On the Flow of Combustible Gas in a Horizontal Pipe

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Abstract: The motion of a combustible gas in a horizontal pipe is studied in the presence of free convection heat transfer. The study determines the velocity and temperature profiles of the flow of a combustible gas in a horizontal pipe and it is effected by assuming that the Reynolds number of the flow is small, hence, solution to the problem by perturbation method.

Key words: Free convection, combustion, reynolds number, horizontal pipe and radiation

INTRODUCTION

Fluid is a substance, which may flow i.e., the constituent particles may change their position relative to one another and it may be subdivided into liquid and gas. The difference being that under ordinary conditions, liquids are difficult to compress and often regarded as incompressible, while gas may be compressed much more readily under the same conditions.

Fluid can be static and in motion. Fluid static deals primarily with fluid at rest and problem in fluid static are much simpler than those with the motion of fluids and exact analytical solutions are possible. Also, shear forces are not involved and all forces due to the pressure of the fluid are normal to the surface in which they act and because there is no relative movement, viscosity is absent.

However, fluid motion is complicated by the introduction of viscosity dependent shear force, which is absent in the fluid static. The motion of fluid is usually extremely complex and mathematical analysis of the problem of fluid flow is generally possible only if certain simplifying assumptions are made. The assumption is that it is an ideal fluid without viscosity, which is clearly untenable if viscosity plays a major role.

In the study of fluid motion, idea of an imaginary curve in the fluid, across which, at that instant, no fluid is flowing can also, be made use of. Such a line is called streamline or line of flow and at that instant, the velocity of every particle on the line is a direction tangential to the line. If a number of streamlines is considered at a particular instant, the pattern they form gives a good indication of the flow that is occurring.

Free or natural convective heat transfer finds a variety of applications in many systems involving multimode heat transfer effects. It provides the largest resistance to heat transfer and therefore, plays an important role in the design or performance or mechanical advantage (which is the ratio of the desired output to the required input) of the system. It is important in transferring heat from electric baseboard heaters or stream radiators to room air and in dissipating heat from the coil of a refrigeration unit to the surrounding air. It is also responsible for oceanic and atmospheric motion in environmental sciences.

Previous study of radiative transfer in a vertical pipe was carried out by Bestman (1996), who analyzed the flow of gas in a vertical cylindrical pipe as a model for a biomass moving bed gasifier. Berman (1953), obtained perturbation solution for low Reynolds number and Eckert et al. (1972), obtained numerical solution for moderate Reynolds number Ali (2003) and Adesanya et al. (2006) studied flow of gas in a vertical channel.

This research considered flow of a combustible gas in a horizontal pipe simply because the flow can withstand large pressure difference between the inside and the outside without undergoing any distortion. The velocity in the pipe changes from zero at the surface to a maximum at the center because of no-slip condition and the friction between the fluid layers does not cause a slight rise in temperature as a result of the mechanical energy being converted to sensible heat energy but rise in temperature due to frictional heating is usually too small thus, it is disregarded.

FORMULATION OF PROBLEM

The non-dimensional equation governing equation for flow of a thermally radiating and chemically reacting gas may be written in the form:

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$$\frac{1}{r}\frac{\partial(ru)}{\partial r}+\frac{1}{r}\frac{\partial v}{\partial \varphi}=0 \tag{1}$$

$$Re\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(u\frac{\partial v}{\partial r}+\frac{v}{r}\frac{\partial v}{\partial \varphi}+\frac{uv}{r}\right)\right]+\frac{1}{r}\frac{\partial}{\partial \varphi}\left(u\frac{\partial u}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial \varphi}-\frac{v^2}{r}\right)\right\}$$

$$=\nabla^2\left(\frac{1}{r}\frac{\partial(rv)}{\partial r}-\frac{1}{r}\frac{\partial u}{\partial \varphi}\right)+Gr\left(\frac{\partial \theta}{\partial r}\sin\varphi+\frac{1}{r}\frac{\partial \theta}{\partial \varphi}\cos\varphi\right)+$$
$$Gc\left(\frac{\partial c}{\partial r}\sin\varphi+\frac{1}{r}\frac{\partial c}{\partial \varphi}\cos\varphi\right) \tag{2}$$

