\beta_x})t \pm \frac{Kx_1\Omega_{\beta_x}\delta_1}{2\gamma c(2\omega_u \pm \Omega_{\beta_x})}\cos(2\omega_u \pm \Omega_{\beta_x})t \pm \frac{Kx_1\Omega_{\beta_x}\delta_2}{2\gamma c(3\omega_u \pm \Omega_{\beta_x})}\cos(3\omega_u \pm \Omega_{\beta_x})t \quad (7)$$

$$\beta^{**} = 1 - \frac{1}{2\gamma^2}\left[1 + \frac{K^2}{2} + \frac{K^2\delta_1^2}{2} + \frac{K^2\delta_2^2}{2} + \frac{x_1^2\Omega_{\beta_x}^2\gamma^2}{2c^2}\right]$$

The spectral properties of the undulator radiation are easily obtained from the Lienard-Wiechert integral (Jackson, 1975):

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2c}\left|\int_{-\infty}^{\infty}\{\hat{n}\times(\hat{n}\times\vec{\beta})\}\exp\left[i\omega\left(t - \frac{z}{c}\right)\right]dt\right|^2 \quad (8)$$

The brightness expression reduced to:

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2T^2}{4\pi^2c}\left|F_m(z)\right|^2\left(\frac{\sin\left(\frac{\nu_0}{2}\right)}{\left(\frac{\nu_0}{2}\right)}\right)^2 \quad (9)$$

Where:

$$F_m(z) = \Bigg[\frac{K}{2\gamma}\{J_{m-1}(0, z_1) + J_{m+1}(0, z_1)\} J_n(0, z_2) J_p(0, z_3) J_q(0, z_4) + \frac{K\delta_1}{2\gamma}\{J_{n-1}(0, z_2) + J_{n+1}(0, z_2)\} J_m(0, z_1) J_p(0, z_3) J_q(0, z_4) +$$
$$\frac{K\delta_2}{2\gamma}\{J_{p-1}(0, z_3) + J_{p+1}(0, z_3)\} J_m(0, z_1) J_n(0, z_2) J_q(0, z_4) + \frac{x_0\Omega_\beta}{2ic}\{J_{q+1}(0, z_4) - J_{q-1}(0, z_4)\} J_m(0, z_1) J_n(0, z_2) J_p(0, z_3)\Bigg] \times$$
$$J_r(z_5, z_6) J_s(z_7, 0) J_{p1}(z_8, 0) J_{p2}(z_9, 0) J_{p3}(z_{10}, 0) J_{p4}(z_{11}, 0) J_{p5}(z_{12}, 0)$$

$$\nu_0 = \frac{\omega}{2\gamma^2}\left[1 + \frac{K^2}{2} + \frac{K^2\delta_1^2}{2} + \frac{K^2\delta_2^2}{2} + \frac{x_1^2\Omega_{\beta_x}^2\gamma^2}{2c^2}\right] - \eta$$

$$\eta = m\omega_u + n2\omega_u + p3\omega_u t + q\Omega_{\beta_x} r\omega_u + s\frac{3}{2}\omega_u + p_1 2\omega_u + p_2\frac{5}{2}\omega_u + p_3(\omega_u \pm \Omega_{\beta_x}) + p_4(2\omega_u \pm \Omega_{\beta_x}) + p_5(3\omega_u \pm \Omega_{\beta_x})$$

With:

$$z_1 = -\frac{\omega K^2}{8\gamma^2\omega_u},\ z_2 = -\frac{\omega K^2\delta_1^2}{16\gamma^2\omega_u},\ z_3 = -\frac{\omega K^2\delta_2^2}{24\gamma^2\omega_u},\ z_4 = \frac{\omega x_1^2\Omega_{\beta_x}}{8c^2},\ z_5 = -\frac{\omega K^2\delta_1(1+\delta_2)}{2\gamma^2\omega_u},\ z_6 = -\frac{\omega K^2\delta_2}{4\gamma^2\omega_u},\ z_7 = -\frac{\omega K^2\delta_1}{6\gamma^2\omega_u},\ z_8 = -\frac{\omega K^2\delta_2}{8\gamma^2\omega_u},$$
$$z_9 = -\frac{\omega K^2\delta_1\delta_2}{10\gamma^2\omega_u},\ z_{10} = \pm\frac{Kx_1\Omega_{\beta_x}}{2\gamma c(\omega_u \pm \Omega_{\beta_x})},\ z_{11} = \pm\frac{Kx_1\Omega_{\beta_x}\delta_1}{2\gamma c(2\omega_u \pm \Omega_{\beta_x})},\ z_{12} = \pm\frac{Kx_1\Omega_{\beta_x}\delta_2}{2\gamma c(3\omega_u \pm \Omega_{\beta_x})}$$

