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Monotonicity-Preserving Using Rational Cubic Spline Interpolation

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Abstract: This study discusses the use of C^1 rational cubic spline interpolant of the form cubic/quadratic with three shape parameters to preserves the monotonicity of the given data sets. The data dependent sufficient conditions for the monotonicity of rational interpolant are derived on one parameter while the other two parameters can be further utilized to changes and modify the final shape of the monotonic interpolating curves. These sufficient conditions will ensure the existence of monotone rational interpolant. Several numerical results are presented to test the capability of the proposed rational interpolant scheme. Comparisons with the existing scheme also have been done. From all numerical results, the new rational cubic spline interpolant gives satisfactory results.

Key words: Monotonicity-preserving, rational cubic spline, parameters, ufficient condition, C^1

INTRODUCTION

Spline functions have been widely being used in many scientific and engineering based problems such as in car design and computer graphics. When a set collection of data sets are given the user may want to interpolate the data sets or approximate the data sets with preserves the original geometric features of the data. For example, if the data is monotone, then the final interpolating/approximating curves must have the same characteristics as in the original data sets namely monotonic. There exist many phenomena where the monotonicity of the data is useful. For examples, the devices in the specification of Digital to Analog Converters (DAC), Analog to Digital Converters (ADC) and sensors are always monotonic and approximation of couples and quasi couples in statistics and the dose-response curves and surfaces in biochemistry and pharmacology are other examples in which the monotonicity exists in the data sets (Beliakov, 2005). Any non-monotonicity existence in the polynomial interpolation or rational interpolation is unacceptable and any unwanted flaw in the resulting interpolating curves must be recovered nicely and the graphics display should be visual pleasing enough.

There exist many methods that can be used for the monotonicity preserving. For example, Dougherty *et al.* (1989) have used cubic and quintic spline polynomials for positivity, monotonicity and convexity-preserving. In their original research, the motonicity can be achieved by modify the first derivative values in which the cubic or quintic spline polynomial interpolation fails to preserves the motonicity of the data. Hayman (1983) has proposed local method to estimate the first derivative for monotonicity preserving by using cubic spline. Both papers are an extension to the researches of Fritsch and Butland (1984) and Fritsch and Carlson (1980). Meanwhile, Passow and Roulier (1977) discussed the monotone and convex shape preserving by using polynomial quadratic spline. Schumaker (1983) and Lahtinen (1996) discussed the shape preserving interpolation using quadratic spline polynomial. They preserved the data (monotone and convex data) by inserting one or two extra knots in the region or interval in which shape violation are found. Sarfraz *et al.* (2005) have discussed the monotonicity preserving by using GC^1 cubic spline polynomial. Karim (2013) has extended this idea by using GC^1 cubic ball spline polynomial for monotonicity preserving. Whereas, Karim and Kong (2012) have proposed GC^1 rational quartic spline with linear denominator for monotonicity preserving. Their methods work well for all tested data sets. Sarfraz (2000, 2003), Sarfraz *et al.* (2001, 2012) and Hussain and Sarfraz (2009) have utilized rational cubic spline (cubic/cubic) for monotonicity preserving. Hussain and Hussain (2007) have utilized the rational cubic spline (cubic/quadratic) of Tian *et al.* (2005) for

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monotonicity preserving of curves and surfaces. Abbas *et al.* (2012) discussed C^2 new rational cubic spline with three parameters for monotonicity preservation. In this study, researchers will use rational cubic spline with three parameters originally proposed by Karim and Kong (2014) to preserves the monotonic data. Similar to the research of the sufficient condition for monotonicity is derived on one parameter while the other two parameters are free to be utilized and can be modify in order to change the final montonic interpolating curves. Under some circumstances, the proposed rational cubic splines will redce to the research of Hussain and Hussain (2007). Researchers identify several nice features of the rational cubic spline for monotonicity preserving. It is summarized:

- In this study a new rational cubic spline (cubic/quadratic) with three parameters (two are free parameters) has been used for monotonicity preserving while by Hussain and Hussain (2007) and Sarfraz *et al.* (1997, 2013) the rational cubic spline (cubic/quadratic) with two parameters haven used for monotonicity preserving
- The rational cubic spline reproduces the rational cubic spline of Tian *et al.* (2005) when one the parameter is equal to zero, i.e., $\gamma_i = 0$. Indeed, when $\gamma_i = 0$ the monotonicity-preserving is reduces to the research of Hussain and Hussain (2007)
- The degree smoothness attained is this study is C^1 whereas by Abbas *et al.* (2012), the degree smoothness attained is C^2. With C^1 continuity, there is no need to solve any simultaneous equation as will be appear for C^2 rational spline
- Numerical comparison between the proposed rational cubic schemes with Hussain and Hussain (2007) and Fritsch and Carlson (1980) (well documented in Matlab as pchip) for monotonicity preserving also has been done. Furthermore, no derivative modification require in the rational cubic scheme while Fritsch and Carlson (1980) and Hayman (1983) the first derivative must be modified when the cubic spline polynomial fails to preserves the monotonicity of the data
- The rational scheme, not required any extra knots. Schumaker (1983) and Lahtinen (1996) require an extra knots to preserves the monotonicity and/or convexity of the given data sets
- The rational scheme is based from rational cubic spline function while by Bashir and Alim (2013) the interpolant is based from rational trigonometric spline. By having trigonometric functions, the computation would be increase compare to the computation when researchers are using rational cubic spline