$$Re\left(u\frac{\partial w}{\partial r}+\frac{v}{r}\frac{\partial w}{\partial \varphi}\right)=\gamma+\nabla^2 ws \tag{3}$$

$$RePr\left(u\frac{\partial \theta}{\partial r}+\frac{v}{r}\frac{\partial \theta}{\partial \varphi}\right)=\nabla^2\theta-\frac{1}{r}\frac{\partial(rq_r)}{\partial r}-\frac{1}{r}\frac{\partial q_\varphi}{\partial \varphi}+QK_r(\theta)C \tag{4}$$

$$\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial(rq_r)}{\partial r}+\frac{1}{r}\frac{\partial q_\varphi}{\partial \varphi}\right]-\frac{3}{N^2}q_r-\frac{3}{B_0}\theta^3\frac{\partial \theta}{\partial r}=0 \tag{5}$$

$$\frac{1}{r}\frac{\partial}{\partial \varphi}\left(\frac{1}{r}\frac{\partial(rq_r)}{\partial r}+\frac{1}{r}\frac{\partial q_\varphi}{\partial \varphi}\right)-\frac{3}{N^2}q_\varphi-\frac{3}{B_0}\theta^3\frac{1}{r}\frac{\partial \theta}{\partial \varphi}=0 \tag{6}$$

$$ReSc\left(u\frac{\partial C}{\partial r}+\frac{v}{r}\frac{\partial C}{\partial \varphi}\right)=\nabla^2 C-K_r(\theta)C \tag{7}$$

Equation 1-3 and 4-7 are continuity, Navier-Stokes and energy equations, respectively and

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}$$

$$K_r(\theta)=K_r^2\theta^n e^{-E/\theta},\gamma=-\frac{\partial P}{\partial z} \tag{8}$$

where:
Re = Reynolds number (plv/μ)
Pr = Prandlt number ($\mu C_p/k$)
Gr = Grashof number ($p_s g Br_0(T_0-T_s/\mu)$)
RePr = Peclet number (vl/α)
Sc = Schmidt number (Sc) (υ/D_{AB})
Q = Enthalpy of formation
C = Concentration
K = Thermal conductivity
N, B_0 = Radiation parameter
q_r, q_ψ = Radiation flux vector

Equation 1-7 are to be solved subject to the boundary conditions

$$U,V,W=0;C=C_w;\theta=\theta_w;$$
$$\frac{q_r}{\varepsilon_w}-\frac{q_r}{2}-\frac{N}{4}\frac{\partial q_r}{\partial r}=0 \text{ on } r=1 \tag{9}$$

Thus, the mathematical statement of the problem is to solve Eq. 1-5 and 7 subject to the condition Eq. 9.

The problem advanced is couple and non-linear and no analytical solution is envisaged. Thus, asymptotic expansion is required.

MATERIALS AND METHODS

A symptotic solution is an approximate form of solution that is obtained by the use of perturbation parameter ε, which is a small number. The resulting series though often divergent is by construction an Asymptotic expansion, which is required to solve this problem. Thus, we make use of Reynolds number for a laminar flow (Re is small).

For small Reynolds number, we seek perturbation solution of the form

$$U=U^{(0)}+ReU^{(1)}+\ldots\ldots\ldots \tag{10}$$

so that we now have from the Eq. 1-7 for order 0

$$\frac{1}{r}\frac{\partial rU^{(0)}}{\partial r}+\frac{1}{r}\frac{\partial V^{(0)}}{\partial \varphi}=0 \tag{11}$$

