Method of Evaluation of Profile Form and Shaped Surfaces with Application of Wavelets

Vadim A. Pechenin, Michael A. Bolotov and Nikolay V. Rusanov
Faculty of Aircraft Engine, Samara State Aerospace University Named after S.P. Korolev
(National Research University), 443086 Samara, Russia

Abstract: In this study, we propose a new method for evaluation of the form deviation of the profiles and surfaces at the example of the compressor blades of gas turbine engine. The approach is based on separating the form and position deviation from the value of the total deviation of the nominal and the measured profile by using wavelet transformation of profiles coordinates and differences of profile normal angles. In the first stage of filter procedure the inclination normals vector and two-dimensional haar wavelet transformation of the point coordinates array are used. In the second stage construction of the linear regression for the final filtration of form deviations is performed. The method increases precision of the results in comparison with traditional methods used by software for coordinate measurement.

Key words: Complex part, location deviation, profile match, NURBS, haar wavelet

INTRODUCTION

Shaped form surfaces are widely used in many industries from the design and manufacture of molds and plastic products to the automobile and aerospace industries. The complex shape in parts is associated with numerous functional requirements and aesthetics. At the design stage, almost all CAD Systems have the ability to design and modeling of free-form surfaces. In the workshops CNC machines are used the manufacture of shaped form surface parts. To ensure the quality of production such details should be monitored after machining process. Particular attention during the course of various studies is given to the experimental data obtaining by measuring of the curved surfaces. For example, it is the study of the plastic properties of alloys (Khaimovich and Balaykin, 2014). After the measurement of complex surfaces the coordinates of the measured and nominal points can be transformed into a parametric form (e.g., for optimizing the turbomachinery parameters (Shabliy and Dmitrieva, 2006)).

Because of the complexity of parts shape it is a difficult task to evaluate it sufficiently. In the case of simple geometric elements such a cylinder the established techniques in industrial production control and measuring equipment are applying. Sufficient control of the complicated surface shape for example in a gas turbine engine airfoil is difficult to provide with standard measurement tools (e.g., patterns). The measurement of such items is possible with developed in the last century coordinate measuring machines which a mathematical model of the product shape in addition to size determination define. The use of the machines increases the precision and reliability of measurement results.

As a result of coordinate measurement a cloud of points is uploaded onto system of coordinate measuring machine. Then the points are compared with the feature points of the CAD-Model. Part form evaluation can be made only after location deviation evaluation (alignment of coordinate system of measured parts and the CAD Model) (Li and Gu, 2004).

Majority of software products for the best match of parts with complicated form use an Iterative Closest Points algorithm (ICP, iterative closest point algorithm), based on the Least-Squares Method (Besl and Mckay, 1992).

In this study, we propose a method of the form deviation evaluation for complex products with application of the wavelet analysis tool and bypassing the best fit.

MATERIALS AND METHODS

The optimization problem of appropriate points search for the best fit: Geometric tolerance is the distance along the normal from the nominal profile to the measured profile after their best fit.

Corresponding Author: Vadim A. Pechenin, Faculty of Aircraft Engine, Samara State Aerospace University Named after S.P. Korolev (National Research University), 443086 Samara, Russia

Curves of nominal and measured profile can be represented as a NURBS curve. This is a typical representation of curves and surfaces in the CAD-Systems (Li and Gu, 2004). NURBS-curves are piecewise splines which can be expressed as:

$$P(u) = \frac{\sum_{i=1}^{N} h_i \cdot P_i \cdot N_{i,k}(u)}{\sum_{i=1}^{N} h_i \cdot N_{i,k}(u)} \qquad (1)$$

where, P_i is a vector $(x_{,i} y_{,i} z)_i$ which combines the coordinates of the ith point in three-dimensional space as well as for non-uniform rational B-splines. The range of values of the parameter is $[t_{k-1}, t_{n+1}]$. h_i is a point weight.