MATERIALS AND METHODS

A review of rational cubic spline interpolant: This study will review the rational cubic spline interpolant with three parameters originally proposed by Karim and Kong (2014). Let us assume that $\{(x_i, f_i), i = 0, 1, ..., n\}$ is a given the set of data where $x_0<x_1<...x_n$. Let $h_i = x_{i+1}-x_i$, $\Delta_i = (f_{i+1}-f_i)/h_i$ and a local variable, $\theta = (x-x_i)/h_i$ where $0\leq\theta\leq 1$. For $x\in[x_i, x_{i+1}]$, $i = 0, 1, 2, ..., n-1$:

$$s(x) \equiv s_i(x) = \frac{P_i(\theta)}{Q_i(\theta)} \tag{1}$$

Where:

$$P_i(\theta) = A_0(1-\theta)^3+A_1\theta(1-\theta)^2+A_2\theta^2(1-\theta)+A_3\theta^3$$
$$Q_i(\theta) = (1-\theta)^2\alpha_i+\theta(1-\theta)(2\alpha_i\beta_i+\gamma_i)+\theta^2\beta_i, \text{where}$$
$$A_0 = \alpha_i f_i,\ A_1 = (2\alpha_i\beta_i+\alpha_i+\gamma_i)f_i+\alpha_i h_i d_i,$$
$$A_2 = (2\alpha_i\beta_i+\beta_i+\gamma_i)f_{i+1}-\beta_i h_i d_{i+1},\ A_3 = \beta_i f_{i+1}$$

The rational interpolant in Eq. 1 satisfy the following C^1 continuity:

$$\begin{aligned} &s(x_i) = f_i, \quad s(x_{i+1}) = f_{i+1}, \\ &s_i^{(1)}(x_i) = d_i, \quad s_i^{(1)}(x_{i+1}) = d_{i+1} \end{aligned} \tag{2}$$

Where:
$s_i^{(1)}(x)$ = Derivative with respect to x
d_i = The derivative value which is given at the knot x_i
i = 0, 1, 2, ..., n

The parameters α_i, $\beta_i>0$ and $\gamma_i\geq 0$. The data dependent sufficient conditions for monotonicity on the parameters α_i and β_i will be developed. This will ensure the positivity preserving of the given data on the entire interval $[x_i, x_{i+1}]$, $i = 0, 1, 2, ..., n-1$. When $\alpha_i = \beta_i = 1$ and $\gamma_i = 0$ the rational cubic interpolant in Eq. 1 is a standard cubic Hermite spline given as follows:

$$s(x) = (1-\theta)^2(1+2\theta)f_i+\theta^2(3-2\theta)f_{i+1}+\theta(1-\theta)^2 d_i-\theta^2(1-\theta)d_{i+1} \tag{3}$$

The shape parameters α_i, β_i i = 0, 1, 2, ..., n-1 are free parameter (independent) while the positivity constrained will be derived from the other parameter γ_i (dependent). The two parameters α_i, β_i can be used to refined and modify the final shape of the interpolating curve. Some observation including shape control analysis can be obtained by Karim and Kong (2014).

Determination of derivatives: Normally, when the scalar data are being interpolated, the first derivative parameters

d_i is not given and it must be estimated or calculated by using some mathematical formulation. The Arithmetic Mean Method (AMM) has been used to estimate the first derivative for positivity preserving (Karim and Kong, 2014). Due to the fact that Geometric Mean Method (GMM) always gives positive value of first derivatives thus this method is suitable for monotonicity preserving (Delbourgo and Gregory, 1985).

Geometric Mean Method (non-linear approximations): At the end points x_0 and x_n:

$$d_0 = \begin{cases} 0 & \Delta_0 = 0 \text{ or } \Delta_{2,0} = 0 \\ \Delta_0^{\left(1+\frac{h_0}{h_1}\right)} \Delta_{2,1}^{\left(-\frac{h_0}{h_1}\right)} & \text{otherwise} \end{cases} \quad (4)$$

$$d_n = \begin{cases} 0 & \Delta_{n-1} = 0 \text{ or } \Delta_{n,n-2} = 0 \\ \Delta_{n-1}^{\left(1+\frac{h_{n-1}}{h_{n-2}}\right)} \Delta_{n,n-2}^{\left(-\frac{h_{n-1}}{h_{n-2}}\right)} & \text{otherwise} \end{cases} \quad (5)$$

At interior points, x_i, i = 1, 2, ..., n-1, the values of d_i are given by:

$$d_i = \Delta_{i-1}^{\left(\frac{h_i}{h_{i-1}+h_i}\right)} \Delta_i^{\left(\frac{h_{i-1}}{h_{i-1}+h_i}\right)} \quad (6)$$

With:

$$\Delta_{2,0} = \frac{f_2-f_0}{x_2-x_0}$$

And:

$$\Delta_{n,n-2} = \frac{f_n-f_{n-2}}{x_n-x_{n-2}}$$

Monotonicity-preserving using rational cubic spline interpolant: Generally, the rational cubic spline interpolant (cubic/quadratic) in study not guarantee to preserves the monotonicity of all given monotone data sets. The cubic spline interpolation also cannot preserve the monotonicity of monotone data sets completely. There might be exists few unwanted oscillation that may destroys the characteristics of the data sets and the results may look not visual pleasing. This shape violation can be seen clearly from Fig. 1-3, respectively. Since, the rational cubic splines have three parameters, the following question is worthy to be asked: How to control the monotonicity of the interpolating curves for monotone data by using the parameters in the description of the rational cubic spline defined by Eq. 1? This important task can be achieved by manipulating the values of the shape parameters α_i, β_i>0 and γ_i>0 for i = 1, 2, ..., n-1. In practice this is not recommended since it requires a lot of time to find the suitable values of each shape parameters. Furthermore, it is hard for the inexperienced user to keep changes the shape parameters values. Now, by following the same idea of Sarfraz (2000, 2003) the automated choice