$$\nabla^2\left[\frac{1}{r}\frac{\partial rV^{(0)}}{\partial r}-\frac{1}{r}\frac{\partial U^{(0)}}{\partial \varphi}\right]+Gr\left[\frac{\partial \theta^{(0)}}{\partial r}\sin\varphi+\frac{1}{r}\frac{\partial \theta^{(0)}}{\partial \varphi}\cos\varphi\right]$$
$$+Gc\left[\frac{\partial C^{(0)}}{\partial r}\sin\varphi+\frac{1}{r}\frac{\partial C^{(0)}}{\partial \varphi}\cos\varphi\right]=0 \tag{12}$$

$$\frac{d^2 W^{(0)}}{dr^2}+\frac{1}{r}\frac{dW^{(0)}}{dr}=-\gamma \tag{13}$$

$$\frac{d^2\theta^{(0)}}{dr^2}+\frac{1}{r}\frac{d\theta^{(0)}}{dr}-\frac{1}{r}\frac{d(rq_r^{(0)})}{dr}+QK_I^2 C^{(0)}=0 \tag{14}$$

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d(rq_r^{(0)})}{dr}\right]-\frac{3}{N^2}q_r^{(0)}-\frac{3}{B_0}\theta^{(0)3}\frac{d\theta^0}{dr}=0 \tag{15}$$

$$\frac{d^2c^{(0)}}{dr^2} + \frac{1}{r}\frac{dc^{(0)}}{dr} - K_l^2 C^{(0)} = 0 \qquad (16)$$

such that

$$U^{(0)} = V^{(0)} = W^{(0)} = 0; C^{(0)} = C_w^{(0)};$$
$$\frac{q_r^{(0)}}{\varepsilon_w} - \frac{q_r^{(0)}}{2} - \frac{N}{4}\frac{\partial q_r^{(0)}}{\partial r} = 0 \text{ on } r = 1 \qquad (17)$$

And

$$\frac{1}{r}\frac{\partial rU^{(1)}}{\partial r} + \frac{1}{r}\frac{\partial V^{(1)}}{\partial \varphi} = 0 \qquad (18)$$

$$\nabla^2\left[\frac{1}{r}\frac{\partial rV^{(1)}}{\partial r} - \frac{1}{r}U^{(1)}\right] + Gr\left[\frac{\partial \theta^{(1)}}{\partial r}\sin\varphi + \frac{\partial \theta^{(1)}}{\partial r}\cos\varphi\right] +$$
$$Gc\left[\frac{\partial c^{(1)}}{\partial r}\sin\varphi + \frac{1}{r}\frac{\partial c^{(1)}}{\partial r}\cos\varphi\right] = 0 \qquad (19)$$

$$\frac{d^2W^{(1)}}{dr^2} + \frac{1}{r}\frac{dW^{(1)}}{dr} = -\gamma \qquad (20)$$

$$\frac{d^2\theta^{(1)}}{dr^2} + \frac{1}{r}\frac{d\theta^{(1)}}{dr} - \frac{1}{r}\frac{d\left(rq_r^{(0)}\right)}{dr} + QK_l^2 C^{(1)} = 0 \qquad (21)$$

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d\left(rq_r^{(1)}\right)}{dr}\right] - \frac{3}{N^2}q_r^1 - \frac{3}{B_0}\theta^{(1)3}\frac{d\theta^1}{dr} = 0 \qquad (22)$$

$$\frac{d^2c^{(1)}}{dr^2} + \frac{1}{r}\frac{dc^{(1)}}{dr} - K_l^2 C^{(1)} = 0 \qquad (23)$$

such that

$$U^{(1)} = V^{(1)} = W^{(1)} = 0; C^{(1)} = C_w^{(1)}\left(\frac{q_r^{(0)}}{\varepsilon_w} - \frac{q_r^{(0)}}{2}\right) -$$
$$\frac{N}{4}\frac{\partial q_r^{(0)}}{\partial r} = 0 \text{ on } r = 1 \qquad (24)$$