In most cases, the best combination of profiles is performed using ICP-algorithm. In general terms the algorithm is an Iterative algorithm which solves the problem of the optimal rotation and movement to align the point cloud to the nominal points of the CAD Model. The task is to find the minimum of function:

$$f(R,t) = \frac{1}{n} \sum_{i=1}^{N} \left\| R \cdot p_i + t - q_i \right\|^2 \qquad (2)$$

Where:

n = A number of fit points (measured points)
p_i = The coordinates of the ith point
R = Point rotation matrix
t = The displacement vector
q_i = The coordinates of a point on the CAD Models

The aim of the best combination is to find t and R of estimated profile relative to the nominal profile and transform the coordinates of the estimated profile eliminating the location error. The coordinates of the real profile best-matched to the nominal profile can be found by matrix product of points coordinates of the real profile of the transformation matrix:

$$P_{np} = P \cdot M \qquad (3)$$

where, M is the transformation matrix.

Mathematical description of the proposed methods for form deviations evaluation: The angle between the measured profile and its CAD Model can be evaluated due to the absolute differences between angles of the normals at the points CAD Model and the measured profile.

Until the profile is matched well with the nominal profile the angles between the normals in appropriate locations will are large enough. If the level of form deviation is low the difference between angles of the normals tends to 0 after the best-match. In the case of form deviations the absolute values of the differences are a cloud of points along the axis OX. We construct points linear regression model of the form Y = kX+b considering the points coordinates so we obtain the constant b which is equal within certain error to the angle of rotation of the measured profile relative to the nominal profile.

Finding the angle of rotation of the profile it is performed a U-turn at an angle b. Then in order to find the profile transposition matrix the method of two-dimensional Haar transform is applied to coordinates obtained after the measured profile turn and the coordinates of the nominal profile.

Dimensional Haar basis consists of a scaling function Eq. 4 and a wavelet function Eq. 5 (Mallat, 1998):

$$\phi(x) = \begin{cases} 1, 0 \leq x < 1, \\ 0, x \notin [0;1), \end{cases} \qquad (4)$$

$$\psi(x) = \begin{cases} 1, & 0 \leq x < 1/2 \\ -1, & 1/2 \leq x \leq 1 \\ 0, & x \notin [0;1) \end{cases} \qquad (5)$$

Two-dimensional scaling function can be defined as the product of one-dimensional scaling functions:

$$\phi(x, y) = \phi(x)\phi(y) \qquad (6)$$

Two-dimensional wavelet function is defined as:

$$\psi^1(x, y) = \phi(x)\psi(y) \qquad (7)$$

$$\psi^2(x, y) = \phi(x)\psi(y) \qquad (8)$$

$$\psi^3(x, y) = \phi(x)\psi(y) \qquad (9)$$

In general formula of total deviation in shape and position of the profile points can be expressed by Eq. 10 (Menq *et al.*, 1992):

$$(Q_{n \times 2} + F_{n \times 2}) \cdot R_{2 \times 2} + T_{n \times 2} - Q_{n \times 2} = dF_{n \times 2} \qquad (10)$$

Where:

$F_{n \times 2}$ = Form deviations matrix
$Q_{n \times 2}$ = Matrix of nominal points coordinates
$T_{n \times 2}$ = Transpose matrix of points (error of position along the axes x and y)

Applying the two-dimensional discrete Haar transformation (Eq. 13), we obtain the system of equations:

$$\begin{cases} H'_{n/2 \times n} \cdot ((Q_{n \times 2} + F_{n \times 2}) \cdot R_{2 \times 2} + T_{n \times 2} - Q_{n \times 2}) \cdot H_{2 \times 2} = H'_{n/2 \times n} \cdot dF_{n \times 2} \cdot H_{2 \times 2} \\ H''_{n/2 \times n} \cdot ((P_{n \times 2} + F_{n \times 2}) \cdot R_{2 \times 2} + T_{n \times 2} - Q_{n \times 2}) \cdot H_{2 \times 2} = H''_{n/2 \times n} \cdot dF_{n \times 2} \cdot H_{2 \times 2} \end{cases}$$

(11)

where, $H_{2 \times 2}$ $H'_{n/2 \times n}$ $H''_{n/2 \times n}$ are a two-dimensional Haar transform matrixes.