$J_m(0, z_1)$,... are the generalized Bessel functions of order, i = m, n, p, q, r, s, p_1, p_2, p_3, p_4, p_5, respectively. The resonance condition in a free electron laser is provided by $\nu_0 = 0$. This provides the central emission frequency as:

$$\omega_i = \frac{2\gamma^2\eta}{\left[1 + \frac{K^2}{2} + \frac{K^2\delta_1^2}{2} + \frac{K^2\delta_2^2}{2} + \frac{x_1^2\Omega_{\beta_x}^2\gamma^2}{2c^2}\right]}$$

Gain: To calculate the gain, let us consider a linear polarized electromagnetic wave with:

$$\vec{E} = E_0\hat{x}\cos(kz-\omega t+\phi) \quad (10)$$

The change in energy of the electron is given by:

$$\frac{d\gamma}{d\tau} = \frac{eE_0K_1L_u}{m_0c^2\gamma}\cos(\zeta_0+\phi) J_q(0, z_4) J_r(z_5, z_6) J_s(z_7, 0) J_{p1}(z_8, 0) J_{p2}(z_9, 0) J_{p3}(z_{10}, 0) J_{p4}(z_{11}, 0) J_{p5}(z_{12}, 0) \times$$
$$\left[\begin{array}{l}\{J_{m-1}(0, z_1) + J_{m+1}(0, z_1)\} J_n(0, z_2) J_p(0, z_3) + \delta_1\{J_{n-1}(0, z_2) + J_{n+1}(0, z_2)\} J_m(0, z_1) J_p(0, z_3) + \\ \delta_2\{J_{p-1}(0, z_3) + J_{p+1}(0, z_3)\} J_m(0, z_1) J_n(0, z_2)\end{array}\right] \quad (11)$$

Where:

$$\xi_0 = \eta\left[(k_1+k_u)\bar{z} - \omega_1 t\right]$$

The pendulum equation is:

$$\frac{d^2\zeta_0}{d\tau^2} = |a|\cos(\zeta_0+\phi) \quad (12)$$

where, the dimensionless field strength is:

$$|a| = \frac{4\pi NeE_0K_1L}{\gamma^2m_0c^2}\eta J_q(0, z_4) J_r(z_5, z_6) J_s(z_7, 0) J_{p1}(z_8, 0) J_{p2}(z_9, 0) J_{p3}(z_{10}, 0) J_{p4}(z_{11}, 0) J_{p5}(z_{12}, 0) \times$$
$$\left[\begin{array}{l}\{J_{m-1}(0, z_1) + J_{m+1}(0, z_1)\} J_n(0, z_2) J_p(0, z_3) + \delta_1\{J_{n-1}(0, z_2) + J_{n+1}(0, z_2)\} J_m(0, z_1) J_p(0, z_3) + \\ \delta_2\{J_{p-1}(0, z_3) + J_{p+1}(0, z_3)\} J_m(0, z_1) J_n(0, z_2)\end{array}\right] \quad (13)$$

In its simplest form the wave equation is written as:

$$\frac{da}{d\tau} = -j\left\langle e^{-i\xi_0}\right\rangle \tag{14}$$

where, j is the dimensionless current density given as:

$$j = \frac{4\pi^2 N e^2 K_1^{\,2} L^2 n_e}{\gamma^3 m_0 c^2}\eta J_q(0, z_4) J_r(z_5, z_6) J_s(z_7, 0) J_{p1}(z_8, 0) J_{p2}(z_9, 0) J_{p3}(z_{10}, 0) J_{p4}(z_{11}, 0) J_{p5}(z_{12}, 0)\times \left[\begin{array}{l}\left\{J_{m-1}(0, z_1)+J_{m+1}(0, z_1)\right\} J_n(0, z_2) J_p(0, z_3)+\delta_1\left\{J_{n-1}(0, z_2)+J_{n+1}(0, z_2)\right\} J_m(0, z_1) J_p(0, z_3)+\\ \delta_2\left\{J_{p-1}(0, z_3)+J_{p+1}(0, z_3)\right\} J_m(0, z_1) J_n(0, z_2)\end{array}\right] \tag{15}$$

