

## Scatter Search for Solving Team Orienteering Problem

<sup>1</sup>Hamzah Ali Alkhazaleh, <sup>1</sup>Masri Ayob and <sup>2</sup>Zulkifli Ahmad

<sup>1</sup>Center for Artificial Intelligence, Faculty of Information Science and Technology,

<sup>2</sup>School of Linguistics and Language Studies, Faculty of Social Sciences and Humanities,  
Universiti Kebangsaan Malaysia, 43600 Bangi, Selangor, Malaysia

---

**Abstract:** This research proposes a scatter search metaheuristic approach for solving Team Orienteering Problem. The goal is to build a particular number of routes that visit some points to maximize the sum of the score while the route's length does not exceeding the time budget. The approach is compared to other state-of-art approaches and tested using a large set of test instances from the literature. The obtained results are competitive comparing the best known results of these heuristics but the computational time is reduced significantly.

**Key words:** Scatter search, team orienteering problem, heuristic search, computational time, metaheuristic approach

---

### INTRODUCTION

Orienteering Problem (OP) originates from sport skiing game of orienteering (Chao *et al.*, 1996) where the players, start at one point and end at other point in a certain amount of time. The player needs to collect as many as possible the scores from the visited points where each point has its own score. The goal is to maximize the total collected scores before reaching the end points within a given amount of timeframe. The Team Orienteering Problem (TOP) (Chao *et al.*, 1996; Tang and Miller-Hooks, 2005) (or called as multiple tour maximum collection problem (Butt and Cavalier, 1994)) is an OP where the goal is to determine the path, P that gives total high scores, limited to  $T_{max}$ . The TOP consists of a team of several players each collecting scores during the same time span.

The OP is also applicable for several problems such as selective travelling salesperson problem (Gendreau *et al.*, 1998), maximum collection problem and intelligent tourist travel guide system (Vansteenwegen *et al.*, 2009b). These problems were studied and many exact and heuristic solutions were used over the past years and were modeled as orienteering problem.

There are several exact methods that have been proposed to solve OP such as branch and cut (Fischetti *et al.*, 1998) and branch and bound (Ramesh *et al.*, 1992). Based on Fischetti *et al.* (1998), instances up to 500 location can be solved by branch and

cut procedure. Golden *et al.* (1988) proved that OP is a Non-deterministic Polynomial-time (NP)-hard problem. Therefore, the exact methods could not solve this problem in a reasonable computation time.

Recently, many researchers focused on applying metaheuristic approaches to solve TOP. For instance, Ke *et al.* (2008) proposed Ant Colony Optimization (ACO) with four methods such as the sequential, deterministic-concurrent, random-concurrent and simultaneous methods to construct candidate solutions. These methods are used to obtain best known solution quality within less than one minute of calculation time. Vansteenwegen *et al.* (2009a) proposed an algorithm that combines Guided Local Search (GLS) and a skewed Variable Neighborhood Search (VNS) algorithm to solve the TOP. More recently, a memetic algorithm was proposed by Bouly *et al.* (2010). They utilize an optimal split procedure for chromosome evaluation and local search techniques for mutation. This approach had shown that it could obtain good solution within less than one minute of computation time.

As far as researchers are concerned, none of the researches were reported to apply scatter search in solving TOP. Scatter search has been proposed by Glover (1998) as a population-based meta-heuristic in which solutions are intelligently combined to yield better solutions (Glover *et al.*, 2003). It has been successfully used to solve a variety of scheduling problem such as exam timetabling (Sabar and Ayob, 2009), course timetabling (Jaradat and Ayob, 2011), traveling

salesman problems (Liu, 2008), vehicle routing problem (Maquera *et al.*, 2011), flow shop scheduling problem (Engin *et al.*, 2009) and nurse rostering problem (Burke *et al.*, 2010). Therefore, this research investigates the performance of applying scatter search algorithm to solve TOP.

### PROBLEM FORMULATION

The TOP is formulated as in Eq. 1 (Vansteenwegen *et al.*, 2009a) where the decision variables:  $x_{ijr} = 1$  if in route  $r$ , a visit to point  $i$  is followed by visit to point  $j$  or 0 otherwise;  $y_{ir} = 1$  if point  $i$  is visited in route  $r$  or 0 otherwise;  $u_{ir}$  = the position of point  $i$  in route  $r$ ,  $t_{ij}$  is the distance between points  $i$  and  $j$ ,  $S_i$  is the score at the  $i$ th point:

$$\text{Max} \sum_{r=1}^R \sum_{i=2}^{N-1} S_i y_{ir} \quad (1)$$

$$\sum_{r=1}^R \sum_{j=2}^N x_{ijr} = \sum_{r=1}^R \sum_{i=2}^{N-1} x_{iNr} = R \quad (2)$$

$$\sum_{r=1}^R y_{mr} \leq 1; \forall m = 2, \dots, N-1 \quad (3)$$

$$\sum_{i=1}^{N-1} x_{ikr} = \sum_{j=2}^N x_{kjr} = y_{kr}; \forall k = 2, \dots, N-1; \forall r = 1, \dots, R \quad (4)$$

$$\sum_{i=1}^{N-1} \sum_{j=2}^N t_{ij} x_{ijr} \leq T_{\max}; \forall r = 1, \dots, R \quad (5)$$

$$2 \leq \mu_{ir} \leq N; \forall i = 2, \dots, N; \forall r = 1, \dots, R \quad (6)$$

$$\mu_{ir} - \mu_{jr} + 1 \leq (N-1)(1-x_{ijr}); \forall i, j = 2, \dots, N; \forall r = 1, \dots, R \quad (7)$$

$$x_{ijr}, y_{ir} \in \{0, 1\}; \forall i, j = 1, \dots, N; \forall r = 1, \dots, R \quad (8)$$

The Eq. 1 is to maximize the total collected score, where  $R$  is the total number of route and  $N$  is the total points to be visited. Constraint (2) is to ensure that each route start at point 1 and ends at point  $N$ . Constraint (3) ensures that each point is only visited one time at most. Constraint (4) ensures that if a point is visited in a given route, it is outstripped and followed by an exact one other visit in the same route. Constraint (5) guarantees that each route does not exceed the time budget  $T_{\max}$ . Constraint (6) and (7) are important to prohibit sub-routes.

These sub-routes elimination constraints are formulated according to Miller-Tucker-Zemlin (MTZ) formulation of the TSP (Miller *et al.*, 1960).

### SCATTER SEARCH ALGORITHM

Scatter Search (SS) Algorithm was introduced by Glover (1977). Scatter search is an evolutionary algorithm which is very flexible in incorporating strategies for search intensification and diversification. Each of its elements can be implemented in a variety of ways according to the specific domain problem (Laguna and Armentano, 2005). Similarly to other evolutionary methods, SS operates with a population of solution rather than with a single solution at a time and employs procedure to combine these solutions to obtain new trial solution. SS explores solutions in the search space and selects a set of reference points. The reference set that consist of elite solutions (with good solution quality pool and diverse pool) is evolved and updated (Laguna *et al.*, 2003; Resende *et al.*, 2010). SS consists of five components (Resende *et al.*, 2010):

- A Diversification Generation Method is used to generate a collection of diverse trial solutions using one or more arbitrary trial solutions as an input
- Improvement method is used to transforms a trial solution into one or more enhanced trial solutions using any S-metaheuristic. In general, a candidate local search algorithm is applied and then a local optimum is generated
- A Reference Set Update Method is used to construct and maintain a reference set consisting of the  $b$  best solutions found (where the value of  $b$  is typically small, e.g., not  $>20$ ), organized to provide efficient access by other component procedure. Several alternative criteria may be used to add solutions to the reference set and delete solutions from the reference set
- A Subset Generation Method is used to operate on the reference set to produce a subset of its solutions as a basis for creating combined solutions. The most common subset generation method is to generate all pairs of reference solutions
- A Solution Combination Method is used to transform a given subset of solutions produced by the Subset Generation Method into one or more combined solutions

Researchers applied Scatter Search algorithm to tackle TOP based on Resende *et al.* (2010). The pseudo code of SS is shown:

**The pseudo code of scatter search for TOP:**

1. Start with  $P = \emptyset$ ,  $|P| = 50$ . Use the Diversification Generation Method to construct a solution and apply the improvement Method. Let  $\chi$  be the resulting solution. If  $\chi \notin P$  then add  $\chi$  to  $P$ , otherwise, discard  $\chi$ . Repeat step 1 until  $|P| = PSize$ .
2. Build reference set  $|RefSet| = 20$ ,  $RefSet = [X^1, \dots, X^b], [Y^1, \dots, Y^a]$  with the best  $b_1$  and divers  $b_2$  solutions in  $P$ . Order the solutions in  $b_1$  according to their objective function value such that  $\chi^1$  is the best solution and  $\chi^b$  the worst.  
while (StoppingCriterion)
  3. Generate NewSubsets with the subset generation Method.  
Select the subset of solutions randomly that contain 2 or 3 or 4 solutions.
  4. Apply the solution combination Method to NewSubsets to obtain one solution  $\chi^*$ .  
Select one the combination point randomly.
  5. Apply the improvement method to the  $\chi^*$ .
  6. Update the reference set. // if  $\chi^*$  better than the worse solution in  $b_1$  replace with  $\chi$  if replace  $\chi^*$  with the worse divers in  $b_2$

End While

**Diversification Generation Method:** SS contains a diversification generation method to generate  $m$  of diverse trial solution ( $m = 50$  in this study) by employing controlled randomization and frequency memory. A large number of diverse solutions have been generated randomly to obtain diverse region of the solution space. All the generated solutions considered in this research are feasible.