However, we use method of substitution for some terms of the same order. First of all put $U^{(0)} = U^{(0)}(r)\cos\varphi$, $V^{(0)} = V^{(0)}(r)\sin\varphi$ into Eq. 11 and 12, we have

$$\frac{1}{r}\frac{\partial}{\partial r}rU^{(0)}\left(r\right)\cos\varphi + \frac{1}{r}\frac{\partial}{\partial \varphi}V^{(0)}\left(r\right)\sin\varphi = 0$$

$$\Rightarrow \frac{1}{r}\frac{d}{dr}rU^{(0)} + \frac{1}{r}V^{(0)} = 0 \qquad (25)$$

and

$$\nabla^2\left[\frac{1}{r}\frac{d}{dr}rV^{(0)}\left(r\right)\sin\varphi - \frac{1}{r}U^{(0)}\left(r\right)\sin\varphi\right] +$$
$$Gr\left(\frac{d\theta^{(0)}}{dr}\sin\varphi\right) + Gc\left(\frac{dc^{(0)}}{dr}\sin\varphi\right) = 0$$

$$\nabla^2\left[\frac{1}{r}\frac{d}{dr}rV^{(0)} - \frac{1}{r}U^{(0)}\right] = -Gr\frac{d\theta^{(0)}}{dr} - Gc\frac{dc^{(0)}}{dr} \qquad (26)$$

Now applying Eq. 8 into 26, we have

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2}\right)\left[\frac{1}{r}\frac{d}{dr}rV^{(0)} + \frac{1}{r}U^{(0)}\right]$$
$$= -Gr\frac{d\theta^{(0)}}{dr} - Gc\frac{dc^{(0)}}{dr} \qquad (27)$$

Calculation of velocity and temperature of the fluid motion in a horizontal pipe for order zero from the given asymptotic expansion when the flow temperature of the fluid and that of the wall are taken to be small for free convective heat transfer to be significant is given. It is assume that the Reynolds number of the flow is small (laminar flow), hence, solution to the given set of problems by perturbation method.

Solution to the governing equation of order zero is obtained by solving for Eq. 16.

Solving by power series solution.

Let $C^{(0)} = \sum a_n r^{m+n}$, then

$$\frac{dC^{(0)}}{dr} = \sum\left(m+n\right)a_n r^{m+n-1}$$

$$\frac{d^2C^{(0)}}{dr^2} = \sum\left(m+n-1\right)\left(m+n\right)a_n r^{m+n-2}$$

and substituting in Eq. 16 and putting n = n + 2 in the first and 2nd series, we have

$$\sum\left(m+n+2\right)^2 a_{n+2} - K_l^2 a_n = 0 \qquad (28)$$

and solving for m in Eq. 28, we have the roots to be m = 0, $a_0 \neq 0$, $a_1 \neq 0$.

Equation 28 is the indicial equation and the recurrence formula is given as

$$a_{n+2} = \frac{K_l^2 a_n}{\left(m+n+2\right)^2} \qquad (29)$$

Therefore,

$$n = 0 \quad \Rightarrow a_2 = \frac{K_1^2 a_0}{(m+2)^2}$$

$$n = 1 \quad \Rightarrow a_3 = \frac{K_1^2 a_1}{(m+3)^2}$$

$$n = 2 \ \ldots\ldots \quad \Rightarrow a_4 = \frac{K_1^2 a_n}{(m+4)^2}$$

Hence, from $C^{(0)} = \Sigma\, a_n\, r^{m+n}$

$$C^{(0)} = a_0 r^m \left\{ 1 + \frac{K_1^2 r^2}{(m+2)^2} + \frac{\left(K_1^2\right)^2 r^4}{(m+2)^2 (m+4)^2} + \ldots\ldots \right\} \tag{30}$$