The gain is:

$$G = \frac{j}{\nu_0^3}\left[2 - 2\cos(\nu_0) - \nu_0\sin(\nu_0)\right] \tag{16}$$

RESULTS AND DISCUSSION

In this study, researchers have examined the use of three frequency undulator scheme for reducing betatron oscillation effects in a free electron laser. In Fig. 1, the linear undulator emits at odd harmonics (m = 1, 3, 5,...). The betatron oscillation reduces the intensity at respective harmonics. Figure 2 represents both odd and even

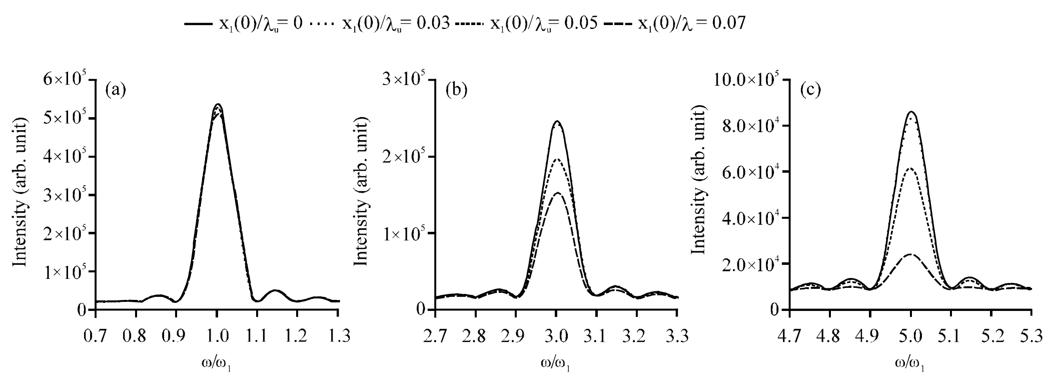

Fig. 1: Intensity vs. frequency with various values of x_1 (0)/λ_u. a) 1st harmonic; b) 3rd harmonic and c) 5th harmonic

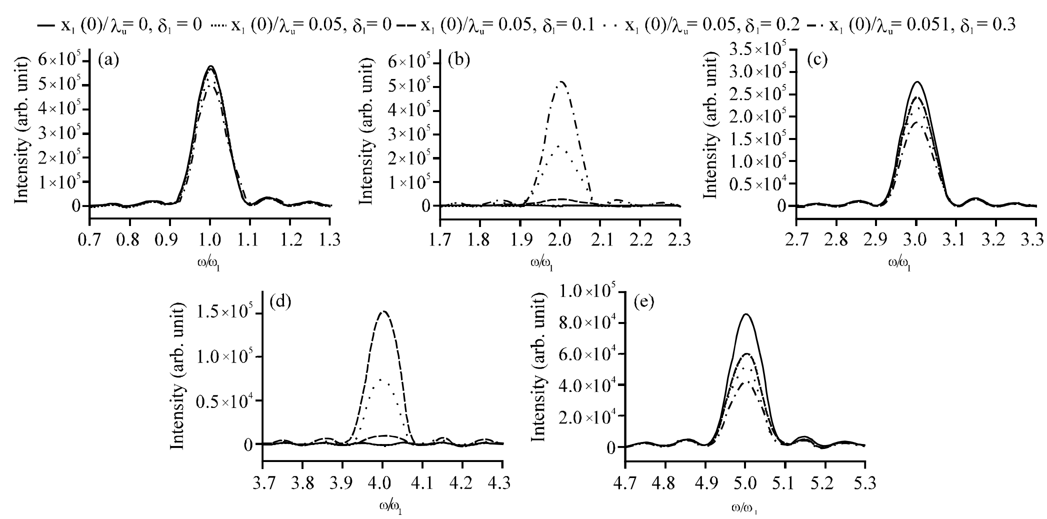

Fig. 2: Intensity vs. frequency for various values of δ_1; a) 1st harmonic; b) 2nd harmonic; c) 3rd harmonic; d) 4th harmonic and e) 5th harmonic