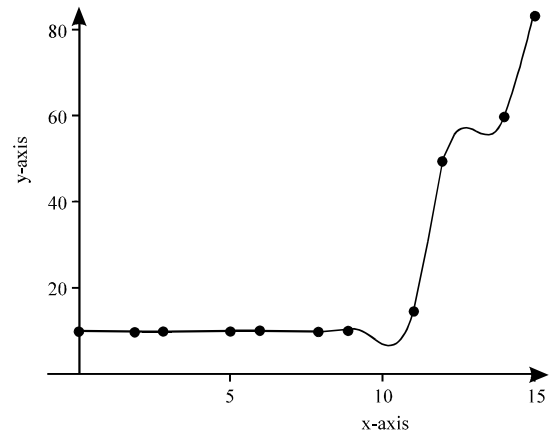

Fig. 1: Default cubic spline polynomial ($\alpha_i = \beta_i = 1$, $\gamma_i = 0$) for data in Table 1

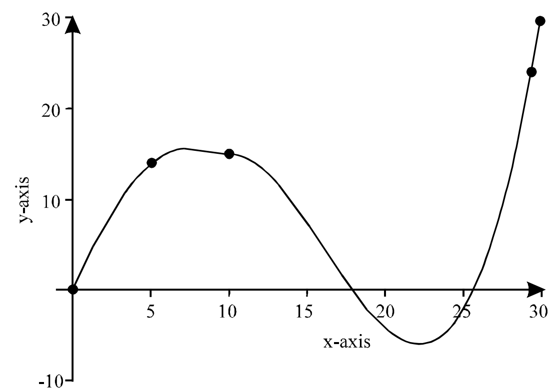

Fig. 2: Default cubic spline polynomial ($\alpha_i = \beta_i = 1$, $\gamma_i = 0$) for data in Table 2

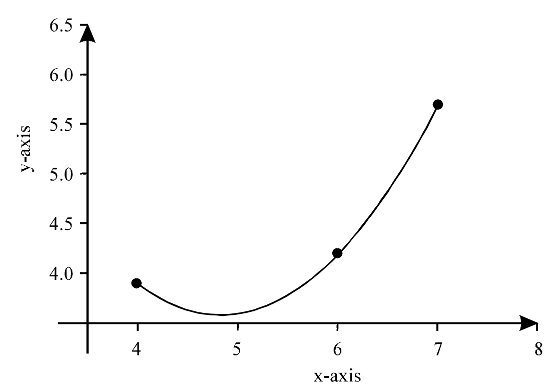

Fig. 3: Default cubic spline polynomial ($\alpha_i = \beta_i = 1$, $\gamma_i = 0$) for data in Table 3

of the shape parameter γ_i will be derived. The data dependents sufficient conditions for monotoncity of the rational interpolant in Eq. 1 will be developed while the other two free parameters α_i, β_i are free to be utilized and its provides greater freedom to the user in controlling the final shape of monotonic interpolating curves. Researchers begin with the following assumption:

Let (x_i, f_i), i = 0, 1, ..., n be a given monotone data set where, $x_0<x_i<,...,<x_n$. For a monotonic increasing (decreasing) the necessary condition should be:

$$f_0 \le f_1 \le, ..., \le f_n \quad (\text{or } f_0 \ge f_1 \ge ... \ge f_n \text{ for monotonic decreasing}) \tag{7}$$

Equation 7 is equivalent with:

$$\Delta_i \ge 0 \quad (\text{or } \Delta_i \le 0 \text{ for monotonic decreasing data}) \tag{8}$$

In this study, the necessary and sufficient condition for the C^1 monotonicity of rational cubic spline cubic interpolant will be derived by finding the conditions on one of shape parameter, γ_i. For a monotone rational cubic interpolant s(x) in Eq. 1, the first derivative must satisfy the following inequalities:

$$d_i \ge 0,\ i = 0, 1, 2, ..., n \left(\text{for monotonic increasing}\right) \tag{9}$$

$$d_i \le 0,\ i = 0, 1, 2, ..., n \left(\text{for monotonic decreasing}\right) \tag{10}$$

Now, s(x) is monotonic increasing if and only if :

$$s^{(1)}(x) \ge 0,\ x_0 \le x \le x_n \tag{11}$$

After some simplification, it can be shown that the first derivative of the rational cubic spline interpolant s(x) is given by:

$$s^{(1)}(x) = \frac{\sum_{j=0}^{4} B_{ij}(1-\theta)^{4-j}\theta^j}{\left[Q_i(\theta)\right]^2} \tag{12}$$

Where:

$$B_{i0} = \alpha_i^2 d_i, B_{i1} = 2\alpha_i(\beta_i((1+2\alpha_i+\gamma_i)\Delta_i - d_{i+1}),$$
$$B_{i3} = 2\alpha_i\beta_i((1+2\beta_i+\gamma_i)\Delta_i - d_i), B_{i4} = \beta_i^2 d_{i+1}$$
$$B_{i2} = 2\alpha_i^2\beta_i^2\beta_i((1+2\beta_i)\Delta_i - d_i)+\gamma_i(\gamma_i\Delta_i - \beta_i d_{i+1} + \beta_i\Delta_i)+ \alpha_i(4\beta_i(1+\gamma_i)\Delta_i + \gamma_i\Delta_i - \gamma_i d_i - \beta_i d_{i+1} - \beta_i d_i + 2\beta_i^2(\Delta_i - d_{i+1}))$$