Let $a_0 = C_w$ and $m = 0$

$$C^{(0)} = C_w \left\{ 1 + \frac{K_1^2}{2^2} + \frac{\left(K_1^2\right)^2 r^4}{2^2 4^2} + \ldots\ldots \right\} \tag{31}$$

which can be written compactly as:

$$C^{(0)} = C_w \frac{I_0\left(K_1^r\right)}{I_0\left(K_1\right)} \tag{32}$$

where, $I_0(K_1)$ is a constant
Also, solving for Eq. 20

$$r \frac{d^2 W^{(0)}}{dr^2} + \frac{dW^{(0)}}{dr} + \gamma = 0$$

using the transformation $r = e^t$

$$\frac{d^2 W^{(0)}}{dr^2} = e^{-2t} \left[\frac{d^2 W}{dt^2} - \frac{dW}{dt} \right] \frac{dW}{dr} = e^{-t} \frac{dW}{dt} \tag{33}$$

substituting Eq. 33 into 20 and multiplying by e^{-t}, we have

$$\frac{d^2 W}{dt^2} + e^{2t} \gamma = 0 \Rightarrow \frac{d^2 W}{dt^2} = -e^{2t} \gamma \tag{34}$$

Therefore,

$$W = -\frac{1}{4} e^{2t} \gamma + C_1 t + C_2$$

but $r = e^t$ and so $t = \ln r$

$$\Rightarrow W = -\frac{1}{4} e^{2\ln r} \gamma + C_1 \ln r + C_2$$

$$= -\frac{1}{4} r^2 \gamma + C_1 \ln r + C_2$$

using the boundary condition on $r = 1$, $w = 0$, we have

$$W = \frac{1}{4} \gamma \left(1 - r^2\right) \tag{35}$$

Next, we solve Eq. 14 and 15 by assuming that the difference between the temperature of fluid and that of the wall is small i.e.,

$$\theta = \theta_w + \psi \tag{36}$$

Now substituting Eq. 36 into 14 and 15 in which $\theta_w > $ order $o(\varphi) > $ order $o(\mathrm{Re})$, we have

$$\frac{d^2 \psi}{dr^2} + \frac{1}{r} \frac{d\psi}{dr} - \frac{dq_r^{(0)}}{dr} - \frac{1}{r} q_r^{(0)} = -QK_1^2 C_w \frac{I_0\left(K_1^2\right)}{I_0\left(K_1\right)} \tag{37}$$

$$\frac{d^2 q_r^{(0)}}{dr^2} + \frac{1}{r} \frac{d\, q_r^{(0)}}{dr} - \left(\frac{1}{r^2} + \frac{3}{N^2} \right) q_r^{(0)} - \frac{3}{B_0} \theta_w^3 \frac{d\psi}{dr} = 0 \tag{38}$$

Making $d\psi/dr$ the subject in Eq. 38 and substitute in Eq. 37, we have

$$\frac{B_0}{3\theta_w^3} \frac{d}{dr} \left\{ \frac{d^2 q_r^{(0)}}{dr^2} + \frac{1}{r} \frac{dq_r^{(0)}}{dr} - \left(\frac{1}{r^2} + \frac{3}{N^2} \right) q_r^{(0)} \right\} +$$

$$\frac{1}{r} \frac{B_0}{3\theta_w^3} \left\{ \frac{d^2 q_r^{(0)}}{dr^2} + \frac{1}{r} \frac{dq_r^{(0)}}{dr} - \left(\frac{1}{r^2} + \frac{3}{N^2} \right) q_r^{(0)} \right\}$$

$$- \frac{dq_r^{(0)}}{dr} - \frac{1}{r} q_r^{(0)} = -QK_1^2 C_w \frac{I_0\left(K_1^r\right)}{I_0\left(K_1\right)}$$

$$\Rightarrow \frac{d}{dr} \left(\frac{d^2 q_r^{(0)}}{dr} \right) + \frac{2}{r} \frac{d^2 q_r^{(0)}}{dr^2} - \frac{1}{r^2} \frac{dq_r^{(0)}}{dr} + \frac{q_r^{(0)}}{r^3} -$$