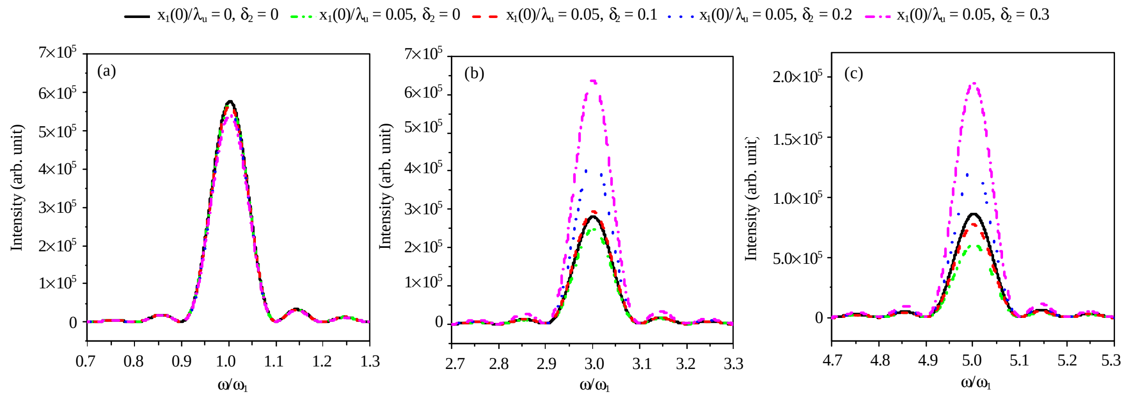

Fig. 3: Intensity vs. frequency for various values of δ_2; a) 1st harmonic; b) 3rd harmonic; c) 5th harmonic

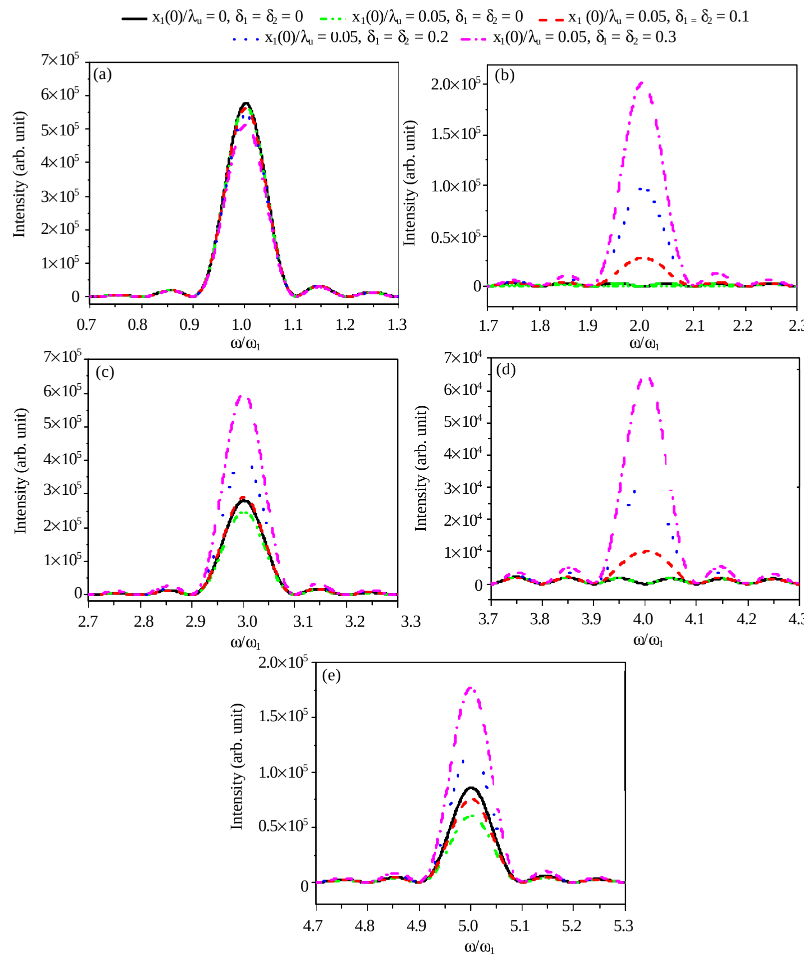

Fig. 4: Intensity vs. frequency for various values of δ_1 and δ_2; a) 1st harmonic; b) 2nd harmonic; c) 3rd harmonic; d) 4th harmonic and e) 5th harmonic

harmonics due to inclusion of frequency. Researchers saw the intensity enhancement for even harmonics as researchers increase value of and odd harmonic intensity decreases. In Fig. 3, researchers saw the intensity of 3rd and 5th harmonic is increases and also one interesting fact is that researchers reach intensity of 3rd harmonic as comparable to first harmonic at δ_2 = 0.3. Figure 4 represents intensity plots for various harmonics and here

