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Stability Analysis of Two-Dimensional Linear Time Invariant Discrete Systems

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Abstract: In this study, an algebraic test is presented to test the stability two-dimensional linear time invariant discrete system represented in the form of its characteristics equation. This characteristics equation is converted into an equivalent one-dimensional equation and the test is applied. Illustrative examples are presented to show the simplicity and application.

Key words: Stability, two-dimensional systems, necessary condition, sufficient condition, linear time invariant systems, discrete systems

INTRODUCTION

Stability of two-dimensional continuous systems (Ooba and Funahashi, 2004) exists is providing a test for a driving point impedance reliability condition using commensurable delay transmission lines and lumped reactance's (Jury, 1988). But stability of two-dimensional digital systems occurs in their design situation. A two-dimensional digital system (Mastorakis, 1998; Bistritz, 2001) arises in image processing and geophysics for processing seismic, gravity as well as magnetic data (Ahmed, 1980). There are many algebraic schemes are available to test the stability of two-dimensional systems (Agathoklis *et al*., 1993) each having its own application and merits. In the present study, a simple and direct scheme (Bose and Jury, 1975; Jury *et al*., 1975; Jury, 1971) is proposed compared to the above methods.

LITERATURE SURVEY

Stability of two-Dimensional (2D) linear discrete-time systems arises in many applications. It is required for the design of digital filters and processing of image, seismic, radar and other types of data in multimedia, geography communication, medicine and more fields. The key for testing stability of 2D discrete systems and subject of this study is to determine whether a 2D polynomial has no zeroes in the closed exterior of the unit bi-circle. Conventional implementation has made use of the two-dimensional convolution algorithms and more recently of the two-dimensional fast fourier transform. The primary problems are stability and synthesis in any two-dimensional discrete time systems was identified by Shanks *et al*. (1972). Huang (1972) clearly investigated the stability of two-dimensional recursive filters in the frequency domain and raised several conjunctures like how to design filters that are guaranteed to be stable and how to stabilize without changing the frequency response of a given filter being unstable. Rajan and Reddy (1989) addresses the procedure to test discrete scattering Hurwitz polynomials which was outlined by employing schurcohn matrix associated with the given polynomial. Asymptotic stability of two-dimensional systems in the state space representation was studied by Ooba and Funahashi (2004) and the stability robustness of a stable FM Model was defined by Lu (1993). Pee *et al*. (1986) and Bisiacco *et al*. (1985) presented a sufficient condition that ensures BIBO stable two-dimensional filters admitting an internally stable statement space realization.

Bistritz (2004) corroborated the testing of the conditions by a 1D stability test leads to tubular 2D stability tests with the term tubular reflects the historic tradition to present 1D stability tests. A new algebraic procedure that solves the problem of stability in a very low count of arithmetic operations was given by Bistritz (2001). Their new procedure profit on the advantages of a recent immittance-type tabular 2D stability test proposed and simplifies it into an even more efficient 2D stability test. Mastorakis (1998) in his study had proposed a new method for computing the stability margins. It was based on a constrained optimization problem of a real positive parameter. Agathoklis *et al*. (1993) and

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Antoniou *et al*. (1990) proved the frequency dependent Lyapuinov equation that was used to obtain necessary and sufficient Algebraic Stability Test which also revealed that 2D stability test can be tested based on the location of the Eigen values of constant matrices and not with the positivity of one or more function. The algorithm for two-dimensional discrete systems was proposed by Kanellakis *et al*. (1989) with its implementation based on expanding the bilinear discrete reactance function into z domain. Jury (1988) had introduced a procedure to determine highly concurrent representation of linear algorithms and this technique replaces matrix/vector operations by sums of small size triple matrix product. He also proved that for processing elements and closed loop data circulation and both the characteristics increases the flexibility and speed of the configuration. Jury and Bauer (1988) in their study of testing, the BIBO stability acclaimed that the 2D continuous system may become unstable after applying a reactance transformation. It was actually a contrast for 2D discrete case. Furthermore, they proved that the DBT does not preserve stability in either direction. It is known that transforming from the continuous domain to the discrete domain might cause stability problems and has shown that transforming from the discrete to the continuous domain can also create such problems. Goodman (1977), Karan and Srivastava (1986) and Woods (1983) in their new stability test for 2D filters had revealed that it was much simpler than the tests hitherto described in the literature and it requires a smaller number of computations that with Marden's table. It may also extend up to multi dimensional filters. Inners of square matrix was introduced by Jury (1971) and for the necessary and sufficient conditions for the roots of real polynomial to be distinct and on the real axis and distinct and on the imaginary axis in the complex plane. Bose and Jury (1975) posed about the test for positive definiteness of an arbitrary form expressed in terms of an inner algorithm for computational aspects. A new formulation of the critical constraints for stability that limits the A matrix and its bi-alternate product was discussed by Jury *et al*. (1975).