**Improvement method:** This component is used to enhance the solution generated via local search procedure which drives the solution to local optima. Researchers randomly generate a neighbor from five neighborhood structures (Vansteenwegen *et al.*, 2009a) at each iteration. Researchers use hill climbing as the local search to search for a better quality solution. Researchers use consecutive non improvement iterations as a termination criterion.

**A Reference Set Update Method:** After building the population (researchers use 50 diverse feasible solutions), researchers apply References Set Update Method to build RefSet which consists of two pools of elites solutions that were selected from the population based on best quality and best diversity. The RefSet was built in the research as follow: selects the best 10 quality solution from the population and store that in  $b_1$  and eliminated the solutions from population. Measure the diversity of the rest of solutions in population with the solution in  $b_1$  by calculating the distance. The distance between two solutions is calculated based on Maquera *et al.* (2011). Researchers calculate the average distance between each solution in the population with the solutions in  $b_1$  then select 10 solutions that have the highest average distance and add it to  $b_2$  as best diversity in the population.

**Subset Generation Method:** Subset Generation Method plays an important role in selecting pair for combination

method to generate new solution. In the research, researchers follow Glover's Method (Glover, 1999) and generate the subsets as 2-4 solution for the following combination.

**Solution Combination Method:** The main goal of this component is to generate new trial solution in another region of the solution space. In this study, the highest quality solution in the subset is selected and then shares 25% from other solutions with the highest quality solution. The new solution has an attributes of quality solution and perhaps infeasible. Repair method is applied on the infeasible solution to restore feasibility by removing the repeated points and the lowest score to fix the exceeding time budget.

The improvement method is employed in new solution to gain better solution quality by exploring its neighborhood. Then, researchers employ the references set update method to update the RefSet to the new solution. If the new solution's quality is better than worse solution in best quality list ( $b_1$ ) the new solution will replace it. If not we will update the best divers list ( $b_2$ ) by measuring the differences with the solutions in best quality list ( $b_1$ ) and compare it with existing solution in best divers list ( $b_2$ ) then researchers replace the worse divers by new solutions.

If there is no more new good solution, researchers will pick up one solution randomly from the population which contain 30 solutions. The operations of subset generation method until references update method are repeated until there is no more updating process on the references set.

## EXPERIMENTAL SETUP AND RESULTS

In this study, scatter search were examined on large numbers of test problem. There are four data sets used to

benchmark different approaches from (Chao *et al.*, 1996) each data sets contain different numbers of locations:  $n = 100$  (date set 4),  $n = 66$  (data set 5),  $n = 64$  (data set 6) and  $n = 102$  (date set 7) including start and end location. Each set contains instances with  $r$  equal to 2, 3 and 4 routes. The time budget  $T_{max}$  differs for each data set. The Scatter Search is coded in Java 1.7 and performed on Intel Pentium (R) Daul-Core 3.0 GHz CPU personal computer with 2 gigabyte RAM, running on Windows Vista operating system (32-bit).

The parameters for SS is set based on the preliminary experiment on the population size, reference set size and the number of non-improvement iteration. These are 50 diverse solutions for the population size, the reference set size = 20 (i.e., 10 good quality and 10 diverse solutions) and the non-improvement iteration = 200 iterations. The experimental results from the SS are compared with the best known results and results of the following research:

- A1: Tabu search by Tang and Miller-Hooks (2005)
- A2: Tabu search with penalty strategy by Archetti *et al.* (2007)
- A3: Tabu search with feasible strategy by Archetti *et al.* (2007)
- A4: Fast Variable Neighborhood by Archetti *et al.* (2007)
- A5: Slow Variable Neighborhood by Archetti *et al.* (2007)
- A6: Guided Local Search by Vansteenwegen *et al.* (2009a)
- A7: Sequential ant colony optimization by Ke *et al.* (2008)
- A8: Deterministic concurrent ant colony optimization by Ke *et al.* (2008)
- A9: Random concurrent ant colony optimization by Ke *et al.* (2008)
- A10: Simultaneous ant colony optimization by Ke *et al.* (2008)
- A11: Skewed Variable Neighborhood search by Vansteenwegen *et al.* (2009b)
- A12: Memetic Algorithm by Bouly *et al.* (2010)
- A13: Fast Path relinking by Souffriau *et al.* (2010)
- A14: Slow Path relinking by Souffriau *et al.* (2010)
- A15: Discrete Particle Swarm Optimization by Muthuswamy and Lam (2011)

Researchers exclude the instance which it the same result was obtained by above algorithms from the comparison. The comparison will be in 158 instances (Archetti *et al.*, 2007).

Scatter search had been runned 31 times for each instance. Table 1-4 summarized the statistical analysis of the results which include minimum and maximum results researchers then calculate the average and standard deviation for each instance. In Table 5-8 each test set will be compared to the best score for the algorithm with the best scores found by algorithms from the past studies.

Table 1: Test set 4

| Instances | Max. | Min. | Avg.   | SD     |
|-----------|------|------|--------|--------|
| p4.2.a    | 206  | 206  | 206.0  | 0.000  |
| p4.2.b    | 341  | 340  | 340.5  | 0.506  |
| p4.2.c    | 452  | 441  | 451.3  | 2.250  |
| p4.2.d    | 531  | 515  | 524.6  | 3.630  |
| p4.2.e    | 618  | 589  | 609.1  | 9.946  |
| p4.2.f    | 678  | 643  | 664.2  | 6.107  |
| p4.2.g    | 750  | 723  | 737.2  | 6.715  |
| p4.2.h    | 821  | 767  | 791.0  | 12.870 |
| p4.2.i    | 892  | 825  | 842.7  | 14.170 |
| p4.2.j    | 937  | 869  | 898.7  | 19.310 |
| p4.2.k    | 979  | 922  | 950.5  | 18.060 |
| p4.2.l    | 1032 | 955  | 993.9  | 18.390 |
| p4.2.m    | 1094 | 1007 | 1043.0 | 23.790 |
| p4.2.n    | 1119 | 1041 | 1088.0 | 18.040 |
| p4.2.o    | 1173 | 1082 | 1127.0 | 21.140 |
| p4.2.p    | 1195 | 1148 | 1174.0 | 12.510 |
| p4.2.q    | 1230 | 1178 | 1202.0 | 12.250 |
| p4.2.r    | 1249 | 1199 | 1226.0 | 14.390 |
| p4.2.s    | 1267 | 1218 | 1247.0 | 11.640 |
| p4.2.t    | 1295 | 1250 | 1275.0 | 8.954  |
| p4.3.c    | 193  | 187  | 188.0  | 2.243  |
| p4.3.d    | 335  | 315  | 324.3  | 5.307  |
| p4.3.e    | 467  | 438  | 450.5  | 8.437  |
| p4.3.f    | 579  | 531  | 562.6  | 13.110 |
| p4.3.g    | 653  | 619  | 642.3  | 7.603  |
| p4.3.h    | 726  | 698  | 716.8  | 6.927  |
| p4.3.i    | 788  | 745  | 771.7  | 13.010 |
| p4.3.j    | 839  | 769  | 812.6  | 17.160 |
| p4.3.k    | 894  | 775  | 862.5  | 28.160 |
| p4.3.l    | 949  | 835  | 899.0  | 23.470 |
| p4.3.m    | 1009 | 905  | 977.5  | 25.240 |
| p4.3.n    | 1078 | 995  | 1044.0 | 22.760 |
| p4.3.o    | 1145 | 1063 | 1110.0 | 19.740 |
| p4.3.p    | 1190 | 1113 | 1160.0 | 19.610 |
| p4.3.q    | 1212 | 1161 | 1190.0 | 11.810 |
| p4.3.r    | 1244 | 1210 | 1223.0 | 8.990  |
| p4.3.s    | 1268 | 1228 | 1243.0 | 9.201  |
| p4.3.t    | 1290 | 1249 | 1268.0 | 9.514  |
| p4.4.e    | 183  | 182  | 182.6  | 0.486  |
| p4.4.f    | 324  | 300  | 321.5  | 6.244  |
| p4.4.g    | 461  | 440  | 457.1  | 6.946  |
| p4.4.h    | 564  | 520  | 543.7  | 10.980 |
| p4.4.i    | 654  | 607  | 640.6  | 13.100 |
| p4.4.j    | 719  | 676  | 702.6  | 12.550 |
| p4.4.k    | 813  | 758  | 793.4  | 19.630 |
| p4.4.l    | 872  | 795  | 858.6  | 16.360 |
| p4.4.m    | 907  | 870  | 892.1  | 9.103  |
| p4.4.n    | 947  | 887  | 927.2  | 12.520 |
| p4.4.o    | 1025 | 975  | 996.5  | 14.350 |
| p4.4.p    | 1090 | 1047 | 1072.0 | 12.810 |
| p4.4.q    | 1130 | 1075 | 1109.0 | 13.540 |
| p4.4.r    | 1185 | 1134 | 1164.0 | 13.050 |
| p4.4.s    | 1232 | 1171 | 1205.0 | 15.670 |
| p4.4.t    | 1271 | 1218 | 1243.0 | 13.680 |

