

Reliability Analysis of Static Pile Capacity for Concrete in Cohesive and Cohesionless Soils

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Abstract: This study examined the reliability assessment of the carrying capacities of piles based on static approach with special consideration to pre-cast concrete as pile type in cohesive and cohesionless soils. The concept of the First-Order Reliability Method (FORM) was adopted for the assessment. The results obtained show that the safety index for piling degenerates with increasing length of piles both in cohesive and cohesionless soils. However, the static pile capacity is grossly conservative in cohesive soils and while it is highly unsafe in cohesionless soils. In fact, the safety index ranges from about 2.5-0 when pile length is between 10 and 30 m. Beyond this range of pile length, piling operation appears to be practically a failure in cohesionless soils.

Key words: Reliability analysis, static pile capacity, concrete piles, cohesive soils, cohesionless soils

INTRODUCTION

Structural reliability can be expressed by defining functional relations between strength (R) and load (L) parameters as follows:

$$M = R - L = g(X_1, X_2, \dots, X_n) \quad (1)$$

In Eq. (1), M is the limit state function and is sometimes referred to as safety margin or performance function, $X = X_1, X_2, \dots, X_n$ denote n basic design random variables and g (X) denotes a function of all design variable. In general, the function g (X) can take any form provided that the failure of the structure is defined when $M \leq 0$ and the survival of the structure is defined when $M > 0$. Therefore, the probability of failure of the structure can be calculated by performing the following integration over the region where, $M \leq 0$.

$$P_f = \iiint \dots \int f_x(X_1, X_2, \dots, X_n) dx_1 dx_2 \dots dx_n \quad (2)$$

where, f_x is the joint probability density function for the basic random variables X_1, X_2, \dots, X_n (Elhewy *et al.*, 2006).

Deterministic methods are very subjective and are generally not based on a systematic assessment of reliability, especially when we consider their use in the entire structure (not only the foundations). The methods can produce structures with either some over-designed or

some under-designed components. The additional experience incurred in constructing the over-designed components probably does not contribute to the overall reliability of the structure, so this is not a very cost-effective way to produce reliable structures. In other words, it would be better to redirect some of the resources used to build the over designed components toward strengthening the under-designed ones. Therefore, there is increasing interest in adopting reliability-based design methods in civil engineering. These methods are intended to quantify reliability and thus, may be used to develop balance designs that are both more reliable and less expensive. Also, to better evaluate the various sources of failure and use this information to develop design and construction methods that are both more reliable and more robust-one that is insensitive to variations in materials, construction techniques and environment (Coduto, 2001).

The reliability of an engineering system can be defined as its ability to fulfill its design purpose for some time period. The theory of probability provides the fundamental basis to measure this ability. The reliability of a structure can be viewed as the probability of its satisfactory performance according to some performance functions under specific service and extreme conditions within a stated time period. In estimating this probability, system uncertainties are modeled using random variables with mean values, variances and probability distribution functions. Many methods have been proposed for structural reliability assessment purposes, such as First

Order Second Moment (FOSM) method, Advanced Second Moment (ASM) method and Computer-based Monte Carlo Simulation (MCS) (Ang and Tang, 1990; Ayyub and Haldar, 1984; White and Ayyub, 1985; Ayyub and McCuen, 1997) as reported by Ayyub and Patev (1998).

Reliability-based design methods could be used to address many different aspects of foundation design and construction. However, most of these efforts to date have focused on geotechnical and structural strength requirements, such as the bearing capacity of shallow foundations, the side friction and toe-bearing capacity of deep foundations and the stresses in deep foundations. All of these are based on the difference between load and capacity, so we can use a more specific definition of reliability as being the probability of the load being less than the capacity for the entire design life of the foundation. According to Coduto (2001), various methods are available to develop reliability-based design of foundations, most notably stochastic methods, the First-Order Second Moment and the Load and Resistance Factor Design method.