PROPOSED METHOD

The transfer function of a two-dimensional discrete system can be written as (Mastorakis, 1998):

$$H(Z_1, Z_2) = \frac{A(Z_1, Z_2)}{B(Z_1, Z_2)} \quad (1)$$

where, A and B non-cancellable polynomials in Z_1 and Z_2. This system is stable if and only if there are no values of Z_1 and Z_2 such that $B(Z_1, Z_2) = 0$ and $|Z_1| \le 1$ and $|Z_2| \le 1$ ((or) equivalently, $B(Z_1, Z_2) \neq 0$ for $|Z_1|$ and $|Z_2|$ simultaneously less than or equal to unity). The necessary condition for stability is:

$$B(Z_1, 0) \neq 0, \ |Z_1| \le 1 \quad (2)$$

Or:

$$B(0, Z_2) \neq 0, \ |Z_2| \le 1 \quad (3)$$

and the sufficient condition is:

$$B(Z_1, Z_2) \neq 0, \ |Z_1| = 1, \ |Z_2| \le 1 \quad (4)$$

Or:

$$B(Z_1, Z_2) \neq 0, \ |Z_1| \le 1, \ |Z_2| = 1 \quad (5)$$

It can be observed that the testing of necessary condition (Agathoklis *et al*., 1993) is easy since it is a one-dimensional equation while tests the sufficient involves complex coefficients will arise in $B(Z_1, Z_2)$ polynomial. In general the following form can also be chosen:

$$B(Z_1, Z_2) = T_0(Z_1)Z_2^n + T_1(Z_1)Z_2^{n-1} \ldots\ldots + T_n(Z_1) = 0 \quad (6)$$

The reciprocals of Z_1 and Z_2 are $1/z_1$ and $1/z_2$ respectively are utilized so that the Eq. 6 is rewritten as:

$$B\left(\frac{1}{Z_1}, \frac{1}{Z_2}\right) = T_0\left(\frac{1}{Z_1}\right)\left(\frac{1}{Z_2}\right)^n + T_1\left(\frac{1}{Z_1}\right)\left(\frac{1}{Z_2}\right)^{n-1} + \ldots + T_n\left(\frac{1}{Z_1}\right) = 0 \quad (7)$$

again Eq. 7 is represented as:

$$M(Z_1, Z_2)\Big|_{Z_1 = Z_2 = x} = F(x) = 0 \quad (8)$$

This F(x) = 0 is one-dimensional equation and for stability $|x|<1$. Then, F(x) can be analyzed by any algebraic method for sufficient condition for stability. Let:

$$F(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_0 = 0 \quad (9)$$

The necessary condition for stability $|a_0|<a_n$ and the sufficient is tested as follows: using the coefficients of F(x) two triangular matrices is written:

$$[t_1] = \begin{pmatrix} a_n & a_{n-1} & a_{n-2} & . & .. & a_0 \\ 0 & a_n & a_{n-2} & . & .. & . \\ 0 & 0 & a_n & . & .. & . \\ 0 & 0 & 0 & a_n & .. & . \\ . & . & . & . & .. & . \\ . & . & . & . & .. & . \\ . & . & . & . & .. & a_n \end{pmatrix} \quad (10)$$

$$[t_1] = \begin{pmatrix} a_n & a_{n-1} & a_{n-2} & . & .. & a_0 \\ a_{n-1} & a_{n-2} & a_{n-3} & . & .. & 0 \\ a_{n-2} & a_{n-3} & a_{n-4} & . & .. & 0 \\ . & . & . & . & .. & 0 \\ . & . & . & . & .. & 0 \\ . & . & . & . & .. & 0 \\ a_0 & 0 & 0 & . & .. & 0 \end{pmatrix} \quad (11)$$