Table 2: Test set 5

| Instances | Max. | Min. | Avg.    | SD    |
|-----------|------|------|---------|-------|
| p5.2.h    | 410  | 345  | 378.23  | 18.51 |
| p5.2.i    | 480  | 410  | 451.94  | 19.26 |
| p5.2.j    | 580  | 550  | 567.26  | 11.75 |
| p5.2.k    | 670  | 625  | 653.71  | 13.41 |
| p5.2.l    | 795  | 735  | 762.26  | 19.36 |
| p5.2.m    | 860  | 810  | 839.52  | 13.93 |
| p5.2.n    | 920  | 895  | 910.32  | 8.56  |
| p5.2.o    | 1015 | 945  | 976.29  | 16.17 |
| p5.2.p    | 1135 | 1050 | 1087.10 | 24.96 |
| p5.2.q    | 1175 | 1135 | 1155.81 | 11.04 |
| p5.2.r    | 1255 | 1205 | 1225.81 | 11.84 |
| p5.2.s    | 1315 | 1275 | 1288.06 | 10.06 |
| p5.2.t    | 1375 | 1335 | 1357.74 | 9.30  |
| p5.2.u    | 1450 | 1395 | 1417.74 | 12.77 |
| p5.2.v    | 1490 | 1450 | 1474.03 | 9.17  |
| p5.2.w    | 1540 | 1510 | 1525.32 | 10.08 |
| p5.2.x    | 1590 | 1565 | 1575.16 | 6.52  |
| p5.2.y    | 1625 | 1605 | 1613.39 | 5.23  |
| p5.2.z    | 1675 | 1635 | 1652.90 | 8.04  |
| p5.3.k    | 495  | 470  | 487.90  | 9.55  |
| p5.3.l    | 595  | 535  | 569.52  | 13.62 |
| p5.3.m    | 650  | 635  | 647.58  | 3.62  |
| p5.3.n    | 755  | 705  | 734.03  | 15.41 |
| p5.3.o    | 870  | 810  | 857.58  | 17.31 |
| p5.3.p    | 990  | 930  | 979.68  | 12.31 |
| p5.3.q    | 1070 | 1020 | 1048.06 | 12.95 |
| p5.3.r    | 1120 | 1090 | 1099.35 | 6.02  |
| p5.3.s    | 1190 | 1145 | 1173.39 | 12.93 |
| p5.3.t    | 1255 | 1210 | 1236.45 | 9.85  |
| p5.3.u    | 1330 | 1285 | 1314.84 | 9.96  |
| p5.3.v    | 1400 | 1360 | 1376.61 | 10.98 |
| p5.3.w    | 1455 | 1410 | 1433.71 | 9.83  |
| p5.3.x    | 1520 | 1485 | 1500.48 | 8.00  |
| p5.3.y    | 1590 | 1545 | 1568.55 | 12.79 |
| p5.3.z    | 1635 | 1590 | 1615.65 | 11.16 |
| p5.4.m    | 555  | 495  | 539.03  | 16.70 |
| p5.4.n    | 620  | 620  | 620.00  | 0.00  |
| p5.4.o    | 690  | 655  | 672.58  | 9.99  |
| p5.4.p    | 760  | 710  | 746.13  | 15.31 |
| p5.4.q    | 860  | 820  | 846.29  | 11.76 |
| p5.4.r    | 960  | 875  | 904.84  | 17.58 |
| p5.4.s    | 1025 | 995  | 1007.74 | 8.84  |
| p5.4.t    | 1160 | 1095 | 1133.87 | 19.65 |
| p5.4.u    | 1285 | 1165 | 1245.97 | 25.90 |
| p5.4.v    | 1320 | 1235 | 1294.84 | 22.34 |
| p5.4.w    | 1370 | 1345 | 1354.35 | 6.29  |
| p5.4.x    | 1430 | 1350 | 1420.00 | 16.02 |
| p5.4.y    | 1500 | 1415 | 1478.23 | 15.73 |
| p5.4.z    | 1595 | 1520 | 1556.45 | 19.76 |

Table 3: Test set 6

| Instances | Max. | Min. | Avg.    | SD   |
|-----------|------|------|---------|------|
| p6.2.d    | 192  | 192  | 192.00  | 0.00 |
| p6.2.e    | 360  | 360  | 360.00  | 0.00 |
| p6.2.f    | 588  | 588  | 588.00  | 0.00 |
| p6.2.g    | 660  | 660  | 660.00  | 0.00 |
| p6.2.h    | 780  | 780  | 780.00  | 0.00 |
| p6.2.i    | 888  | 888  | 888.00  | 0.00 |
| p6.2.j    | 948  | 936  | 939.10  | 3.42 |
| p6.2.k    | 1032 | 1020 | 1028.32 | 4.29 |
| p6.2.l    | 1116 | 1092 | 1102.06 | 5.23 |
| p6.2.m    | 1182 | 1158 | 1167.68 | 7.70 |
| p6.2.n    | 1260 | 1230 | 1246.45 | 8.05 |
| p6.3.g    | 282  | 276  | 281.23  | 2.04 |

Table 3: Continue

| Instances | Max. | Min. | Avg.    | SD   |
|-----------|------|------|---------|------|
| p6.3.h    | 444  | 438  | 442.45  | 2.67 |
| p6.3.i    | 642  | 618  | 630.39  | 7.89 |
| p6.3.j    | 828  | 816  | 824.90  | 4.06 |
| p6.3.k    | 894  | 858  | 882.39  | 7.89 |
| p6.3.l    | 990  | 960  | 973.16  | 7.81 |
| p6.3.m    | 1074 | 1038 | 1059.87 | 8.13 |
| p6.3.n    | 1164 | 1134 | 1151.42 | 9.21 |
| p6.4.j    | 366  | 366  | 366.00  | 0.00 |
| p6.4.k    | 528  | 516  | 521.03  | 3.83 |
| p6.4.l    | 696  | 678  | 692.52  | 4.59 |
| p6.4.m    | 912  | 882  | 899.61  | 9.42 |
| p6.4.n    | 1068 | 1050 | 1064.32 | 4.82 |

Table 4: Test set 7

| Instances | Max. | Min. | Avg.    | SD    |
|-----------|------|------|---------|-------|
| p7.2.d    | 190  | 179  | 184.68  | 5.59  |
| p7.2.e    | 290  | 289  | 289.58  | 0.50  |
| p7.2.f    | 387  | 382  | 385.87  | 1.59  |
| p7.2.g    | 459  | 456  | 456.81  | 1.17  |
| p7.2.h    | 521  | 517  | 517.90  | 0.98  |
| p7.2.i    | 579  | 530  | 569.16  | 15.56 |
| p7.2.j    | 641  | 597  | 625.10  | 9.05  |
| p7.2.k    | 699  | 682  | 691.90  | 4.16  |
| p7.2.l    | 758  | 714  | 740.84  | 10.69 |
| p7.2.m    | 827  | 776  | 801.90  | 13.76 |
| p7.2.n    | 888  | 840  | 866.45  | 10.75 |
| p7.2.o    | 939  | 892  | 919.19  | 13.13 |
| p7.2.p    | 988  | 938  | 971.35  | 13.67 |
| p7.2.q    | 1029 | 990  | 1009.16 | 12.33 |
| p7.2.r    | 1078 | 1041 | 1058.16 | 10.71 |
| p7.2.s    | 1122 | 1095 | 1103.71 | 6.66  |
| p7.2.t    | 1159 | 1131 | 1145.94 | 7.06  |
| p7.3.h    | 425  | 422  | 424.68  | 0.79  |
| p7.3.i    | 487  | 482  | 486.16  | 1.29  |
| p7.3.j    | 564  | 547  | 558.87  | 4.51  |
| p7.3.k    | 632  | 609  | 627.74  | 6.23  |
| p7.3.l    | 684  | 668  | 680.81  | 3.70  |
| p7.3.m    | 762  | 739  | 754.00  | 5.95  |
| p7.3.n    | 820  | 783  | 799.55  | 10.26 |
| p7.3.o    | 874  | 832  | 846.39  | 8.14  |
| p7.3.p    | 929  | 908  | 917.26  | 3.71  |
| p7.3.q    | 977  | 956  | 965.03  | 4.49  |
| p7.3.r    | 1015 | 996  | 1005.55 | 4.86  |
| p7.3.s    | 1072 | 1038 | 1053.71 | 6.66  |
| p7.3.t    | 1116 | 1096 | 1103.39 | 3.39  |
| p7.4.g    | 217  | 209  | 212.23  | 2.60  |
| p7.4.h    | 285  | 283  | 283.97  | 1.02  |
| p7.4.i    | 366  | 366  | 366.00  | 0.00  |
| p7.4.j    | 462  | 459  | 461.90  | 0.54  |
| p7.4.k    | 518  | 514  | 516.90  | 1.66  |
| p7.4.l    | 590  | 575  | 581.52  | 4.79  |
| p7.4.m    | 646  | 627  | 641.10  | 4.99  |
| p7.4.n    | 726  | 706  | 720.29  | 6.38  |
| p7.4.o    | 777  | 744  | 767.10  | 8.17  |
| p7.4.p    | 840  | 808  | 824.87  | 7.33  |
| p7.4.q    | 899  | 877  | 888.52  | 5.70  |
| p7.4.r    | 960  | 931  | 950.10  | 6.36  |
| p7.4.s    | 1006 | 977  | 987.87  | 7.96  |
| p7.4.t    | 1046 | 1006 | 1031.74 | 7.97  |