Now, $s^{(1)}(x)\ge 0$ if then the rational interpolant s(x) will be monotonic increasing for all $x\epsilon[x_0, x_n]$. For, $x\epsilon[x_i, x_{i+1}]$ the denominator in Eq. 12 is always positive, therefore researchers only consider the numerator in Eq. 12 to find the sufficient condition for monotonicity. Since, $s^{(1)}(x)$ is non-negative if and only if $B_{ij}\ge 0$, j = 0, 1, 2, 3, 4. The necessary conditions for monotonicity are $d_i\ge 0$, $d_{i+1}\ge 0$ and the sufficient condition for monotonicity of rational interpolant in Eq. 1 can be determined from the condition: $B_{ij}\ge 0$, j = 0, 1, 2, 3, 4. It is obvious that $B_{i0}\ge 0$, $B_{i4}\ge 0$. If the given data is strictly monotone (i.e., $\Delta_i>0$), then from $B_{i1}\ge 0$, $B_{i2}\ge 0$will gives us the following conditions:

$$B_{i1} \ge 0 \text{ if } 2\alpha_i(\beta_i((1+2\alpha_i)\Delta_i - d_{i+1})+\gamma_i\Delta_i) > 0 \tag{13}$$

And:

$$B_{i3} \ge 0 \text{ if } 2\alpha_i\beta_i((1+2\beta_i+\gamma_i)\Delta_i - d_i) > 0 \tag{14}$$

Equation 13 and 14 provide the following conditions (Eq. 15 and 16), respectively:

$$\gamma_i > \frac{d_{i+1}}{\Delta_i} - 2\alpha_i \tag{15}$$

$$\gamma_i > \frac{d_i}{\Delta_i} - 2\beta_i \tag{16}$$

The conditions in Eq. 15 and 16 satisfy $B_{i2}>0$. Therefore, $s^{(1)}(x)\ge 0$ if Eq. 15 and 16 holds. Theorem 1 gives the sufficient condition for monotonicity preserving by using rational cubic spline (cubic/quadratic) interpolant.

Theorem 1: Given a strictly monotonic increasing set of data satisfying Eq. 7 or 8, there exists a class of monotonic rational (of the form cubic/quadratic) interpolating spline $s(x)\epsilon C^1[x_0, x_n]$ involving free parameters α_i, β_i and γ_i provided that if it satisfy the following sufficient conditions:

$$\alpha_i > 0, \beta_i > 0$$
$$\gamma_i > \text{Max}\left\{0, \frac{d_{i+1}}{\Delta_i} - 2\alpha_i, \frac{d_i}{\Delta_i} - 2\beta_i\right\} \tag{17}$$

Remark 1: If the data are constant on certain interval, i.e., $\Delta_i = 0$, then it is necessary to set $d_i = d_{i+1} = 0$, hence $s(x) = f_i = f_{i+1}$ is a constant on the interval $[x_i, x_{i+1}]$, i = 0, 1, 2, ..., n-1. This shows that the rational interpolant is monotone.

Remark 2: The sufficient condition for monotonicity in Eq. 17 can be rewritten as follows:

$$\gamma_i = \delta_i + \text{Max}\left\{0, \frac{d_{i+1}}{\Delta_i} - 2\alpha_i, \frac{d_i}{\Delta_i} - 2\beta_i\right\}, \delta_i > 0, \Delta_i > 0 \tag{18}$$

An algorithm to generate C^1 positivity-preserving curves using the results in Theorem 1 is given.

Algorithm for monotonicity-preserving:
Input: Data points, d_i, shape parameters $\alpha_i>0$, $\beta_i>0$
Output: γ_i and piecewise monotonic interpolating curves.

1. For i = 0, 1, ..., n, input data points (x_i, f_i) with condition: $\Delta_i>0$.
2. For i = 0, 1, ..., n, estimate d_i using Geometric Mean Method (GMM).
3. For i = 0, 1, ..., n-1
   - Calculate h_i and Δ_i
   - Choose any suitable values of $\alpha_i>0$, $\beta_i>0$
   - Calculate the shape parameter γ_i using Eq. 18 with suitable choices of $\delta_i>0$
   - Calculate the inner control ordinates A_1 and A_2
4. For i = 0, 1, ..., n-1
   Construct the piecewise monotonic interpolating curves using Eq. 1.
   Modify the value of α_i, β_i, $\delta_i>0$ to obtain difference curves.
   Repeat step 1 until step 4 for each tested data sets.

Remark 3: When $\gamma_i = 0$, the rational cubic spline will reduce to the positivity preserving by using rational cubic spline interpolant (cubic/quadratic) with two parameters discussed in details by Hussain and Hussain (2007). Hussain and Hussain (2007) have proposed monotonicity preserving by using the rational cubic spline (Tian *et al.*, 2005). For the purpose of numerical comparison later, researchers cited the sufficient condition for monotonicity preserving by Hussain and Hussain (2007).

Theorem 2 (Hussain and Hussain, 2007): The piecewise rational cubic function in Eq. 1 with $\gamma_i = 0$ preserves the monotonicity of the data if in subinterval the free parameters α_i and β_i satisfies the following sufficient conditions:

$$\alpha_i > \text{Max}\left\{0, \frac{d_i}{2\Delta_i}\right\}, \beta_i > \text{Max}\left\{0, \frac{d_{i+1}}{2\Delta_i}\right\} \tag{19}$$

The conditions in Eq. 19 can be obtained by rearrange Eq. 15 and 16 by setting $\gamma_i = 0$. In the research of Hussain and Hussain (2007), there is no free parameter(s) to refine the final shape of the curve; meanwhile there exists two free parameters α_i, β_i by using the proposed rational cubic spline proposed in this study for monotonicity-preserving.

RESULTS AND DISCUSSION

In order to illustrate the shape preserving interpolation by using the proposed rational cubic spline interpolation (cubic/quadratic), three sets of monotone data taken from Akima (1970), Sarfraz (2000) and Hussain *et al.* (2010) were used.