Adding (t_1) and (t_2), a square matrix is formed:

$$[S] = [t_1] + [t_2] \quad (12)$$

This square matrix S is said to be positive inner wise when all the determinants with the center element(s) and proceeding out wards up to the entire matrix are positive (Jury, 1988). This is used for testing the sufficient condition that $|x| < 1$. For example for:

$$[S] = \begin{pmatrix} b_1 & b_2 & b_3 & b_4 & b_5 \\ c_1 & c_2 & c_3 & c_4 & c_5 \\ d_1 & d_2 & d_3 & d_4 & d_5 \\ e_1 & e_2 & e_3 & e_4 & e_5 \\ f_1 & f_2 & f_3 & f_4 & f_5 \end{pmatrix} \quad (13)$$

The determinants (indicated by dashed lines) are:

- d_3
- $\begin{vmatrix} C_2 & C_3 & C_4 \\ d_2 & d_3 & d_4 \\ e_2 & e_3 & e_4 \end{vmatrix}$
- $|S|$

It can be observed that the first determinant is a scalar if f(x) = 0 is of even degree and is a 2×2 determinant for odd degree of f(x) = 0. The proposed procedure is applied for the following illustrative examples.

Illustraions 1: Let: $B(Z_1, Z_2) = 1\text{-}0.75Z_1\text{-}0.5Z_2\text{+}0.3Z_1Z_2 = 0$ with $Z_2 = 0$, $B(Z_1) = 1\text{-}0.75Z_1\text{-}0$ and its reciprocal is Huang, 1972):

$$B\left(\frac{1}{Z_1}\right) = Z_1 - 0.75 = 0 \quad (14)$$

Thus, $|Z_1|<1$ which satisfies the necessary condition for $B(Z_1, Z_2)$ to leave roots outside unit circle. The sufficiency test is carried out using the one dimensional equivalent equation f(x) = 0:

$$\begin{aligned} F(x) &= x^2 - 0.75x - 0.5x + 0.3 = 0 \\ &= x^2 - 1.25x + 0.3 = 0 \end{aligned} \quad (15)$$

The two triangular matrices along with their square matrix are given as:

$$[t_1] = \begin{pmatrix} 1 & -1.25 & 0.30 \\ 0 & 1 & -1.25 \\ 0 & 0 & 1 \end{pmatrix} \quad (16)$$

$$[t_2] = \begin{pmatrix} 1 & -1.25 & 0.30 \\ -1.25 & 0.30 & 0 \\ -0.30 & 0 & 0 \end{pmatrix} \quad (17)$$

And:

$$[S] = \begin{pmatrix} 2 & -2.5 & 0.6 \\ -1.25 & 1.3 & -1.25 \\ 0.3 & 0 & 1 \end{pmatrix} \quad (18)$$

The inner determinants are written as: $\nabla_1 = 1.3>0$ and $\nabla_3 = 2(1.3)+1.25(-2.5)+0.3(2.5\times1.25-0.78) = 0.1785$. Since, $\nabla_3>0$, thus f(x) have all its two roots with $|x|<1$. Hence, the original two-dimensional $B(Z_1, Z_2) = 0$ has both the roots outside the unit circle. Thus, the system represented by $B(Z_1, Z_2) = 0$ is stable.

Illustrations 2: From Huang (1972), the two-dimensional characteristic polynomial is written as:

$$B(Z_1, Z_2) = (1\text{-}0.95Z_1)\text{-}(0.95\text{-}0.5Z_1)Z_2$$

Test for necessary condition:

$$B(Z_1, 0) = 1\text{-}0.95Z_1 \neq 0 \text{ for } |Z_1| \leq 1$$

In other words:

$$B\left(\frac{1}{Z_1}, 0\right) = Z_1\text{-}0.95$$

Thus, $|Z_1|<1$ is assured for necessary condition for stability.