The number of times the best results for each algorithm equals the best known result will be summarized in Table 9. It includes also the average gaps

Table 5: Results test set 4

| Instances | Best | SS   | A1   | A2   | A3   | A4   | A5   | A6   | A7   | A8   | A9   | A10  | A11  | A12  | A13  | A14  | A15  |
|-----------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| p4.2.a    | 206  | 206  | 202  | 206  | 206  | 206  | 206  | 206  | 206  | 206  | 206  | 206  | 202  | 206  | 206  | 206  | 206  |
| p4.2.b    | 341  | 341  | 341  | 341  | 341  | 341  | 341  | 303  | 341  | 341  | 341  | 341  | 341  | 341  | 341  | 341  | 341  |
| p4.2.c    | 452  | 452  | 438  | 452  | 452  | 452  | 452  | 447  | 452  | 452  | 452  | 452  | 452  | 452  | 452  | 452  | 452  |
| p4.2.d    | 531  | 531  | 517  | 530  | 531  | 531  | 531  | 526  | 531  | 531  | 530  | 531  | 528  | 531  | 531  | 531  | 527  |
| p4.2.e    | 618  | 618  | 593  | 618  | 613  | 618  | 618  | 602  | 618  | 600  | 600  | 613  | 593  | 618  | 612  | 618  | 600  |
| p4.2.f    | 687  | 678  | 666  | 687  | 676  | 684  | 687  | 651  | 687  | 672  | 672  | 672  | 675  | 687  | 687  | 687  | 685  |
| p4.2.g    | 757  | 750  | 749  | 751  | 756  | 750  | 753  | 734  | 757  | 756  | 756  | 756  | 750  | 757  | 757  | 757  | 737  |
| p4.2.h    | 835  | 821  | 827  | 795  | 820  | 827  | 835  | 797  | 827  | 819  | 819  | 820  | 819  | 835  | 835  | 835  | 820  |
| p4.2.i    | 918  | 892  | 915  | 882  | 899  | 916  | 918  | 826  | 918  | 900  | 918  | 918  | 916  | 918  | 918  | 918  | 901  |
| p4.2.j    | 965  | 937  | 914  | 946  | 962  | 962  | 962  | 939  | 965  | 962  | 962  | 962  | 962  | 964  | 962  | 965  | 953  |
| p4.2.k    | 1022 | 979  | 963  | 1013 | 1013 | 1019 | 1022 | 994  | 1022 | 1016 | 1016 | 1016 | 1007 | 1022 | 1013 | 1022 | 1009 |
| p4.2.l    | 1074 | 1032 | 1022 | 1061 | 1058 | 1073 | 1074 | 1051 | 1071 | 1070 | 1071 | 1069 | 1051 | 1071 | 1064 | 1074 | 1058 |
| p4.2.m    | 1132 | 1094 | 1089 | 1106 | 1098 | 1132 | 1132 | 1051 | 1130 | 1115 | 1119 | 1113 | 1051 | 1132 | 1130 | 1132 | 1095 |
| p4.2.n    | 1174 | 1119 | 1150 | 1169 | 1171 | 1159 | 1171 | 1117 | 1168 | 1149 | 1158 | 1169 | 1124 | 1174 | 1161 | 1173 | 1118 |
| p4.2.o    | 1218 | 1173 | 1175 | 1180 | 1192 | 1216 | 1218 | 1191 | 1215 | 1209 | 1198 | 1210 | 1195 | 1217 | 1206 | 1218 | 1184 |
| p4.2.p    | 1242 | 1195 | 1208 | 1226 | 1239 | 1239 | 1241 | 1214 | 1242 | 1229 | 1233 | 1239 | 1237 | 1242 | 1240 | 1242 | 1200 |
| p4.2.q    | 1267 | 1230 | 1255 | 1252 | 1255 | 1265 | 1263 | 1248 | 1263 | 1253 | 1252 | 1260 | 1239 | 1267 | 1257 | 1263 | 1237 |
| p4.2.r    | 1292 | 1249 | 1277 | 1281 | 1283 | 1283 | 1286 | 1267 | 1288 | 1278 | 1278 | 1279 | 1279 | 1292 | 1278 | 1286 | 1251 |
| p4.2.s    | 1304 | 1267 | 1294 | 1296 | 1299 | 1300 | 1301 | 1286 | 1304 | 1304 | 1303 | 1304 | 1295 | 1304 | 1293 | 1296 | 1275 |
| p4.2.t    | 1306 | 1295 | 1306 | 1306 | 1306 | 1306 | 1306 | 1294 | 1306 | 1306 | 1306 | 1306 | 1305 | 1306 | 1299 | 1306 | 1293 |
| p4.3.c    | 193  | 193  | 192  | 193  | 193  | 193  | 193  | 193  | 193  | 193  | 193  | 193  | 193  | 193  | 193  | 193  | 191  |
| p4.3.d    | 335  | 335  | 333  | 335  | 335  | 335  | 335  | 335  | 335  | 333  | 333  | 335  | 331  | 335  | 333  | 335  | 333  |
| p4.3.e    | 468  | 467  | 465  | 468  | 468  | 468  | 468  | 444  | 468  | 468  | 468  | 468  | 460  | 468  | 468  | 468  | 458  |
| p4.3.f    | 579  | 579  | 579  | 579  | 579  | 579  | 579  | 564  | 579  | 579  | 579  | 579  | 556  | 579  | 579  | 579  | 560  |
| p4.3.g    | 653  | 653  | 646  | 651  | 652  | 653  | 653  | 644  | 653  | 652  | 653  | 652  | 651  | 653  | 653  | 653  | 646  |
| p4.3.h    | 729  | 726  | 709  | 722  | 727  | 724  | 729  | 706  | 720  | 713  | 713  | 713  | 718  | 725  | 725  | 729  | 718  |
| p4.3.i    | 809  | 788  | 785  | 806  | 806  | 806  | 807  | 806  | 796  | 793  | 793  | 786  | 807  | 809  | 797  | 809  | 795  |
| p4.3.j    | 861  | 839  | 860  | 858  | 858  | 861  | 861  | 826  | 861  | 857  | 855  | 858  | 854  | 861  | 858  | 861  | 845  |
| p4.3.k    | 919  | 894  | 906  | 919  | 918  | 919  | 919  | 864  | 918  | 913  | 910  | 910  | 902  | 919  | 918  | 918  | 896  |
| p4.3.l    | 979  | 949  | 951  | 976  | 973  | 975  | 978  | 960  | 979  | 958  | 976  | 966  | 969  | 974  | 968  | 979  | 955  |
| p4.3.m    | 1063 | 1009 | 1005 | 1034 | 1049 | 1056 | 1063 | 1030 | 1053 | 1039 | 1028 | 1046 | 1047 | 1063 | 1043 | 1063 | 1015 |
| p4.3.n    | 1121 | 1078 | 1119 | 1108 | 1115 | 1111 | 1121 | 1113 | 1121 | 1109 | 1112 | 1103 | 1106 | 1121 | 1108 | 1120 | 1077 |
| p4.3.o    | 1172 | 1145 | 1151 | 1156 | 1157 | 1172 | 1170 | 1121 | 1170 | 1163 | 1167 | 1165 | 1136 | 1172 | 1165 | 1170 | 1147 |
| p4.3.p    | 1222 | 1190 | 1218 | 1207 | 1221 | 1208 | 1222 | 1190 | 1221 | 1202 | 1207 | 1207 | 1200 | 1222 | 1209 | 1220 | 1190 |
| p4.3.q    | 1253 | 1212 | 1249 | 1237 | 1241 | 1250 | 1251 | 1210 | 1252 | 123  | 1239 | 1238 | 1236 | 1252 | 1246 | 1253 | 1234 |
| p4.3.r    | 1273 | 1244 | 1265 | 1224 | 1269 | 1272 | 1272 | 1239 | 1267 | 1263 | 1263 | 1263 | 1250 | 1273 | 1257 | 1272 | 1264 |
| p4.3.s    | 1295 | 1268 | 1282 | 1250 | 1294 | 1289 | 1293 | 1279 | 1293 | 1291 | 1289 | 1291 | 1280 | 1295 | 1276 | 1287 | 1275 |
| p4.3.t    | 1305 | 1290 | 1288 | 1303 | 1304 | 1298 | 1304 | 1290 | 1305 | 1304 | 1303 | 1304 | 1299 | 1304 | 1294 | 1299 | 1291 |
| p4.4.e    | 183  | 183  | 182  | 183  | 183  | 183  | 183  | 183  | 183  | 183  | 183  | 183  | 183  | 183  | 183  | 183  | 182  |
| p4.4.f    | 324  | 324  | 315  | 324  | 324  | 324  | 324  | 312  | 324  | 324  | 324  | 324  | 324  | 324  | 324  | 324  | 324  |
| p4.4.g    | 461  | 461  | 453  | 461  | 461  | 461  | 461  | 461  | 461  | 461  | 460  | 460  | 461  | 461  | 461  | 461  | 460  |
| p4.4.h    | 571  | 564  | 554  | 571  | 571  | 571  | 571  | 565  | 571  | 556  | 556  | 556  | 571  | 571  | 571  | 571  | 555  |
| p4.4.i    | 657  | 654  | 627  | 655  | 657  | 657  | 657  | 657  | 657  | 653  | 653  | 653  | 653  | 657  | 653  | 657  | 641  |
| p4.4.j    | 732  | 719  | 732  | 731  | 731  | 732  | 732  | 691  | 732  | 731  | 731  | 731  | 732  | 732  | 732  | 732  | 724  |
| p4.4.k    | 821  | 813  | 819  | 821  | 816  | 821  | 821  | 815  | 821  | 820  | 818  | 818  | 820  | 821  | 820  | 821  | 804  |
| p4.4.l    | 880  | 872  | 875  | 878  | 878  | 879  | 880  | 852  | 880  | 877  | 875  | 875  | 875  | 879  | 875  | 879  | 843  |
| p4.4.m    | 919  | 907  | 910  | 916  | 918  | 916  | 919  | 910  | 918  | 911  | 911  | 911  | 914  | 918  | 914  | 919  | 898  |
| p4.4.n    | 977  | 947  | 977  | 972  | 976  | 968  | 968  | 942  | 961  | 956  | 956  | 956  | 953  | 969  | 953  | 969  | 957  |
| p4.4.o    | 1061 | 1025 | 1014 | 1057 | 1057 | 1051 | 1061 | 937  | 1036 | 1030 | 1029 | 1029 | 1033 | 1061 | 1033 | 1057 | 998  |
| p4.4.p    | 1124 | 1090 | 1056 | 1120 | 1120 | 1120 | 1120 | 1091 | 1111 | 1108 | 1110 | 1110 | 1098 | 1124 | 1098 | 1122 | 1086 |
| p4.4.q    | 1161 | 1130 | 1124 | 1148 | 1157 | 1160 | 1161 | 1106 | 1145 | 1150 | 1148 | 1148 | 1149 | 1161 | 1139 | 1160 | 1131 |
| p4.4.r    | 1216 | 1185 | 1165 | 1203 | 1211 | 1207 | 1203 | 1148 | 1200 | 1195 | 1194 | 1194 | 1193 | 1216 | 1196 | 1213 | 1156 |
| p4.4.s    | 1260 | 1232 | 1243 | 1245 | 1256 | 1259 | 1255 | 1242 | 1249 | 1256 | 1252 | 1252 | 1213 | 1259 | 1231 | 1250 | 1230 |
| p4.4.t    | 1285 | 1271 | 1255 | 1279 | 1285 | 1282 | 1279 | 1250 | 1281 | 1281 | 1281 | 1281 | 1281 | 1284 | 1256 | 1280 | 1253 |