Figure 1-3 show the default cubic spline interpolation for data given in Table 1-3, respectively. Figure 4-6 shows the shape preserving interpolation using Hussain and Hussain (2007) for data listed in Table 1-3, respectively. Figure 7-9 show the shape preserving by using the rational cubic spline for monotone data in Table 1. Figure 10 shows the graphs of motone interpolating curves using the proposed rational cubic spline with two choices of free parameters and Hussain and Hussain (2007) shown in black color.

Table 1: A monotone data from Akima (1970)

x_i	f_i	d_i
0	10.0	0.000
2	10.0	0.000
3	10.0	0.000
5	10.0	0.000
6	10.0	0.000
8	10.0	0.000
9	10.5	1.350
11	15.0	14.021
12	50.0	18.297
14	60.0	14.620
15	85.0	36.596

Table 2: A monotone data from Sarfraz (2000)

x_i	f_i	d_i
0	0.01	5.3756
6	15.00	0.0000
10	15.00	0.0000
29.5	25.00	9.2843
30	30.00	10.6867

Table 3: A monotone data from (Hussain *et al.*, 2010)

x_i	f_i	d_i
4.0	3.9	0.009375
6.0	4.2	0.696240
7.0	5.7	2.371710

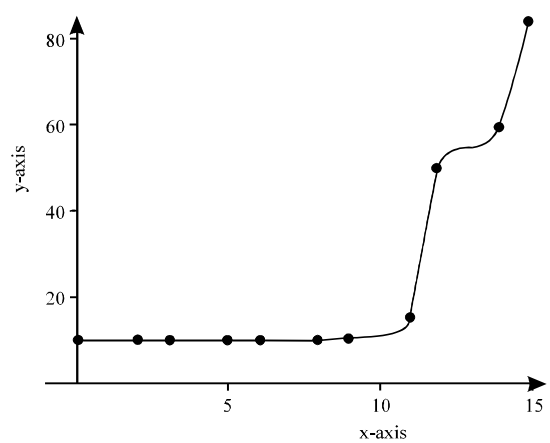

Fig. 4: Shape preserving using Hussain and Hussain (2007) for data in Table 1

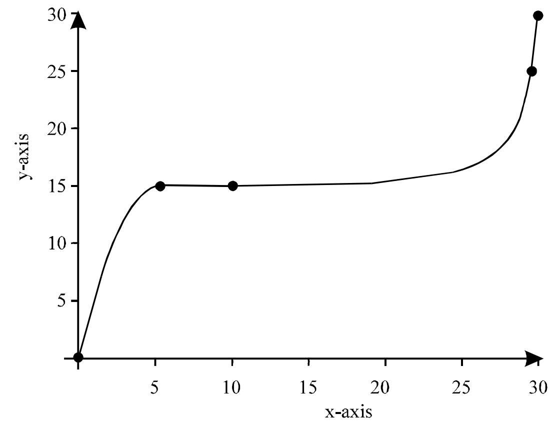

Fig. 5: Shape preserving using Hussain and Hussain (2007) for data in Table 2

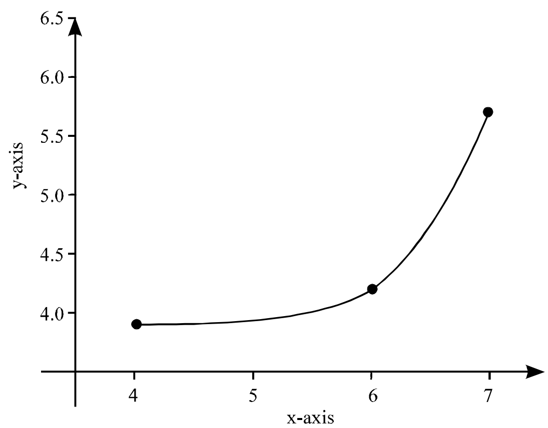

Fig. 6: Shape preserving using Hussain and Hussain (2007) for data in Table 3

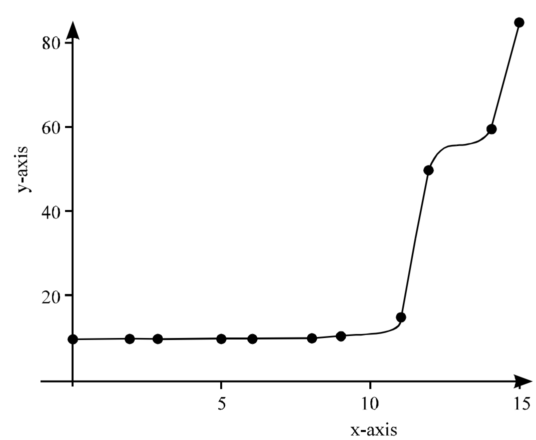

Fig. 7: Shape preserving using the proposed rational spline with $\alpha_i = \beta_i = 1$, $\delta_i = 0.25$ for data in Table 1

Meanwhile, Fig. 11-13 show shape preserving interpolation using the rational cubic spline with various

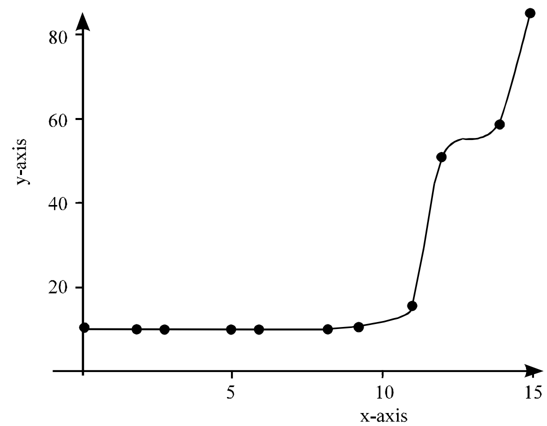

Fig. 8: Shape preserving using the proposed rational spline with ($\alpha_i = \beta_i = 0.5$, $\delta_i = 0.25$) for data in Table 1