Test for sufficiency: The two-dimensional polynomial is converted into its one-dimensional equivalent polynomial in x as:

$$F(x) = x^2\text{-}1.9x\text{+}0.5$$

Utilizing the concept of two-triangular matrices, their square matrix is formed as:

$$[S] = \begin{pmatrix} 2 & -3.8 & 1 \\ -1.9 & \boxed{1.5} & -1.9 \\ 0.5 & 0 & 1 \end{pmatrix} \quad (19)$$

From [S], the two inner determinants are: $\nabla_1 = 1.5>0$ and $\nabla_3 = -1.36<0$. Since, ∇_3 is negative, the equivalent one-dimensional system is unstable this shows that the two-dimensional discrete system is also unstable.

Illustrations 3: Let the polynomial be (Rajan and Reddy, 1989):

$$B(Z_1, Z_2) = Z_2^2 + (Z_1+2)Z_2 + 4Z_1\text{-}8$$

Test for necessary condition:

$$B(Z_1, 0) = 4Z_1\text{-}8 \neq 0, |Z_1| \leq 1$$

Using the reciprocal:

$$B\left(\frac{1}{Z_1}, 0\right) = \text{-}8Z_1 + 4$$

It is found that $|Z_1|<1$ which satisfies the necessary condition. Test for sufficiency:

$$F(x) = 8x^2\text{-}6x\text{-}2$$

The square matrix S is formed as:

$$[S] = \text{-}6 \begin{pmatrix} 16 & -12 & -4 \\ -6 & \boxed{6} & -6 \\ -2 & 0 & 8 \end{pmatrix} \quad (20)$$

The two inner determinants are:

$$\nabla_1 = 6 > 0$$
$$\nabla_3 = 16(48) + 6(\text{-}96)\text{-}2(72+24)$$
$$= 768\text{-}576\text{-}192$$
$$= 768\text{-}768 = 0$$

Since, ∇_3 = zero, the one-dimensional equivalent system is unstable which ensures that the given two-dimensional discrete system is also unstable (not a very strict Hurwitz Polynomial).

Illustrations 4: Let the characteristic polynomial representing an all pole second order two-dimensional discrete filter be (Shanks *et al.*, 1972):

$$B(Z_1, Z_2) = A_2(Z_1)Z_2^2 + A_1(Z_1)Z_2 + A_0(Z_1)$$

Where:
$A_2(Z_1) = 0.29Z_1^2 - 0.75Z_1 + 0.5$
$A_1(Z_1) = -0.72Z_1^2 + 1.8Z_1 - 1.2$
$A_0(Z_1) = 0.6Z_1^2 - 1.5Z_1 + 1$

Test for necessary condition:

$$B(Z_1, 0) = A_0(Z_1) = 0.6Z_1^2\text{-}1.5Z_1 + 1 \neq 0 \text{ for } |Z_1| \leq 1$$

The reciprocal of $B(Z_1, 0)$ is:

$$B\left(\frac{1}{Z_1}\right) = Z_1^2\text{-}1.5Z_1 + 0.6 = 0 \text{ for } |Z_1| \leq 1$$

From the two triangular matrices the square matrix:

$$[S] = \begin{pmatrix} 2 & -3 & 0.6 \\ -1.5 & 1.6 & -1.5 \\ 0.6 & 0 & 1 \end{pmatrix}$$

From [S] the two inner determinants are formed: $\nabla_1 = 1.6>0$ and $\nabla_3 = 0.348>0$. Thus, the necessary condition is satisfied.