The first equation contains factors responsible for the high-frequency approximation and vertical high-frequency decomposition. The second one includes the high-frequency coefficients of horizontal and diagonal decomposition. Matrix products in the left sides of (Eq. 11) are arranged on the sum of matrices. In the matrix product $H'_{n/2 \times n} \cdot (Q_{n \times 2} + F_{n \times 2}).R_{2 \times 2}.H_{2 \times 2}$ matrix R can be adopted as a unit if the steering angle is considered by the difference between the angles of normals inclination. The matrix $T_{n \times 2}$ does not change the values in rows so in the expression $H'_{n/2 \times n}.T_{n \times 2}.H_{2 \times 2}$ the lines are the same as well. We obtain the system:

$$\begin{cases} H'_{n/2 \times n} \cdot F_{n \times 2} \cdot H_{2 \times 2} + H'_{n/2 \times n} \cdot T_{n \times 2} \cdot H_{2 \times 2} = H'_{n/2 \times n} \cdot dF_{n \times 2} \cdot H_{2 \times 2} \\ H''_{n/2 \times n} \cdot F_{n \times 2} \cdot H_{2 \times 2} + H''_{n/2 \times n} \cdot T_{n \times 2} \cdot H_{2 \times 2} = H''_{n/2 \times n} \cdot dF_{n \times 2} \cdot H_{2 \times 2} \end{cases}$$

(12)

Then linear regression models were constructed on the basis of the coefficients of the left sides of the equations in the form of $Y = k'X + b'$ so, we obtain the parameters b' which are equal to a constant $H'_{n/2 \times n}.T_{n \times 2}.H_{2 \times 2}$. When, we subtract these constants from Eq. 11 and obtain the coefficients of haar responsible for the form deviation. After procedures of inverse discrete wavelet transform is performed we obtain the form deviation at the nominal profile points.

Methods of assessing the precision of the results: To asess the precision of the best match it is more appropriate to use the relative values of the comparison of its parameters (Zhang, 1994). The accuracy of the ICP algorithm can be characterized by deviation of rotation profile matrix and transposition matrix. The deviation of the rotation matrix is calculated as:

$$e_R = \frac{|R - R'|}{|R|} \cdot 100\%$$

(13)

Where:
R = Planted rotation profile matrix
R' = Planted rotation profile as a result of the algorithm calculating

Transposition matrix of deviation is calculated as follows:

$$e_T = \frac{|T - T'|}{|T|} \cdot 100\%$$

(14)

Where:
T = Planted transpose profile matrix
T' = A calculated transpose profile matrix as a result of the algorithm

Similarly, evaluates of the deviation of parameter deviations can be calculated average relative precision of the form deviation evaluation:

$$e_F = \frac{1}{N} \sum_1^N \frac{|\Delta F_i - \Delta F_i'|}{|\Delta F_i'|} \cdot 100\%$$

(15)

Where:
ΔF = The mortgaged deviation of the profile at the point
$\Delta F'$ = The calculated deviation of the profile at the point
N = Number of profile points

RESULTS AND DISCUSSION

The experiments were performed on the profiles of compressor blades. Different values of rotation and profile displacement and the coordinates of profile points with a position deviation and form deviation were simulated (Eq. 10). X offset x varies from 0.1-0.3 mm. Y offset is from 0.1-0.5 mm, the rotation angle varies from 0.1-0.7°. The two types of form deviation were analyzed (Fig. 1). Deviations amplitude ranges from 3-77 microns.

Table 1 shows the results of the error of transposition matrix and profile (e_R and e_T) turn matrix which were obtained by the ICP Method. The method performs the best match for nominal and measured points. Match quality has influence on the precision of the profile form evaluation.

As the table shows match errors increase while profile form deviation is increasing.

Table 2 shows the ratio error of determining the blades profiles form deviation using wavelet decomposition method ($e_{F, Wav}$) and the conventional method (e_F). It is calculated by the Eq. 12 using the simulated combinations of position deviations and profile form deviation.