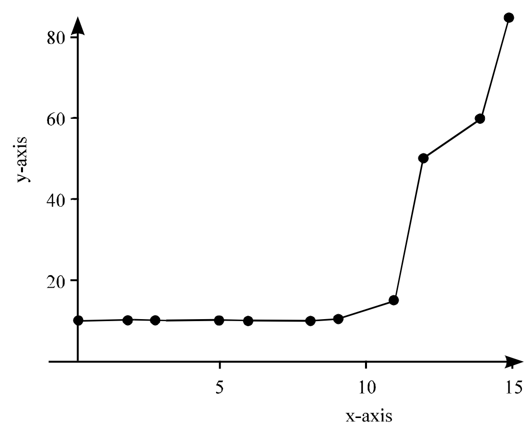

Fig. 9: Shape preserving using the proposed rational spline ($\alpha_i = \beta_i = 0.05$, $\delta_i = 0.25$) with for data in Table 1

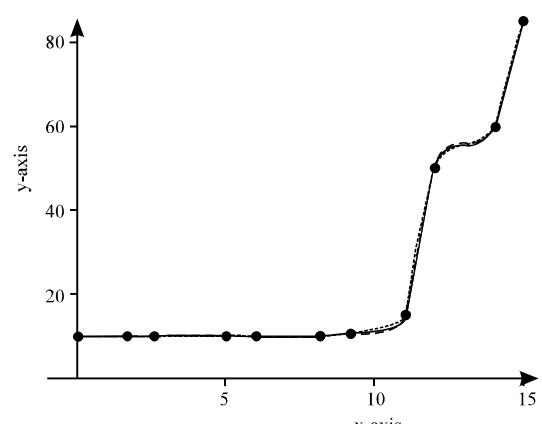

Fig. 10: Shape preserving with $\alpha_i = \beta_i = 1$ (blue), $\alpha_i = \beta_i = 0.5$, (red) with $\delta_i = 0.25$ and Hussain and Hussain (2007) (black) for data in Table 1

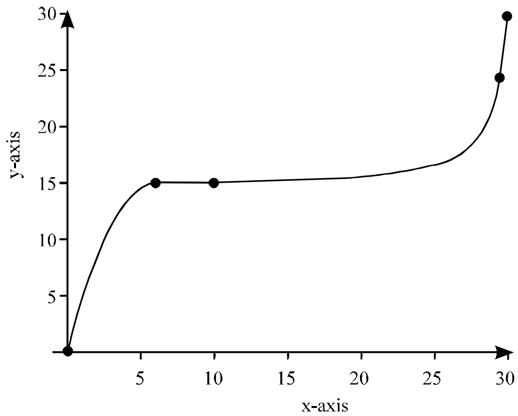

Fig. 11: Shape preserving using the proposed rational spline with $\alpha_i = \beta_i = 1$, $\delta_i = 0.25$ for data in Table 2

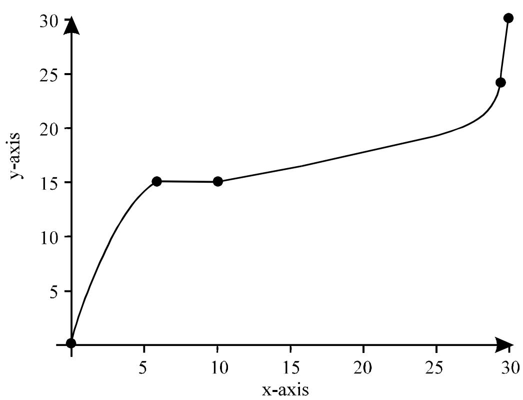

Fig. 12: Shape preserving using the proposed rational spline with $\alpha_i = \beta_i = 0.5$, $\delta_i = 0.25$ for data in Table 2

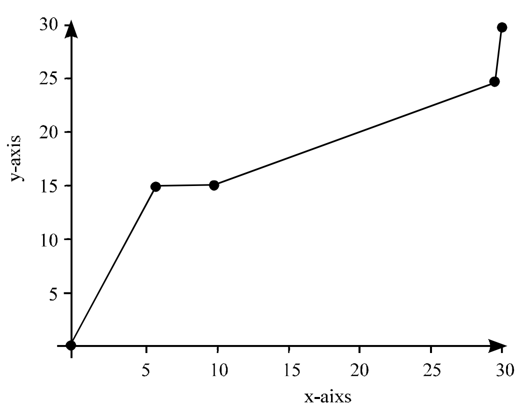

Fig. 13: Shape preserving using the proposed rational spline with $\alpha_i = \beta_i = 0.05$, $\delta_i = 0.25$ for data in Table 2

free parameters values for data in Table 2. Figure 14 shows the comparison between the rational cubic spline

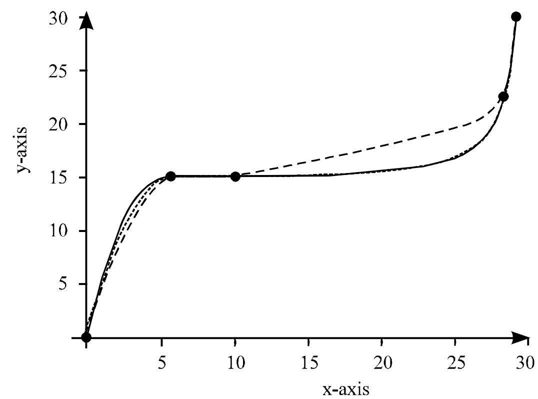

Fig. 14: Shape preserving with $\alpha_i = \beta_i = 1$, $\delta_i = 0.25$ (blue), $\alpha_i = \beta_i = 0.5$, $\delta_i = 0.25$ (red) and Hussain and Hussain (2007) (black) for data in Table 2