Test for sufficiency: The one-dimensional equivalent polynomial obtained for $B(Z_1, Z_2)$ is:

$$F(x) = x^4\text{-}2.7x^3 + 2.9x^2\text{-}1.47x + 0.29$$

The square matrix obtained for F(x) is:

$$[S] = \begin{pmatrix} 2 & -5.4 & 5.8 & -2.94 & 0.58 \\ -2.7 & 3.9 & -4.17 & 3.19 & -1.47 \\ 2.9 & -1.47 & 1.29 & -2.7 & 2.9 \\ -1.47 & 0.29 & 0 & 1 & -2.7 \\ 0.29 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (22)$$

From [S] the inner determinants are evaluated: $\nabla_1 = 1.29>0$, $\nabla_3 = 0.9728>0$, $\nabla_5 = 0.0112>0$. Since, all inner determinants are positive the sufficiency condition is satisfied for stability. Thus, the original two-dimensional discrete system is found to be stable.

Illustrations 5: Consider the second order two-dimensional discrete polynomial by Shanks *et al.* (1972):

$$B(Z_1, Z_2) = A_2(Z_1)Z_2^2 + A_1(Z_1)Z_2 + A_0(Z_1)$$

Where:

$A_2(Z_1) = 0.25Z_1^2 - 0.75Z_1 + 0.5$
$A_1(Z_1) = -0.72Z_1^2 + 1.8Z_1 - 1.2$
$A_0(Z_1) = 0.6Z_1^2 - 1.5Z_1 + 1$

Test for necessary condition:

$$B(Z_1, 0) = A_0(Z_1) = 0.6Z_1^2 \text{-} 1.5Z_1 + 1 \neq 0 \text{ for } |Z_1| \leq 1$$

The reciprocal of $B(Z_1, 0)$ is:

$$B\left(\frac{1}{Z_1}\right) = Z_1^2 \text{-} 1.5Z_1 + 0.6$$

The square matrix for the above polynomial S is same as the previous example in which the necessary condition for stability is satisfied.

Test for sufficiency: The one-dimensional equivalent polynomial obtained for $B(Z_1, Z_2)$ is:

$$F(x) = x^4 \text{-} 2.7x^3 + 2.9x^2 \text{-} 1.47x + 0.25$$

with the help of two triangular matrices the square matrix is formed:

$$[S] = \begin{pmatrix} 2 & -5.4 & 5.8 & -2.94 & 0.5 \\ -2.7 & 3.9 & -4.17 & 3.15 & -1.47 \\ 2.9 & -1.47 & 1.25 & 2.7 & 2.9 \\ -1.47 & 0.25 & 0 & 1 & -2.7 \\ 0.25 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (23)$$

From [S], the inner wise determinants are calculates as $\nabla_1 = 1.25>0$, $\nabla_3 = -5.0540<0$ and $\nabla_5 = 5.9291>0$. Since $\nabla_3<0$, it is inferred that the one-dimensional equivalent system is unstable which indicates that the given two-dimensional system is also unstable.

Illustrations 6: The two-dimensional discrete polynomial available by Tzafestas (1986) is used as:

$$B(Z_1, Z_2) = A_3(Z_1)Z_2^3 + A_2(Z_1)Z_2^2 + A_1(Z_1)Z_2 + A_0(Z_1)$$

Where:

$A_3(Z_1) = 0.10Z_1^3 + 2.0Z_1^2 + 1.3Z_1 + 1.1$
$A_2(Z_1) = -0.9Z_1^3 - 2.0Z_1^2 + 1.5Z_1 + 0.25$
$A_1(Z_1) = 1.7Z_1^3 - 0.85Z_1^2 + 1.25Z_1 + 0.10$
$A_0(Z_1) = -0.25Z_1^3 - 0.40Z_1^2 + 0.70Z_1 + 1.0$

Test for necessary condition:

$$B(Z_1, 0) = \text{-}0.25Z_1^3 \text{-} 0.40Z_1^2 + 0.70Z_1 + 1.0 \neq 0 \text{ for } |Z_1| \leq 1$$

The reciprocal polynomial with simplification is:

$$B\left(\frac{1}{Z_1}\right) = Z_1^3 + 0.7Z_1^2 \text{-} 0.4Z \text{-} 0.25 = 0 \text{ for } |Z_1| \leq 1$$