Table 6: Results test set 5

| Instances | Best | SS   | A1   | A2   | A3   | A4   | A5   | A6   | A7   | A8   | A9   | A10  | A11  | A12  | A13  | A14  | A15  |
|-----------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| p5.2.h    | 410  | 410  | 410  | 410  | 410  | 410  | 410  | 385  | 410  | 410  | 410  | 410  | 395  | 410  | 410  | 410  | 410  |
| p5.2.j    | 580  | 580  | 560  | 580  | 580  | 580  | 580  | 580  | 580  | 580  | 580  | 580  | 580  | 580  | 580  | 580  | 580  |
| p5.2.k    | 670  | 670  | 670  | 670  | 670  | 670  | 670  | 665  | 670  | 670  | 670  | 670  | 670  | 670  | 670  | 670  | 670  |
| p5.2.l    | 800  | 795  | 770  | 800  | 800  | 800  | 800  | 760  | 800  | 800  | 800  | 800  | 770  | 800  | 800  | 800  | 770  |
| p5.2.m    | 860  | 860  | 860  | 860  | 860  | 860  | 860  | 830  | 860  | 860  | 860  | 860  | 860  | 860  | 860  | 860  | 855  |
| p5.2.n    | 925  | 920  | 920  | 925  | 925  | 925  | 925  | 920  | 925  | 920  | 920  | 925  | 920  | 925  | 925  | 925  | 920  |
| p5.2.o    | 1020 | 1015 | 975  | 1020 | 1020 | 1020 | 1020 | 1010 | 1020 | 1020 | 1010 | 1010 | 1020 | 1020 | 1020 | 1020 | 1010 |
| p5.2.p    | 1150 | 1135 | 1090 | 1130 | 1150 | 1150 | 1150 | 1030 | 1150 | 1150 | 1150 | 1150 | 1150 | 1150 | 1150 | 1150 | 1140 |
| p5.2.q    | 1195 | 1175 | 1185 | 1195 | 1195 | 1195 | 1195 | 1145 | 1195 | 1195 | 1195 | 1195 | 1195 | 1195 | 1195 | 1195 | 1190 |
| p5.2.r    | 1260 | 1255 | 1260 | 1260 | 1260 | 1260 | 1260 | 1225 | 1260 | 1260 | 1260 | 1260 | 1260 | 1260 | 1260 | 1260 | 1255 |
| p5.2.s    | 1340 | 1315 | 1310 | 1330 | 1340 | 1340 | 1340 | 1325 | 1340 | 1330 | 1330 | 1330 | 1325 | 1330 | 1340 | 1340 | 1315 |
| p5.2.t    | 1400 | 1375 | 1380 | 1380 | 1400 | 1400 | 1400 | 1360 | 1400 | 1400 | 1400 | 1400 | 1380 | 1400 | 1400 | 1400 | 1380 |
| p5.2.u    | 1460 | 1450 | 1445 | 1440 | 1460 | 1460 | 1460 | 1460 | 1460 | 1460 | 1460 | 1460 | 1450 | 1460 | 1460 | 1460 | 1450 |