Table 2 shows calculated deviations of the same order for the form deviation of the first type. Besides, the method of form evaluation by taking into account deviations of the normal vectors and using the Haar wavelet transform method is more accurate in the case of form deviations of the second kind. This is particularly evident at high angles of rotation of the profile.
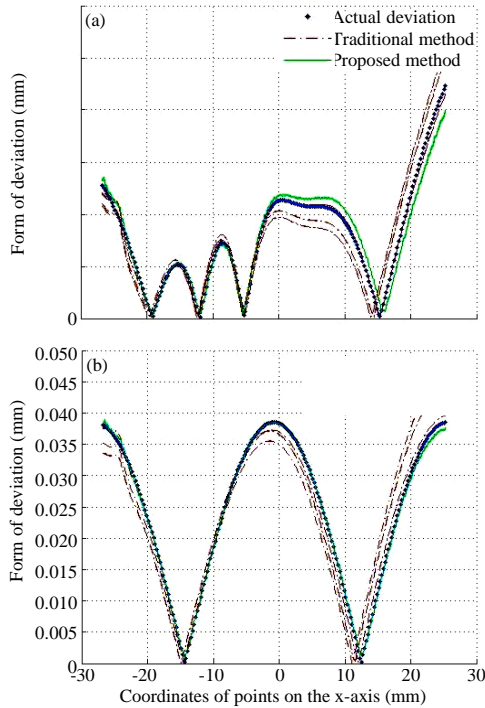
Fig. 1: Absolute values of evaluated form deviation according to traditional methods and the wavelet filter within error the a) first and b) second type
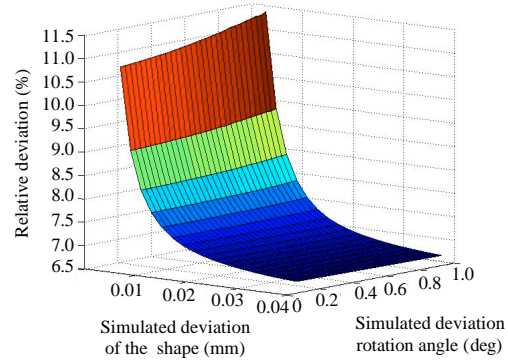

Fig. 2: The relative error of the evaluated deviations of the form by the method of wavelet analysis at different angles of rotation and simulated deflection shapes

Figure 2 represents the surface of relative deviations in the angle error $e_{F, Wav}$. The deviation of rotation is along axis X and the form deviation is along axis Y. We can see that the amplitude of the evaluated form deviations influences the precision of its determination. Accuracy is growing faster than the pledged deviation increases.

The error of the measured profile form estimation depends on the assessment of position deviation. Determination of the profile position deviations by the proposed method is carried out by constructing a linear regressive dependence. Therefore, the precision of the estimation of the profile position deviation is determined by the number of profile points involved in the assessment and the form.

It is typically as form deviation amplitude increases 2-folds the relative error is reduced by 1.5-3 times (depending on the simulated deviations). At high values of position deviations the method precision is higher in comparison with the method of ICP (Table 2). For the profiles the error of position deviation evaluation is 9% using the traditional method, the form deviation error is about 35-60% at fractional deviations and 10-36% at significant deviations. Using supposed method the form deviation error is about 11-50% at fractional deviationsand 7-20% at significant deviations. Thus, the evaluation precision of form deviation with application of supposed method is higher by 25-30% then traditional method precision. The accuracy of the method decreases with increasing angle of measured profile rotation and the

Table 1: Ratio error of the profile fit parameters according to the algorithm ICP

Ranges of deviation along the x and y	The form deviation (mm)	The angle of rotation is 0.1°		The angle of rotation is 0.4°		The angle of rotation is 0.7°	
		e_T (%)	e_R (%)	e_T (%)	e_R (%)	e_T (%)	e_R (%)
The form deviation of the first type							
x = 0.1, ..., 0.3 (mm)	0.0034	4.38	2.02	5.05	0.51	5.70	0.31
y = 0.1, ..., 0.5 (mm)	0.0096	4.75	2.26	5.43	0.63	6.07	0.34
	0.0158	5.70	3.18	6.36	0.80	7.05	0.43
	0.0220	6.37	3.75	7.02	0.94	7.65	0.55
	0.0281	7.03	4.33	7.68	1.08	8.30	0.64
	0.0343	7.69	4.90	8.34	1.29	8.96	0.72
The form deviation of the second type							
x = 0.1. .... 0.3 (mm)	0.0077	4.36	1.90	5.07	0.47	5.72	0.29
y = 0.1. .... 0.5 (mm)	0.0216	4.71	1.94	5.39	0.49	6.12	0.31
	0.0354	5.63	2.24	6.31	0.62	6.96	0.34
	0.0493	6.26	2.44	6.95	0.92	7.60	0.37
	0.0631	6.90	2.64	7.59	1.24	8.24	0.40
	0.0770	7.53	2.85	8.22	1.62	8.88	0.42