The purpose of design is the achievement of acceptable probabilities that the structure being designed will not become unfit in any way for the use for which it is intended. Engineering problems of this structure, however, often involved multiple failure modes; that is, there may be several potential modes of failure, in which the occurrence of any one of the potential failure modes will constitute non-performance of the system or component. Recent researches in the area of structural reliability and probabilistic analysis have centered around the development of probabilistic-based design procedures. These include load modeling, ultimate and service load performance and evaluation of current levels of safety/reliability in design (Farid Uddin, 2000; Afolayan, 1999, 2003; Afolayan and Abubakar, 2003).

It is increasingly required that the hazard and risk associated with engineered constructions be quantified. Geotechnical hazard assessment in the context of a risk framework, using concepts of uncertainties, reliability, safety and risk shows that the use of this approach is exemplified for offshore facilities, including piled foundations, jack-up structures, gravity foundations and under water slopes. The application demonstrates that probabilistic analyses complement the conventional deterministic safety factor and deformation-based analyses and contribute to achieving safe and optimum design. The probabilistic approach adds value to the results with a modest additional effort. The conclusions emphasize the usefulness of risk assessment, the importance of engineering judgment in the assessment

and the need for involving multi-disciplinary competences to achieve reliable estimate of hazard and risk (Lacasse and Nadim, 2007).

Patev (1995) made a reliability assessment of timber pile-founded navigation structures without the loss of support. The techniques developed to perform the reliability assessment employed a capacity/demand relationship for limit states of axial capacity, lateral deflections, axial deflections and combined bending. Monte Carlo Simulation and First Order Second Moment were utilized to calibrate the limit states to an instrumented pile load test. In this study, a First-order reliability assessment of static pile capacity for concrete in cohesive and cohesionless soils is reported.

MATERIALS AND METHODS

All static pile capacities can be computed by the following equations (Bowles, 1988):

$$P_u = P_{pu} + \sum P_{si} \text{ (Compression)} \quad (3)$$

and

$$T_u = \sum P_{si} + W \text{ (Tension)} \quad (4)$$

where:

- P_u : Ultimate (maximum) pile capacity in compression.
- T_u : Ultimate pullout capacity.
- P_{pu} : Ultimate point capacity.
- $\sum P_{si}$: Skin (or shaft friction) resistance contribution from several strata penetrated.
- W : Weight of pile.

The allowable pile capacity P_a or T_a is obtained from applying a suitable factor of safety (SF) on the parts as:

$$P_a = \frac{P_{pu}}{SF_p} + \frac{\sum P_{si}}{SF_s} \quad (5)$$

Or using a single value SF (most common) to obtain

$$P_a = \frac{P_u}{SF} \text{ and } T_a = \frac{T_u}{SF}$$

This value of P_a or T_a should be compatible with the capacity based on the pile material (timber, concrete, or steel) and SF, represents the safety factor which commonly range from 2.0-4.0 or more depending on designer uncertainties.

The ultimate static pile point capacity can be computed using the bearing capacity Eq. 6:

$$P_{pu} = A_p (cN'_c + \eta qN'_q + \frac{1}{2\gamma} - BN_\gamma s_\gamma) \quad (6)$$

where:

- A_p : Area of pile point effective in bearing.
- c : Cohesion (or undrained shear strength, S_u).
- B : Base of pile (usually used only when point is enlarged).
- N'_c : Bearing capacity factor for cohesion adjusted for shape and depth. When $\phi = 0$, we have $c = S_u$ and N'_c is often taken as 9.
- N'_q : Bearing capacity factor for overburden effects, $q = \gamma L$ and includes shape and depth effects.
- N'_r : Bearing capacity factor for base width = N_r since it is not affected by depth effects.
- q : Effective vertical stress (overburden pressure) at pile point.
- η : 1.

The skin resistance capacity is currently computed using either a combination of total and effective or only effective stresses. There are three procedures currently used with the two general methods for computing the skin resistance of piles in cohesive soils. These are called the α -, γ - and β - methods. The β - method is also used for piles in cohesionless soils. In all cases the skin resistance capacity is computed as:

$$P_{st} = \sum A_s f_s \quad (7)$$

where:

- A_s : Effective pile surface area on which f_s acts and commonly computed as the product of perimeter and embedment increment ΔL .
- f_s : Skin resistance and Σ = summation of contributions from several strata or pile segments.