Table 6: Continue

| Instances | Best | SS   | A1   | A2   | A3   | A4   | A5   | A6   | A7   | A8   | A9   | A10  | A11  | A12  | A13  | A14  | A15  |
|-----------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| p5.2.v    | 1505 | 1490 | 1500 | 1490 | 1505 | 1500 | 1505 | 1500 | 1505 | 1495 | 1500 | 1495 | 1500 | 1505 | 1505 | 1505 | 1495 |
| p5.2.w    | 1565 | 1540 | 1560 | 1555 | 1565 | 1560 | 1560 | 1560 | 1560 | 1555 | 1555 | 1555 | 1560 | 1560 | 1560 | 1560 | 1560 |
| p5.2.x    | 1610 | 1590 | 1610 | 1595 | 1610 | 1590 | 1610 | 1610 | 1610 | 1610 | 1610 | 1610 | 1600 | 1610 | 1610 | 1610 | 1580 |
| p5.2.y    | 1645 | 1625 | 1630 | 1635 | 1635 | 1635 | 1635 | 1630 | 1645 | 1645 | 1645 | 1645 | 1630 | 1645 | 1645 | 1645 | 1645 |
| p5.2.z    | 1680 | 1675 | 1680 | 1670 | 1680 | 1670 | 1670 | 1680 | 1680 | 1680 | 1680 | 1680 | 1665 | 1680 | 1670 | 1680 | 1660 |
| p5.3.k    | 495  | 495  | 495  | 495  | 495  | 495  | 495  | 470  | 495  | 495  | 495  | 495  | 495  | 495  | 495  | 495  | 495  |
| p5.3.l    | 595  | 595  | 575  | 595  | 595  | 595  | 595  | 545  | 595  | 595  | 595  | 595  | 595  | 595  | 595  | 595  | 595  |
| p5.3.n    | 755  | 755  | 755  | 755  | 755  | 755  | 755  | 720  | 755  | 755  | 755  | 755  | 755  | 755  | 755  | 755  | 755  |
| p5.3.o    | 870  | 870  | 835  | 870  | 870  | 870  | 870  | 870  | 870  | 870  | 870  | 870  | 870  | 870  | 870  | 870  | 870  |
| p5.3.q    | 1070 | 1070 | 1065 | 1070 | 1070 | 1070 | 1070 | 1045 | 1070 | 1065 | 1065 | 1065 | 1065 | 1070 | 1070 | 1070 | 1070 |
| p5.3.r    | 1125 | 1120 | 1115 | 1110 | 1125 | 1125 | 1125 | 1090 | 1125 | 1120 | 1125 | 1125 | 1125 | 1125 | 1125 | 1125 | 1115 |
| p5.3.s    | 1190 | 1190 | 1175 | 1185 | 1190 | 1190 | 1190 | 1145 | 1190 | 1190 | 1190 | 1185 | 1185 | 1190 | 1185 | 1190 | 1175 |
| p5.3.t    | 1260 | 1255 | 1240 | 1250 | 1260 | 1260 | 1260 | 1240 | 1260 | 1250 | 1255 | 1260 | 1260 | 1260 | 1260 | 1260 | 1260 |
| p5.3.u    | 1345 | 1330 | 1330 | 1340 | 1345 | 1345 | 1345 | 1305 | 1345 | 1330 | 1335 | 1335 | 1345 | 1345 | 1335 | 1345 | 1340 |
| p5.3.v    | 1425 | 1400 | 1410 | 1420 | 1425 | 1425 | 1425 | 1425 | 1425 | 1425 | 1425 | 1420 | 1425 | 1425 | 1420 | 1425 | 1405 |
| p5.3.w    | 1485 | 1455 | 1465 | 1485 | 1485 | 1485 | 1485 | 1460 | 1485 | 1465 | 1465 | 1465 | 1475 | 1485 | 1465 | 1485 | 1455 |
| p5.3.x    | 1555 | 1520 | 1530 | 1555 | 1555 | 1555 | 1555 | 1520 | 1540 | 1535 | 1540 | 1540 | 1535 | 1555 | 1540 | 1550 | 1515 |
| p5.3.y    | 1595 | 1590 | 1580 | 1590 | 1595 | 1595 | 1595 | 1590 | 1590 | 1590 | 1590 | 1590 | 1580 | 1590 | 1590 | 1590 | 1540 |
| p5.3.z    | 1635 | 1635 | 1635 | 1625 | 1635 | 1635 | 1635 | 1635 | 1635 | 1635 | 1635 | 1635 | 1635 | 1635 | 1635 | 1635 | 1620 |
| p5.4.m    | 555  | 555  | 555  | 555  | 555  | 555  | 555  | 550  | 555  | 555  | 555  | 555  | 550  | 555  | 555  | 555  | 555  |
| p5.4.o    | 690  | 690  | 680  | 690  | 690  | 690  | 690  | 680  | 690  | 690  | 690  | 690  | 690  | 690  | 690  | 690  | 690  |
| p5.4.p    | 765  | 760  | 760  | 765  | 765  | 765  | 765  | 760  | 765  | 760  | 760  | 760  | 760  | 760  | 760  | 760  | 765  |
| p5.4.q    | 860  | 860  | 860  | 860  | 860  | 860  | 860  | 830  | 860  | 860  | 860  | 860  | 835  | 860  | 860  | 860  | 860  |
| p5.4.r    | 960  | 960  | 960  | 960  | 960  | 960  | 960  | 890  | 960  | 960  | 960  | 960  | 960  | 960  | 960  | 960  | 960  |
| p5.4.s    | 1030 | 1025 | 1000 | 1025 | 1030 | 1030 | 1030 | 1020 | 1030 | 1030 | 1030 | 1030 | 1020 | 1030 | 1005 | 1025 | 1030 |
| p5.4.t    | 1160 | 1160 | 1100 | 1160 | 1160 | 1160 | 1160 | 1160 | 1160 | 1160 | 1160 | 1160 | 1160 | 1160 | 1160 | 1160 | 1160 |
| p5.4.u    | 1300 | 1285 | 1275 | 1300 | 1300 | 1300 | 1300 | 1300 | 1300 | 1300 | 1300 | 1300 | 1300 | 1300 | 1300 | 1300 | 1300 |
| p5.4.v    | 1320 | 1320 | 1310 | 1320 | 1320 | 1320 | 1320 | 1245 | 1320 | 1320 | 1320 | 1320 | 1320 | 1320 | 1320 | 1320 | 1290 |
| p5.4.w    | 1390 | 1370 | 1380 | 1375 | 1390 | 1390 | 1390 | 1330 | 1390 | 1380 | 1390 | 1380 | 1380 | 1380 | 1380 | 1390 | 1385 |
| p5.4.x    | 1450 | 1430 | 1410 | 1440 | 1450 | 1450 | 1450 | 1410 | 1450 | 1450 | 1450 | 1450 | 1440 | 1450 | 1430 | 1450 | 1430 |
| p5.4.y    | 1520 | 1500 | 1520 | 1520 | 1520 | 1520 | 1520 | 1485 | 1520 | 1510 | 1510 | 1500 | 1500 | 1520 | 1520 | 1520 | 1520 |
| p5.4.z    | 1620 | 1595 | 1575 | 1620 | 1620 | 1620 | 1620 | 1590 | 1620 | 1620 | 1575 | 1580 | 1600 | 1620 | 1620 | 1620 | 1575 |

Table 7: Results test set 6

| Instances | Best | SS   | A1   | A2   | A3   | A4   | A5   | A6   | A7   | A8   | A9   | A10  | A11  | A12  | A13  | A14  | A15  |
|-----------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| p6.2.d    | 192  | 192  | 192  | 192  | 192  | 192  | 192  | 180  | 192  | 192  | 192  | 192  | 192  | 192  | 192  | 192  | 192  |
| p6.2.j    | 948  | 948  | 936  | 948  | 948  | 948  | 948  | 948  | 948  | 948  | 948  | 948  | 948  | 948  | 948  | 948  | 948  |
| p6.2.l    | 1116 | 1116 | 1116 | 1098 | 1116 | 1116 | 1116 | 1104 | 1116 | 1110 | 1116 | 1116 | 1116 | 1116 | 1110 | 1116 | 1116 |
| p6.2.m    | 1188 | 1182 | 1188 | 1164 | 1188 | 1188 | 1188 | 1164 | 1188 | 1188 | 1188 | 1188 | 1188 | 1188 | 1188 | 1188 | 1188 |
| p6.2.n    | 1260 | 1260 | 1260 | 1242 | 1260 | 1260 | 1260 | 1254 | 1260 | 1260 | 1254 | 1260 | 1248 | 1260 | 1260 | 1260 | 1242 |
| p6.3.g    | 282  | 282  | 282  | 282  | 282  | 282  | 282  | 264  | 282  | 282  | 282  | 282  | 276  | 282  | 282  | 282  | 282  |
| p6.3.h    | 444  | 444  | 444  | 444  | 444  | 444  | 444  | 444  | 444  | 444  | 438  | 438  | 444  | 444  | 444  | 444  | 444  |
| p6.3.i    | 642  | 642  | 612  | 642  | 642  | 642  | 642  | 642  | 642  | 642  | 642  | 642  | 642  | 642  | 642  | 642  | 636  |
| p6.3.k    | 894  | 894  | 876  | 894  | 894  | 894  | 894  | 882  | 894  | 888  | 888  | 894  | 894  | 894  | 894  | 894  | 894  |
| p6.3.l    | 1002 | 990  | 990  | 1002 | 1002 | 1002 | 1002 | 990  | 1002 | 1002 | 1002 | 1002 | 996  | 1002 | 1002 | 1002 | 1002 |
| p6.3.m    | 1080 | 1074 | 1080 | 1080 | 1080 | 1080 | 1080 | 1068 | 1080 | 1074 | 1080 | 1080 | 1080 | 1080 | 1080 | 1080 | 1080 |
| p6.3.n    | 1170 | 1164 | 1152 | 1170 | 1170 | 1170 | 1170 | 1140 | 1170 | 1164 | 1164 | 1164 | 1152 | 1170 | 1164 | 1170 | 1158 |
| p6.4.j    | 366  | 366  | 366  | 366  | 366  | 366  | 366  | 360  | 366  | 366  | 366  | 366  | 366  | 366  | 366  | 366  | 366  |
| p6.4.k    | 528  | 528  | 522  | 528  | 528  | 528  | 528  | 528  | 528  | 528  | 528  | 528  | 528  | 528  | 528  | 528  | 522  |
| p6.4.l    | 696  | 696  | 696  | 696  | 696  | 696  | 696  | 678  | 696  | 696  | 696  | 696  | 678  | 696  | 696  | 696  | 696  |

to the best known result and the average CPU time in seconds. Table 10 includes the execution time needed for each algorithm to solve each data set.

The basic scatter search is capable of gaining 60 times of best known solution and achieves solutions that depart on average 0.69% from the best known solution within 15.08 sec. Finally, Table 11 expresses the number of results that were achieved better, equals and worse using scatter search compared to the algorithms in the earlier

studies. Based on this comparison, the quality of results A2, A4 and A5 proposed by Archetti *et al.* (2007); A7, A8, A9 and A10 proposed by Ke *et al.* (2008); A13 and A14 proposed by Souffriau *et al.* (2010) and A11 proposed by Bouly *et al.* (2010) is better than the SS. On the other hand, the average execution time of scatter search is 15.08 sec which appears to be much better.