Table 2: Ratio error the blades profiles form deviation

Ranges of deviation along the x and y	The form deviation (mm)	The angle of rotation is 0.1°		The angle of rotation is 0.4°		The angle of rotation is 0.7°	
		e_T (%)	e_R (%)	e_T (%)	e_R (%)	e_T (%)	e_T (%)
The form deviation of the first type							
x = 0.1, ..., 0.3 (mm)	0.0034	38.85	61.42	44.21	59.85	50.98	58.57
y = 0.1, ..., 0.5 (mm)	0.0096	22.42	37.45	23.19	37.03	24.89	37.60
	0.0158	20.81	14.68	21.03	28.81	21.54	29.64
	0.0220	20.44	12.61	20.60	27.36	20.78	27.86
	0.0281	20.34	11.50	20.47	26.49	20.61	26.81
	0.0343	20.32	10.81	20.42	36.43	20.53	26.14
The form deviation of the second type							
x = 0.1, ..., 0.3 (mm)	0.0077	11.17	17.17	11.44	31.30	11.75	35.37
y = 0.1, ..., 0.5 (mm)	0.0216	7.68	12.57	7.72	12.34	7.79	32.07
	0.0354	7.07	11.69	7.06	11.54	7.08	31.34
	0.0493	6.86	11.32	6.84	11.21	6.84	26.79
	0.0631	6.76	11.12	6.74	11.04	6.73	26.40
	0.0770	6.69	11.02	6.67	10.94	6.66	10.87

parameters of the matrix transpose. Thus, the estimation error form has a complex dependence on many factors. Hence, the form evaluation error has a complicated dependence on many factors.

CONCLUSION

This study proposes an alternative to the existing method of evaluation of complex form deviation using the Haar wavelet transform of the measured data.

The main part of the existing methods for assessing deviations of the form based on two stages: the best-match procedure for calculating of position deviations parameters; the calculation of form deviation as the distance from the nominal profile point to the measured profile points.

The proposed method is based on the geometric features of the profile such as normal angles and wavelet spectrum array. The process of form deviation evaluation also includes the step of location devaluation evaluation and form evaluating.

Comparison of traditional and proposed approaches shows similar values of the methods accuracy. When simulated form deviation is symmetric (Fig. 1b) wavelet transform performs more accurate assessment of the deviation in comparison to the traditional method. Form evaluation accuracy by the proposed method depends on the nature and value of the form deviation (Fig. 1 and 2). Thus, the new method of assessing the form deviation may be used when measuring parts have a complicated surface shape. The use of wavelets in the proposed method offers great potential to improve the accuracy. In the next study, we plan method improvement in order to improve its accuracy by application of adaptive mechanisms which include analysis of the profile features and its partial filtering during evaluating of the location.

REFERENCES

Besl, P.J. and N.D. McKay, 1992. A method for registration of 3-D shapes. IEEE Trans. Pattern Anal. Mach. Intell., 14: 239-256.

Khaimovich, A.I. and A.V. Balaykin, 2014. Analysis of titanium alloys plastic properties under severe deformation conditions in machining. ARPN J. Eng. Applied Sci., 9: 1828-1833.

Li, Y. and P. Gu, 2004. Free-form surface inspection techniques state of the art review. Comput. Aided Des., 36: 1395-1417.

Mallat, S.G., 1998. A Wavelet Tour of Signal Processing. 2nd Edn., Academic Press, London, UK., ISBN-13: 9780124666054, Pages: 577.

Menq, C.H., H.T. Yau and G.Y. Lai, 1992. Automated precision measurement of surface profile in CAD-directed inspection. IEEE Trans. Rob. Autom., 8: 268-278.

Shabliy, L.S. and I.B. Dmitrieva, 2006. Conversion of the blade geometrical data from points cloud to the parametric format for optimization problems. ARPN J. Eng. Applied Sci., 9: 1849-1853.

Zhang, Z., 1994. Iterative point matching for registration of free-form curves and surfaces. Int. J. Comput. Vision, 13: 119-152.