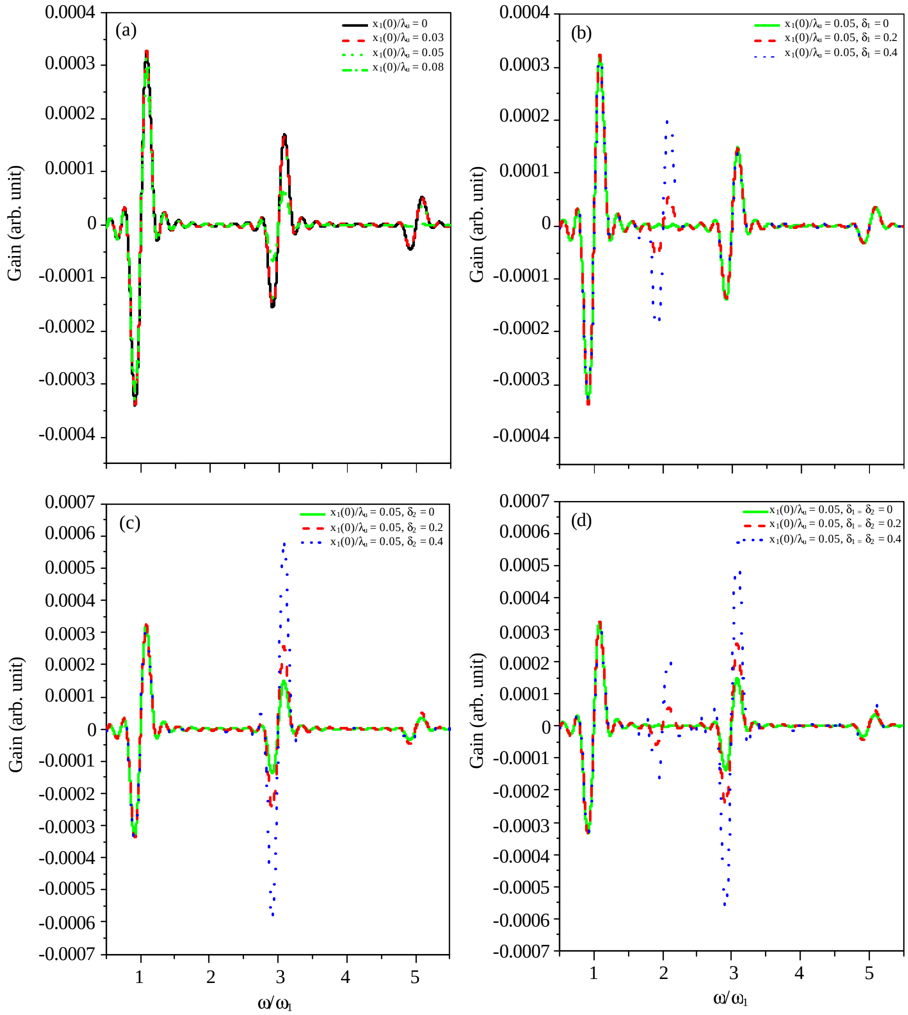

Fig. 5: a-d) Gain curve versus frequency for various harmonics

researchers saw that intensity of all the higher harmonics either odd or even get enhanced with certain combination of δ_1 and δ_2. In Fig. 5, researhers plot gain curves versus frequency. Figure 5a shows gain degradation due to inclusion of betatron motion of electron. Figure 5b enhances gain for even harmonics and Fig. 5c enhances gain for odd harmonics. Figure 5d enhances gain for odd and even harmonics except first harmonic for various values of δ_1 and δ_2. The role of three frequency undulator scheme concludes that researchers can achieve higher intensity and gain at higher harmonics.

CONCLUSION

The analysis reflects the view that the betatron oscillations reduce the intensity and gain. However, in the case of a three frequency undulator one gets substantial enhanced intensity and gain at a selected harmonic in comparison to a regular planar undulator field.

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