$$\frac{3}{N^2} \left(\frac{dq_r^{(0)}}{dr} + \frac{q_r^{(0)}}{r} \right) - \frac{3\theta_w^3}{B_0} \left(\frac{dq_r^{(0)}}{dr} + \frac{q_r^{(0)}}{r} \right) =$$

$$- \frac{3\theta_w^3}{B_0} QK_1^2 C_w \frac{I\left(K_1^r\right)}{I_0\left(K_1\right)}$$

$$\Rightarrow \frac{d}{dr} \left\{ r \left[\frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{1}{r^2} - 3 \left(\frac{1}{N^2} + \frac{\theta_w^3}{B_0} \right) \right] q_r^{(0)} \right\} =$$

$$- QK_1^2 C_w \frac{3\theta_w^3 I_0\left(K_1^r\right)}{B_0 I_0\left(K_1\right)} \tag{39}$$

Now integrating Eq. 39, we have

$$\left\{ \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \left(\frac{1}{r^2} + \frac{3}{N^2}\right) - \frac{3\theta_w^3}{B_0} \right\}$$
$$q_r^{(0)} = -QK_1^2 C_w \frac{3\theta_w^3}{B_0} I_1(K_1 r) \qquad (40)$$

However, from Eq. 38, we have

$$\frac{3\theta_w^3}{B_0}\frac{d\psi}{dr} = \frac{d^2 q_r^{(0)}}{dr^2} + \frac{1}{r}\frac{dq_r^{(0)}}{dr} - \left(\frac{1}{r^2} + \frac{3}{N^2}\right) q_r^{(0)}$$

Therefore, Eq. 40 becomes

$$-\frac{QK_1^2 C_w 3\theta_w^3}{I_0(K_1)B_0} I_1(K_1 r) + \frac{3\theta_w^3}{B_0} q_r^{(0)} - \frac{3\theta_w^3}{B_0}\frac{d\psi}{dr} = 0 \qquad (41)$$

Using power series to the differential Eq. 40, we obtained the recurrence equation

$$a_{n+2} = \frac{3\left(\frac{1}{N^2} + \frac{\theta_w^3}{B_0}\right) a_n}{(c+n+1)(c+n+3)}$$

which, gives $c = 1$, $a_0 \neq 0$, $a_1 = 0$ from the indicial equation, so that from the power series from the Eq. 42, we have

$$q_r^{(0)} = a_0 r^c \left[1 + 3\left[\frac{\left(\frac{1}{N^2} + \frac{\theta_w^3}{B_0}\right)}{(c+1)(c+3)}\right] r^2 + 3\left[\frac{\frac{1}{N^2} + \frac{\theta_w^3}{B_0}}{(c+1)(c+3)^2(c+5)}\right]^2 r^4 + ... \right] \qquad (42)$$

Let $a_0 = A_1$ and $c = 1$, then

$$q_r^{(0)} = A_1 r \left[1 + 3\left[\frac{\left(\frac{1}{N^2} + \frac{\theta_w^3}{B_0}\right)}{2.4}\right] + r^2 + 3\left[\frac{\frac{1}{N^2} + \frac{\theta_w^3}{B_0}}{2^2.4^2.6}\right]^2 r^4 + ... \right] = -\frac{QK_1^2 C_w 3\theta_w^3}{I_0(K_1)B_0} I_1(K_1 r)$$

$$(43)$$

Thus, the non-linear Eq. 15 can be reduced to a non-linear integral equation using the transformation,

$$\frac{3\theta_w^3}{B_0}\frac{d\theta^{(0)}}{dr} = \frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r q_r^{(0)}\right)\right] - \frac{3}{N^2} q_r^{(0)}$$

which can be written as:

$$\frac{d\theta^{(0)}}{dr} = -QK_1 C_w \frac{I(K_1 r)_1}{I_0(K_1)} + q_r^{(0)} \qquad (44)$$