The contribution of this study is to show that basic scatter search is able to tackle TOP within competitive

Table 8: Results test set 7

| Instances | Best | SS   | A1   | A2   | A3   | A4   | A5   | A6   | A7   | A8   | A9   | A10  | A11  | A12  | A13  | A14  | A15  |
|-----------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| p7.2.d    | 190  | 190  | 190  | 190  | 190  | 190  | 190  | 190  | 190  | 190  | 190  | 190  | 182  | 190  | 190  | 190  | 190  |
| p7.2.e    | 290  | 290  | 290  | 290  | 290  | 289  | 290  | 279  | 290  | 290  | 290  | 290  | 289  | 290  | 290  | 290  | 290  |
| p7.2.f    | 387  | 387  | 382  | 387  | 387  | 387  | 387  | 340  | 387  | 387  | 387  | 387  | 387  | 387  | 387  | 387  | 387  |
| p7.2.g    | 459  | 459  | 459  | 456  | 459  | 459  | 459  | 440  | 459  | 459  | 459  | 459  | 457  | 459  | 459  | 459  | 459  |
| p7.2.h    | 521  | 521  | 521  | 520  | 520  | 521  | 521  | 517  | 521  | 521  | 521  | 521  | 521  | 521  | 521  | 521  | 521  |
| p7.2.i    | 580  | 579  | 578  | 579  | 579  | 575  | 579  | 568  | 580  | 579  | 579  | 579  | 579  | 579  | 578  | 580  | 578  |
| p7.2.j    | 646  | 641  | 638  | 643  | 644  | 643  | 644  | 633  | 646  | 646  | 646  | 646  | 632  | 646  | 646  | 646  | 623  |
| p7.2.k    | 705  | 699  | 702  | 702  | 705  | 704  | 705  | 691  | 705  | 704  | 704  | 704  | 700  | 704  | 702  | 705  | 698  |
| p7.2.l    | 767  | 758  | 767  | 758  | 767  | 759  | 767  | 748  | 767  | 767  | 767  | 767  | 758  | 767  | 759  | 767  | 755  |
| p7.2.m    | 827  | 827  | 817  | 827  | 824  | 824  | 827  | 798  | 827  | 827  | 827  | 827  | 827  | 827  | 816  | 827  | 807  |
| p7.2.n    | 888  | 888  | 864  | 884  | 888  | 883  | 888  | 861  | 888  | 878  | 878  | 878  | 866  | 888  | 888  | 888  | 868  |
| p7.2.o    | 945  | 939  | 914  | 933  | 945  | 945  | 945  | 897  | 945  | 945  | 940  | 941  | 928  | 945  | 932  | 945  | 908  |
| p7.2.p    | 1002 | 988  | 987  | 1000 | 1002 | 1002 | 1002 | 954  | 1002 | 991  | 993  | 993  | 955  | 1002 | 993  | 1002 | 970  |
| p7.2.q    | 1044 | 1029 | 1017 | 1041 | 1043 | 1038 | 1044 | 1031 | 1043 | 1042 | 1043 | 1043 | 1029 | 1044 | 1043 | 1044 | 1020 |
| p7.2.r    | 1094 | 1078 | 1067 | 1091 | 1088 | 1094 | 1094 | 1075 | 1094 | 1093 | 1088 | 1094 | 1069 | 1094 | 1076 | 1094 | 1066 |
| p7.2.s    | 1136 | 1122 | 1116 | 1123 | 1128 | 1136 | 1136 | 1102 | 1136 | 1136 | 1134 | 1131 | 1118 | 1136 | 1125 | 1136 | 1097 |
| p7.2.t    | 1179 | 1159 | 1165 | 1172 | 1174 | 1168 | 1179 | 1142 | 1179 | 1179 | 1179 | 1179 | 1154 | 1179 | 1168 | 1175 | 1129 |
| p7.3.f    | 247  | 247  | 247  | 247  | 247  | 247  | 247  | 247  | 247  | 247  | 247  | 247  | 247  | 247  | 247  | 247  | 240  |
| p7.3.h    | 425  | 425  | 416  | 425  | 425  | 425  | 425  | 418  | 425  | 425  | 425  | 425  | 425  | 425  | 425  | 425  | 419  |
| p7.3.i    | 487  | 487  | 481  | 487  | 487  | 487  | 487  | 480  | 487  | 487  | 486  | 487  | 480  | 487  | 485  | 487  | 485  |
| p7.3.j    | 564  | 564  | 563  | 564  | 564  | 562  | 564  | 539  | 564  | 564  | 564  | 564  | 543  | 563  | 560  | 564  | 557  |
| p7.3.k    | 633  | 632  | 632  | 633  | 633  | 632  | 633  | 586  | 633  | 632  | 633  | 633  | 633  | 633  | 633  | 633  | 633  |
| p7.3.l    | 684  | 684  | 681  | 683  | 679  | 681  | 681  | 668  | 684  | 683  | 684  | 684  | 681  | 683  | 684  | 684  | 681  |
| p7.3.m    | 762  | 762  | 756  | 749  | 755  | 745  | 762  | 735  | 762  | 762  | 762  | 762  | 743  | 762  | 762  | 762  | 754  |
| p7.3.n    | 820  | 820  | 789  | 810  | 811  | 814  | 820  | 789  | 820  | 819  | 819  | 820  | 804  | 820  | 813  | 820  | 813  |
| p7.3.o    | 874  | 874  | 874  | 873  | 865  | 871  | 874  | 833  | 874  | 874  | 874  | 874  | 841  | 874  | 859  | 874  | 848  |
| p7.3.p    | 929  | 929  | 922  | 917  | 923  | 926  | 927  | 912  | 929  | 925  | 926  | 925  | 918  | 927  | 925  | 927  | 919  |
| p7.3.q    | 987  | 977  | 966  | 976  | 987  | 978  | 987  | 945  | 987  | 987  | 987  | 987  | 966  | 987  | 970  | 987  | 960  |
| p7.3.r    | 1026 | 1015 | 1011 | 1018 | 1022 | 1024 | 1022 | 1015 | 1026 | 1024 | 1021 | 1022 | 1009 | 1024 | 1017 | 1021 | 1017 |
| p7.3.s    | 1081 | 1072 | 1061 | 1081 | 1081 | 1079 | 1079 | 1054 | 1081 | 1081 | 1081 | 1077 | 1070 | 1081 | 1076 | 1081 | 1064 |
| p7.3.t    | 1120 | 1116 | 1098 | 1114 | 1116 | 1112 | 1115 | 1080 | 1118 | 1117 | 1103 | 1117 | 1109 | 1120 | 1111 | 1118 | 1093 |
| p7.4.g    | 217  | 217  | 217  | 217  | 217  | 217  | 217  | 209  | 217  | 217  | 217  | 217  | 217  | 217  | 217  | 217  | 211  |
| p7.4.h    | 285  | 285  | 285  | 285  | 285  | 285  | 285  | 285  | 285  | 285  | 285  | 285  | 283  | 285  | 285  | 285  | 283  |
| p7.4.i    | 366  | 366  | 359  | 366  | 366  | 366  | 366  | 359  | 366  | 366  | 366  | 366  | 364  | 366  | 366  | 366  | 349  |
| p7.4.k    | 520  | 518  | 503  | 520  | 520  | 518  | 520  | 511  | 520  | 520  | 520  | 520  | 518  | 518  | 518  | 518  | 516  |
| p7.4.l    | 590  | 590  | 576  | 590  | 588  | 588  | 590  | 573  | 590  | 590  | 590  | 590  | 575  | 590  | 581  | 590  | 578  |
| p7.4.m    | 646  | 646  | 643  | 644  | 646  | 646  | 646  | 638  | 646  | 644  | 646  | 646  | 639  | 646  | 646  | 646  | 633  |
| p7.4.n    | 730  | 726  | 726  | 723  | 721  | 715  | 730  | 698  | 730  | 725  | 725  | 726  | 723  | 726  | 723  | 730  | 712  |
| p7.4.o    | 781  | 777  | 776  | 772  | 778  | 770  | 781  | 761  | 781  | 778  | 781  | 778  | 778  | 779  | 780  | 780  | 776  |
| p7.4.p    | 846  | 840  | 832  | 841  | 839  | 846  | 846  | 803  | 846  | 846  | 838  | 842  | 841  | 846  | 846  | 846  | 819  |
| p7.4.q    | 909  | 899  | 905  | 902  | 898  | 899  | 906  | 899  | 909  | 909  | 909  | 909  | 896  | 907  | 902  | 907  | 890  |
| p7.4.r    | 970  | 960  | 966  | 970  | 969  | 970  | 970  | 937  | 970  | 970  | 970  | 970  | 964  | 970  | 961  | 970  | 948  |
| p7.4.s    | 1022 | 1006 | 1019 | 1021 | 1020 | 1021 | 1022 | 1005 | 1022 | 1019 | 1021 | 1019 | 1019 | 1022 | 1022 | 1022 | 988  |
| p7.4.t    | 1077 | 1046 | 1067 | 1071 | 1071 | 1077 | 1077 | 1020 | 1077 | 1072 | 1077 | 1077 | 1073 | 1077 | 1066 | 1077 | 1066 |