The two triangular matrices are formed as:

$$[t_1] = \begin{pmatrix} 1 & 0.7 & -0.4 & -0.25 \\ 0 & 1 & 0.7 & -0.4 \\ 0 & 0 & 1 & 0.7 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (24)$$

$$[t_2] = \begin{pmatrix} 1 & 0.7 & -0.40 & -0.25 \\ 0.7 & 0.4 & -0.25 & 0 \\ 0.4 & -0.25 & 0 & 0 \\ -0.25 & 0 & 0 & 0 \end{pmatrix} \quad (25)$$

Their corresponding square matrix:

$$[S]=\begin{pmatrix} 2 & 0.4 & -0.8 & -0.5 \\ 0.7 & 1.4 & 0.45 & -0.4 \\ 0.4 & -0.25 & 1 & 0.7 \\ -0.25 & 0 & 0 & 1 \end{pmatrix} \quad (26)$$

The inner wise determinants are evaluated as $\nabla_2 = 1.5125>0$ and $\nabla_4 = 3.1222>0$. Since, ∇_2 and ∇_4 are positive, the necessary condition for stability is satisfied.

Test for sufficiency: The one-dimensional equivalent polynomial for $B(Z_1, Z_2)$ in x:

$$F(x) = x^6+1.4x^5+1.1x^4+1.5x^3+1.6x^2+1.1x+0.1$$

Forming the two triangular matrices the square matrix for F(x) is:

$$[S]=\begin{pmatrix} 2 & 2.8 & 2.2 & 3 & 3.2 & 2.2 & 0.2 \\ 1.4 & 2.1 & 2.9 & 2.7 & 2.6 & 1.7 & 1.1 \\ 1.1 & 1.5 & 2.6 & 2.5 & 1.2 & 1.5 & 1.6 \\ 1.5 & 1.6 & 1.1 & 1.1 & 1.4 & 1.1 & 1.5 \\ 1.6 & 1.1 & 0.1 & 0 & 1 & 1.4 & 1.1 \\ 1.1 & 0.1 & 0 & 0 & 0 & 1 & 1.4 \\ 0.1 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (27)$$

The inner wise determinants of [S] are evaluated: $\nabla_1 = 1.1>0$, $\nabla_3 = 0.3280>0$, $\nabla_5 = -0.2614<0$, $\nabla_7 = 0.7471>0$. Since, $\nabla_5<0$, the one-dimensional equivalent discrete system is unstable and hence the original two-dimensional discrete is unstable.

Note that ∇_7 need not be evaluated since, $\nabla_5<0$ thus minimizing the computations; the one-dimensional equivalent polynomial F(x) can directly be analyzed for necessary and sufficient conditions. As per this from Agathoklis *et al.* (1993), it is observed that $F(1)>0$, $F(-1)<0$ and $0.1<1$. The second value shows unstable situation. Hence, the test for sufficiency can be stopped, thus minimizing further more computations

Illustrations 7: For the given (Shanks *et al.*, 1972):

$$B(Z_1, Z_2) = 1-aZ_1-bZ_2+cZ_1Z_2$$

It is proposed to make a choice of the values for a, b and c for marginal condition. The one-dimensional equivalent discrete polynomial can be formed as:

$$F(x) = x^2-x(a+b)+c$$

with the help of two triangular matrices, the square matrix is:

$$[S]=\begin{pmatrix} 2 & -2(a+b) & c \\ -(a+b) & 1+c & -(a+b) \\ c & 0 & 1 \end{pmatrix} \quad (28)$$

The inner wise determinates are written as:

$$\nabla_1 = 1+c$$
$$\nabla_3 = \left\lfloor 2(a+b)^2(c-1)\right\rfloor-\left[(c+1)(c^2+2)\right]$$

If the value of $\nabla_1>0$, then the given one dimensional equivalent system is stable otherwise the system is not stable. If the value of $\nabla_3>0$, then the given system is stable otherwise is unstable. In ∇_1, the value of $c\leq 1$ which indicates the given system is unstable.

CONCLUSION

From the illustrative examples, it is easily observed that the proposed necessary and sufficient conditions are sufficient enough to test the stability of Linear Time Invariant Discrete Systems. The proposed test procedures are direct and simple in application compare to other methods given by Jury and Bauer (1988) and Pee *et al.* (1986).

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