First Order Reliability Method (FORM) is the technique adopted to perform the reliability analysis of static pile capacity for concrete piling in cohesive and cohesionless soils. FORM is an approximate computation of general probability integrals over region domains with locally smooth boundaries. The concept of FORM is, essentially based on the approximate solution to Eq. (1) such that:

$$P_f = P(X \in F) = P(g(X) \leq 0) = \int_{g(x) \leq 0} dF_X(x) = \Phi(-\beta_R) \quad (8)$$

in which β_R = the reliability or safety index.

The general problem to which FORM provides an approximate solution is as follows. The state of a system is a function of many variables some of which are uncertain. These uncertain variables are random with joint distribution function

$$F_x(x) = P(\prod_{i=1}^n \{X_i \leq x_i\})$$

defining the stochastic model. For FORM, it is required that $F_x(x)$, is at least locally continuously differentiable, i. e., that probability densities exist. The random variables $X = (X_1, \dots, X_n)^T$ are called basic variables. The locally sufficiently smooth (at least once differentiable) state function is denoted by $g(X)$. It is defined such that $g(X) > 0$ corresponds to favourable (safe, intact, acceptable) state. $g(X) = 0$ denotes the so-called limit state or the failure boundary. Therefore, $g(X) < 0$ (sometimes also $g(X) \leq 0$) defines the failure (unacceptable, adverse) domain, F . The function $g(X)$ can be defined as an analytic function or an algorithm (e.g., a finite element code). In the context of FORM it is convenient but necessary only locally that $g(X)$ is a monotonic function in each component of X . Among other useful information FORM produces an approximation to Eq. (8) (Melchers, 2002).

RESULTS AND DISCUSSION

Limit state for static for static pile capacity of concrete piling in cohesionless soils: The functional relationship between the allowable design load and the allowable pile capacity can be expressed as follows:

$G(X)$ = Allowable Design Load-Allowable Pile Capacity, So that,

$$G(X) = 0.33f_{cu} \frac{\pi D_1^2}{4} + 0.4f_y \frac{\pi D_2^2}{4} - \left\{ \frac{\pi D_1^2}{4} (9S_u) + \pi D_1 L_b \alpha S_u \right\} / SF \quad (9)$$

where:

- f_{cu} : Characteristic strength of concrete, D_1 = pile diameter, D_2 = steel diameter.
- f_y : Characteristic strength of steel, S_u = cohesion, L_b = pile length, α = adhesion factor and SF = factor of safety.

In Table 1, the statistical and probabilistic descriptions of the variables in the functional relations are presented.

Table 1: Stochastic model for concrete piling in cohesive soils

Variables	Probability density functions	Mean values	Coefficients of variations
f_{cu}	Lognormal	40 MN m ⁻²	0.15
D_1	Normal	0.3 m	0.06
f_y	Lognormal	460 MN m ⁻²	0.15
D_2	Normal	25×10 ⁻³ m	0.06
S_u	Lognormal	12 kN m ⁻¹	0.15
α	Lognormal	1.00	0.15
L_b	Normal	23.0 m	0.06
SF	Lognormal	3.0	0.15

Table 2: Stochastic model for concrete piling in cohesionless soils

Variables	Probability density function	Mean values	Coefficients of variations
f_{cu}	Lognormal	40 MN m ⁻²	0.15
D_1	Normal	4.5×10 ⁻¹ m	0.06
f_y	Lognormal	460 MN m ⁻²	0.15
D_2	Normal	25×10 ⁻³	0.06
γ	Lognormal	17.3 kN m ⁻³	0.15
L	Normal	12 m	0.06
ϕ	Lognormal	34°	0.15
ϕ'	Lognormal	34°	0.15
δ	Lognormal	22°	0.15
SF	Lognormal	3.0	0.15