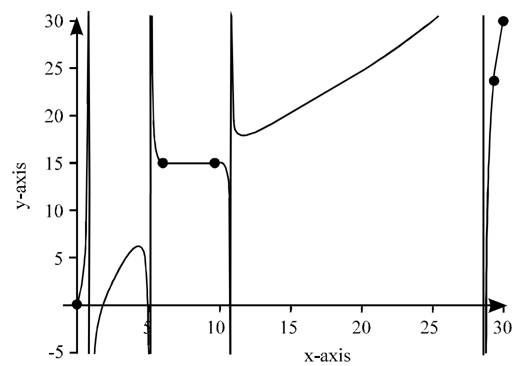

Fig. 15: Interpolating curves using the proposed rational spline ($\alpha_i = \beta_i = -1$, $\delta_i = 0.25$) with for data in Table 2

and the research of Hussain and Hussain (2007). Figure 15 shows the rational curves when both free parameters are negative (<0). Figure 16-19 show monotonic interolating curves using the rational cubic spline with respective free parameters values as indicated in the figures title. Figure 20 shows the comparison between the rational scheme and the research of Hussain and Hussain (2007).

Finally, Fig. 21-23 show the monotonic interpolating curves by using cubic spline polynomial riginally proposed by Fritsch and Carlson (1980). Figure 21-23 show the monotonicity preserving by using cubic spline polynomial proposed by Fritsch and Carlson (1980). This Fig. 21-23 is generate by using pchip function in Matlab. It can be seen clearly that the proposed rational cubic spline with three parameters in this study has clear advantage compare with the research of Fritsch and Carlson (1980). Researchers can change the final shape of the monotonic interpolating curves by choosing different values of shape parameters (with same data sets) while

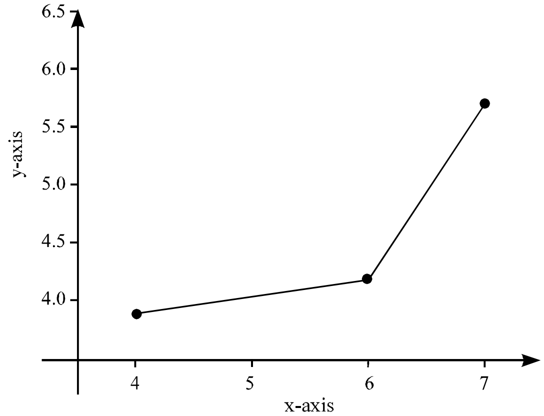

Fig. 16: Shape preserving using the proposed rational spline with $\alpha_i = \beta_i = 0.1$, $\delta_i = 0.25$ for data in Table 3

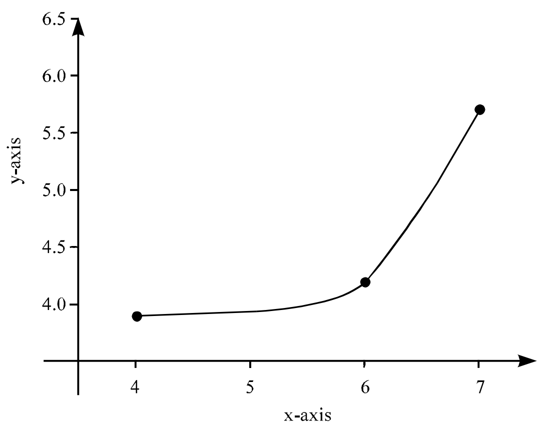

Fig. 17: Shape preserving using the proposed rational spline with $\alpha_i = 2$, $\beta_i = 1$, $\delta_i = 0.25$ for data in Table 3

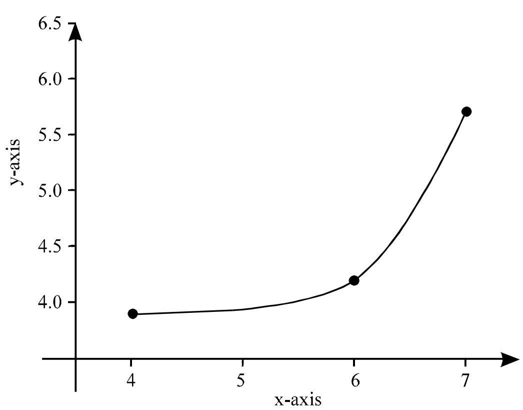

Fig. 18: Shape preserving using the proposed rational spline with $\alpha_i = 0.1$, $\beta_i = 5$, $\delta_i = 0.25$ for data in Table 3

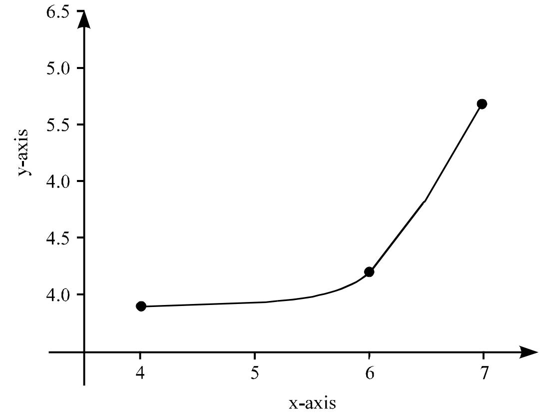

Fig. 19: Shape preserving using the proposed rational spline with $\alpha_i = \beta_i = 2.5$, $\delta_i = 0.25$ for data in Table 3