Integrating Eq. 44, we obtain

$$\theta^{(0)} = \frac{-QKC_{w1}}{I_0(K_1)}\left\{\frac{r^2}{2^2} + \frac{K_1^2 r^4}{2^2.4^2} + ...\right\} +$$
$$A_1\left\{\frac{r^2}{2^2} + 3\left(\frac{\frac{1}{N^2} + \frac{\theta_w^3}{B_0}}{2^2.4.3}\right)r^3 + 3\left(\frac{\frac{1}{N^2} + \frac{\theta_w^3}{B_0}}{2^2.4^2.5.6}\right)r^5 + ...\right\} - (45)$$
$$\frac{QK_1 C_w 3\theta_w^3}{I_0(K_1)B_0}\left(\frac{r^2}{2^2} + \frac{K_1^2 r^4}{2^2.4^2} + ...\right)$$

Therefore, Eq. 27 can be written as

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2}\right)\left[\frac{1}{r}\frac{d}{dr}rV^{(0)} + \frac{1}{r}U^{(0)}\right] = -A_1 I_1\left[3^{\frac{1}{2}}\left(\frac{1}{N^2} + \frac{\theta_w^3}{B_0}\right)^{\frac{1}{2}}r\right]$$
$$+ A_2\frac{QK_1 C_w I_1(Kr)}{I_0(Kr)} - C_w K\frac{I_1(Kr)}{I_0(Kr)}$$

$$(46)$$

Now, solving this equation together with the continuity Eq. 24 using numerical approach with the Rugge-Kutta method of solution that reduces higher differential equations to first order making use of the boundary conditions.

The fourth other Rugge-Kutta method is given by the formular

$$y_{n+2} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where:

$$k_1 = f(x_n + y_n); \quad k_2 = f(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_1)$$
$$k_3 = f(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_2), \quad k_4 = f(x_n + h, y_n + hk_3)$$

$$(47)$$

The results are presented graphically thus, using the standard values of some terms.

RESULTS AND DISCUSSION

In flow of high temperature combustible gas in a horizontal pipe, the temperature distribution and free convection heat transfer takes the pride of place. And as a result of free convection in the limit of creeping flow, axial velocity is induced.

For a given value of the free convection and radiation parameters, the velocity predicted by the asymptotic solution is higher than that obtained from the numerical solution of the general differential approximation (Fig. 1).

When the free convection parameters are increased, or radiation parameters are reduced, the velocity increases
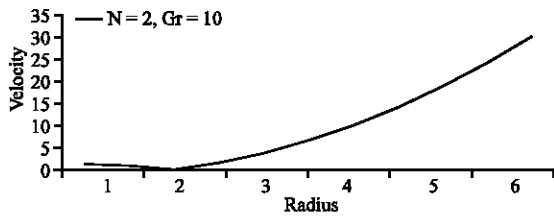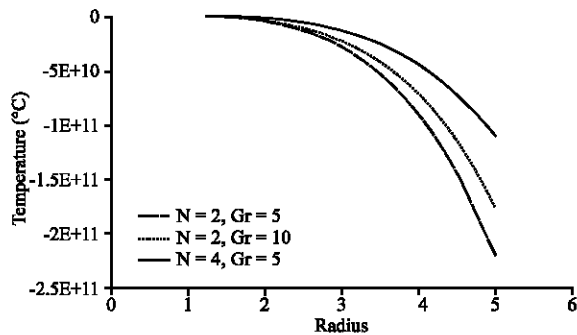

Fig. 1: Axial velocity

Fig. 2: Temperature distribution

and when there is decrease in the radiation parameters, then there is increase in the convection parameters. Similarly, the temperature distribution follows a similar pattern (Fig. 2).

CONCLUSION

When a combustible gas is flowing through a horizontal pipe, decrease in the radiation parameter, N, leads to an increase in velocity of gas. Considering the free convection parameters $G_c = G_r$, an increase in G_r gives an increase in the velocity of the gas. Similarly, for the temperature it follows the same pattern as that of the velocity of the gas. Thus, the conclusion of the velocity and temperature distribution of the flow of a combustible gas in horizontal pipe in the presence of free convection heat transfer.

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