Limit state for static pile capacity of concrete piling in cohesionless soils: Similar to the functional relationship between allowable design load and the allowable pile capacity expressed for cohesive soils, we also have:

$$G(X) = \left[\frac{0.259f_{cu}D_1^2 + 0.314f_yD_2^2 - k_f}{SF} \right] \quad (10)$$

in which

$$k_f = 0.785\gamma LD_1^2 \left(\frac{[e^{(2.36+0.5\phi)\tan\phi}]^2}{2\cos^2(45-0.5\phi)} \right) + 3.142D_1L_b + \gamma L(1 - \sin\phi')\tan\delta$$

as the expression for assessing the performance of concrete piling in cohesionless soils. In Eq. (10), the additional variables not in Eq. (9) are: γ = unit weight of soil, ϕ = drained angle, ϕ' = effective stress angle and δ = effective friction angle. The assumed statistical values and their corresponding probability distributions are shown in Table 2.

Starting with the assumed statistical values and the probability distributions given in Tables 1 and 2, the formulae for concrete piling in both cohesive and cohesionless soils are rated. As is common in practice, the length and diameter of piles are subjected to variations and the results of the assessment are as displayed in Fig. 1 and 2.

From Fig. 1, it can be seen that the safety level, β_R , increases with the diameter of the pile, though it decreases with pile length. At any rate, the safety level is

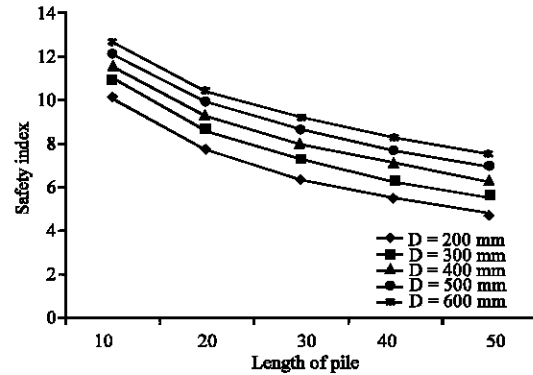


Fig. 1: Safety index (β_R) against length of Pile with varying diameters in cohesive soils

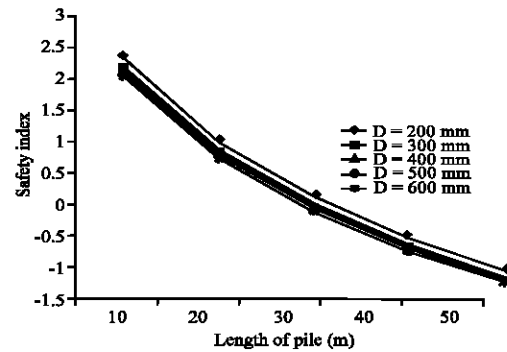


Fig. 2: Safety index (β_R) against length of pile with varying diameters in cohesionless soils

generally high for all the range of values for pile length, implying that the estimated pile capacity is highly conservative.

In safety classification, piling attracts a high degree of safety and as such, any design procedure should be able to afford safety index not less than 4.2 (NBK, 1978). By this standard, the current design formulae for static piling employing pre-cast concrete are grossly inadequate in cohesionless soils. This follows from the results in Fig. 2. It is observable that the safety index rapidly degenerates with increasing pile length even when the pile diameter is increased. This is very much unlike the design formulae recommended for concrete piles in cohesive soils (Fig. 1). In cohesive soils, pile length that is as much as 50 m is still theoretically, if not economically, admissible. Most practical designs adopt pile length between 10-30 m. From the results of the current assessment, pile length of about 30 m in cohesionless soils may lead to catastrophic failure if pre-cast concrete is used. The authors are working on the assessment of the reliability of alternate materials in cohesionless soils.

CONCLUSION

The First-Order Reliability Method has been employed to rate static pile capacity for concrete in cohesive and cohesionless soils. All relevant variables are considered random with assumed probability density distributions. From the results, it can be concluded that concrete piling should be discouraged in cohesionless soils and if it must be used at all, the pile length should not exceed 20 m.

Also, even in cohesive soils where the piling safety is grossly conservative, the static pile capacity equations are very expensive.

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