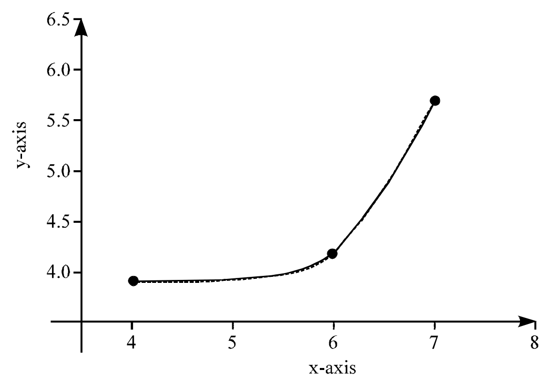

Fig. 20: Shape preserving with $\alpha_i = \beta_i = 1$, $\delta_i = 0.25$ (red) and Hussain and Hussain (2007) (blue) for data in Table 3

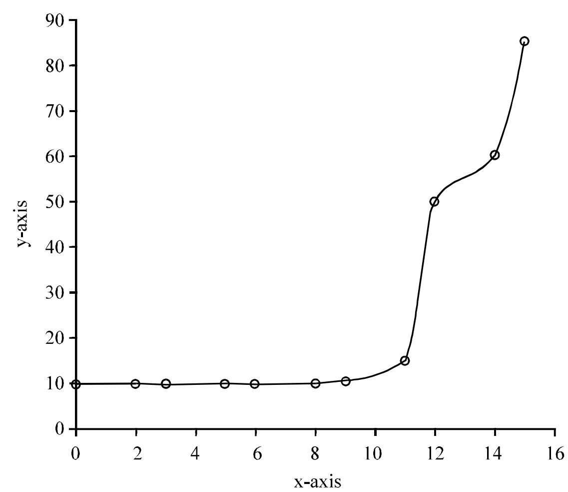

Fig. 21: Shape preserving interpolation using pchip in Matlab for data in Table 1

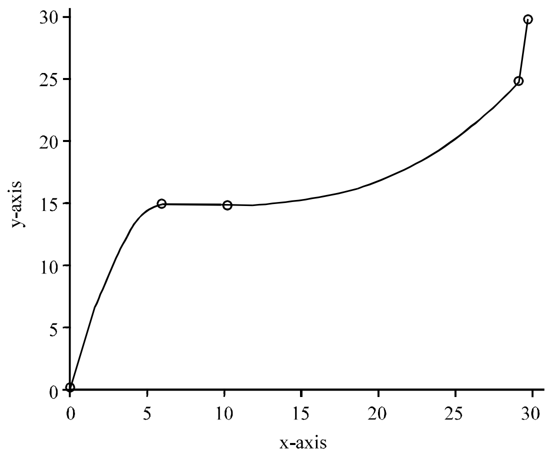

Fig. 22: Shape preserving interpolation using pchip in Matlab for data in Table 2

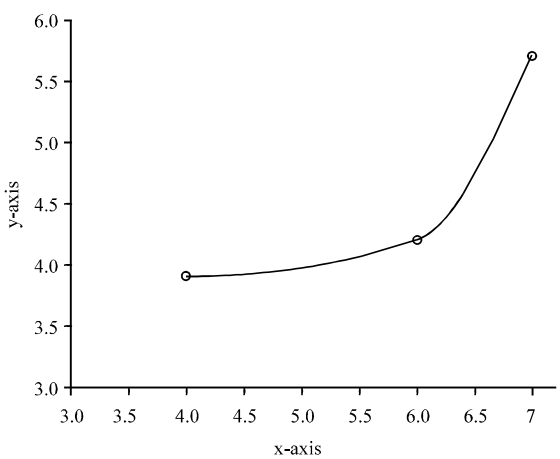

Fig. 23: Shape preserving interpolation using pchip in Matlab for data in Table 3

researchers cannot changes the shape of monotonic interpolating curves by Fritsch and Carlson (1980) without having to change its control points or data points. For example for data in Table 1, Fritsch and Carlson (1980) Method require the modification of the first derivative on interval [9, 11] and [12, 14], respectively. Whereas the proposed rational cubic spline with three parameters can be used to preserves the monotone data without the need to modify the first derivative values. Furthermore, from Fig. 21-23, it can be seen clearly that monotonic interpolating curves by using (Fritsch and Carlson, 1980) tend to overshoot the interpolating curves thus it looks not visual pleasing enough even though the monotonicity of the data sets are preserved. For example, in the interval [29.5, 30], the 4th segment of the curves, not too smooth when researchers compare with monotonicity preserving by using the rational cubic spline shown in Fig. 11 and 12. The rational cubic spline of Karim and Kong (2014) gives satisfactory results.

CONCLUSION

Shape preserving for monotonicity data was important task in scientific visualization and computer graphics. This study has successfully applied rational cubic spline (with three parameters) of Karim and Kong (2014) to preserves the monotonicity of the given monotone data sets. The sufficient conditions for monotonicity have been derived on one of the parameter while the other two parameters α_i, β_i are free parameters γ_i that can be used for the refining processes. From numerical results, it can be seen clearly that by changing the value of two shape parameters, researchers may have variety shape of monotonic interpolating curves. In the research of Hussain and Hussain (2007) the sufficient conditions are also data dependent but no free parameters. Meanwhile, the rational cubic scheme has two free parameters. These free parameters are useful for designing purposes as researchers can see clearly through numerical examples shown in study. Researchers conclude that the developed scheme works well and is comparable to the existing schemes namely the research of Hussain and Hussain (2007). Finally, the researcher is the final process to comple te the research for positivity and convexity preserving for data in 2D and 3D environment. Researches on rational cubic spline also is in progress. All the findings will be reported in near future.

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