Table 9: Summary of results

| Algorithm | # best | Avg. gap (%) | Avg. CPU (s) |
|-----------|--------|--------------|--------------|
| SS        | 61     | 0.69         | 15.08        |
| A1        | 36     | 1.03         | 336.58       |
| A2        | 70     | 0.40         | 100.00       |
| A3        | 95     | 0.17         | 146.61       |
| A4        | 95     | 0.15         | 18.94        |
| A5        | 128    | 0.04         | 268.64       |
| A6        | 22     | 1.93         | 7.83         |
| A7        | 129    | 0.08         | 252.28       |
| A8        | 81     | 0.70         | 213.80       |
| A9        | 81     | 0.31         | 204.75       |
| A10       | 85     | 0.28         | 214.98       |
| A10       | 46     | 0.75         | 3.78         |
| A12       | 131    | 0.03         | 63.76        |
| A13       | 79     | 0.33         | 4.98         |
| A14       | 127    | 0.04         | 212.43       |
| A15       | 40     | 1.01         | -            |

Table 10: Execution time (sec)

| Algorithm | Set 4  | Set 5  | Set 6  | Set 7  | Avg.   |
|-----------|--------|--------|--------|--------|--------|
| SS        | 13.89  | 15.79  | 12.36  | 18.29  | 15.08  |
| A1        | 796.70 | 71.30  | 45.70  | 432.60 | 336.58 |
| A2        | 105.29 | 69.45  | 66.29  | 158.97 | 100.00 |
| A3        | 282.92 | 26.55  | 20.11  | 256.76 | 146.61 |
| A4        | 22.52  | 34.17  | 8.74   | 10.34  | 18.94  |
| A5        | 457.89 | 158.93 | 147.88 | 309.87 | 268.64 |
| A6        | 11.40  | 3.50   | 4.30   | 12.10  | 7.83   |
| A7        | 370.90 | 173.60 | 161.10 | 303.50 | 252.28 |
| A8        | 317.90 | 150.60 | 140.80 | 245.90 | 213.80 |
| A9        | 307.40 | 143.30 | 135.20 | 233.10 | 204.75 |
| A10       | 320.40 | 151.30 | 141.70 | 246.50 | 214.98 |
| A11       | 7.40   | 1.50   | 1.90   | 4.30   | 3.78   |
| A12       | 125.26 | 23.96  | 15.53  | 90.30  | 63.76  |
| A13       | 8.60   | 2.90   | 2.10   | 6.30   | 4.98   |
| A14       | 367.40 | 119.90 | 89.60  | 272.80 | 212.43 |
| A15       | -      | -      | -      | -      | -      |

Table 11: SS compared to literature

| Algorithm | Better | Equal | Worse |
|-----------|--------|-------|-------|
| A1        | 80     | 32    | 46    |
| A2        | 31     | 51    | 76    |
| A3        | 15     | 53    | 90    |
| A4        | 16     | 56    | 86    |
| A5        | 4      | 60    | 94    |
| A6        | 103    | 25    | 30    |
| A7        | 1      | 62    | 95    |
| A8        | 19     | 59    | 80    |
| A9        | 22     | 55    | 81    |
| A10       | 16     | 59    | 83    |
| A11       | 54     | 41    | 63    |
| A12       | 4      | 63    | 91    |
| A13       | 24     | 54    | 80    |
| A14       | 1      | 64    | 93    |
| A15       | 77     | 37    | 44    |

time. Scatter search has a reference set component that contain a set of elite solution and a set of divers solution which were combined in systematic way to reduce the probability of getting stuck in local optimas.

## CONCLUSION

This study presents scatter search algorithm to tackle TOP. Applying scatter search to solve TOP is obviously a very promising method. Scatter search can achieves good solutions during small amount of calculation time that depart on average 0.69% from the best known solution within 15.08 sec average time. The proposed algorithm is compared with several promising algorithms from the literature. The results prove that this algorithm can rival any other popular algorithms. The advantages of the proposed algorithm are efficient, effective and simple. Scatter search algorithm can be hybridized with other local search to increase intensification search in each region to get the best solution in the neighborhood and to reduce execution time.

## ACKNOWLEDGEMENT

The researchers wish to thank Ministry of Higher Education for supporting this research under the FRGS Research Grant Scheme (FRGS/1/2012/SG05/UKM/02/11).

## REFERENCES

Archetti, C., A. Hertz and M.G. Speranza, 2007. Metaheuristics for the team orienteering problem. *J. Heurist.*, 13: 49-76.

Bouly, H., D.C. Dang and A. Moukrim, 2010. A memetic algorithm for the team orienteering problem. *4OR*, 8: 49-70.

Burke, E.K., T. Curtois, R. Qu and G.V. Berghe, 2010. A scatter search methodology for the nurse rostering problem. *J. Oper. Res. Soc.*, 61: 1667-1679.

Butt, S.E. and T.M. Cavalier, 1994. A heuristic for the multiple tour maximum collection problem. *Comput. Oper. Res.*, 21: 101-111.

Chao, I., B.L. Golden and E.A. Wasil, 1996. A fast and effective heuristic for the orienteering problem. *Eur. J. Oper. Res.*, 88: 475-489.

Engin, O., C. Kahraman and M.K. Yilmaz, 2009. A scatter search method for multiobjective fuzzy permutation flow shop scheduling problem: A real world application. *Comput. Intel. Flow Shop Job Shop Scheduling*, 230: 169-189.

Fischetti, M., J.J.S. Gonzalez and P. Toth, 1998. Solving the orienteering problem through branch-and-cut. *INFORMS J. Comput.*, 10: 133-148.

Gendreau, M., G. Laporte and F. Semet, 1998. A tabu search heuristic for the undirected selective travelling salesman problem. *Eur. J. Oper. Res.*, 106: 539-545.

Glover, F., 1977. Heuristics for integer programming using surrogate constraints. *Decis. Sci.*, 8: 156-166.

Glover, F., 1998. A template for scatter search and path relinking. <http://leeds-faculty.colorado.edu/glover/SS-PR%20Template.pdf>.

Glover, F., 1999. Scatter Search and Path Relinking. In: *New Ideas in Optimization*, Corne, D., M. Dorigo and F. Glover (Eds.). McGraw-Hill Ltd., UK., pp: 297-316.

Glover, F., M. Laguna and R. Marti, 2003. *Scatter Search*. Springer-Verlag, USA., pp: 519-537.

Golden, B.L., Q. Wang and L. Liu, 1988. A multifaceted heuristic for the orienteering problem. *Naval Res. Logistics*, 35: 359-366.

Jaradat, G.M. and M. Ayob, 2011. Scatter search for solving the course timetabling problem. *Proceedings of the 3rd Conference on Data Mining and Optimization (DMO)*, June, 28-29, 2011, Malaysia, pp: 213-218.

Ke, L., C. Archetti and Z. Feng, 2008. Ants can solve the team orienteering problem. *Comput. Ind. Eng.*, 54: 648-665.

Laguna, M. and V. Armentano, 2005. Lessons from Applying and Experimenting with Scatter Search. In: *Metaheuristic Optimization via Memory and Evolution*, Sharda, R., S. Vob, C. Rego and B. Alidaee (Eds.). Vol. 30, Springer, USA., pp: 229-246.

Laguna, M., R. Marti and R.C. Marti, 2003. *Scatter Search: Methodology and Implementations*. Springer, Netherlands.

Liu, Y.H., 2008. Diversified local search strategy under scatter search framework for the probabilistic traveling salesman problem. *Eur. J. Oper. Res.*, 191: 332-346.

- Maquera, G., M. Laguna, D.A. Gandelman and A.P. Sant'Anna, 2011. Scatter search applied to the vehicle routing problem with simultaneous delivery and pickup. *Int. J. Applied Metaheuristic Comput.*, 2: 1-20.
- Miller, C., A.W. Tucker and R. Zemlin, 1960. Integer programming formulation of traveling salesman problems. *J. ACM*, 7: 326-329.
- Muthuswamy, S. and S.S. Lam, 2011. Discrete particle swarm optimization for the team orienteering problem. *Memetic Comput.*, 3: 287-303.
- Ramesh, R., Y.S. Yoon and M.H. Karwan, 1992. An optimal algorithm for the orienteering tour problem. *ORSA J. Comput.*, 4: 155-165.
- Resende, M.G.C., C.C. Ribeiro, F. Glover and R. Marti, 2010. Scatter Search and Path-Relinking: Fundamentals, Advances and Applications. In: *Handbook of Metaheuristics*, Gendreau, M. and J.Y. Potvin (Eds.). Springer, USA., pp: 87-107.
- Sabar, N.R. and M. Ayob, 2009. Examination timetabling using scatter search hyper-heuristic. *Proceedings of the 2nd Conference on Data Mining and Optimization*, October 27-28, 2009, Kajand, pp: 127-131.
- Souffriau, W., P. Vansteenwegen, G.V. Berghe and D. van Oudheusden, 2010. A path relinking approach for the team orienteering problem. *Comput. Oper. Res.*, 37: 1853-1859.
- Tang, H. and E. Miller-Hooks, 2005. A tabu search heuristic for the team orienteering problem. *Comput. Oper. Res.*, 32: 1379-1407.
- Vansteenwegen, P., W. Souffriau, G.V. Berghe and D.V. Oudheusden, 2009a. Metaheuristics for tourist trip planning. *Metaheuristics Serv. Ind.*, 624: 15-31.
- Vansteenwegen, P., W. Souffriau, G.V. Berghe and D.V. Oudheusden, 2009b. A guided local search metaheuristic for the team orienteering problem. *Eur. J. Oper. Res.*, 196: 118-127.