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Assessment of Fatigue Life Behaviour of Cylinder Block for Free Piston Engine Using Frequency Response Approach

M.M. Rahman and B.A.B. Rosli
Faculty of Mechanical Engineering, University of Malaysia Pahang,
Locked Bag 12, 25000 Kuantan, Pahang, Malaysia

Abstract: This study presents the finite element analysis technique to predict the fatigue life using narrow band frequency response approach. The life prediction results are worthy for improving the component design at the very early developing stage. This approach is adequate for periodic loading, however, requires very large time records to accurately describe random loading processes. This study describes how this technique can be implemented in the finite element environment to rapidly identify the critical areas of the structure. Fatigue damage in conventionally determined from time signals of the loading, frequently in the form of stress and strain. However, there are scenarios when a spectral form of loading is more appropriate. In this case the loading is defined in terms of its magnitude at different frequencies in the form of a Power Spectral Density (PSD) plot. The vibration fatigue calculation can be utilized where the random loading and response are categorized using power spectral density functions and the dynamic structure is modeled as a linear transfer function. This study also investigates the effect of the mean stress on the fatigue life prediction using the random loading. The acquired results indicate that the Goodman mean stress correction method gives the most conservative results with the Gerber and no (zero) mean stress method. The proposed technique is capable of determining premature products failure phenomena. Therefore, it can be reduce cost, time to market, improve the product reliability and finally the user confidence.

Key words: Fatigue life, vibration fatigue, power spectral density functions, frequency response, cylinder block, free piston

INTRODUCTION

Structures and mechanical components are frequently subjected to the oscillating loads which are random in nature. Random vibration theory has been introduced for more then three decades to deal with all kinds of random vibration behaviour. Since, fatigue is one of the primary causes of the component failure, fatigue life prediction has become a most important issue in almost any random vibration problem (Bolotin, 1984; Crandell and Mark, 1973; Soong and Grigoriu, 1993; Newland, 1993). Nearly all structures or components have been designed using the time based structural and fatigue analysis methods. However, by developing a frequency based fatigue analysis approach, the accurate composition of the random stress or strain responses can be retained within a greatly optimized fatigue design process.

The time domain fatigue approach consists of 2 major steps. Firstly, the numbers of stress cycle in the response time history (Suresh, 2002; Stephen *et al*., 2001; Bishop and Sherrat, 1990) are counted. This is conducted through a process called rainflow cycle counting. Secondly, the damage from each cycle is determined, normally from the S-N curve. The damage is then summed over all cycles using the linear damage summation techniques to determine the total life. The purpose of presenting these basic fatigue concepts is to emphasize that fatigue analysis is generally thought of as a time domain approach, that is, all of the operations are based on time descriptions of the load function. This study demonstrates that an alternative frequency domain (Newland, 1993; Wirsching *et al*., 1995; Bishop, 1988) fatigue approach can be more appropriate.

A vibration analysis is generally carried out to ensure that structural natural frequencies or resonant modes are not excited by the frequencies present in the applied load. Sometimes, this is not possible and designers then have to estimate the maximum response at the resonance caused by the loading. These are the best performed in the frequency domain using the power spectral density

Corresponding Author: M.M. Rahman, Faculty of Mechanical Engineering, University of Malaysia Pahang, Locked Bag 12, 25000 Kuantan, Pahang, Malaysia

functions of input loading and stress response. It is often easier to obtain a PSD of stress rather than a time history (Bishop and Sherratt, 2000a, b). The dynamic analysis of complicated finite element models is considered in this study. It is valuable to carryout the frequency response analysis instead of a computationally intensive transient dynamic analysis in the time domain. A finite element analysis based on the frequency domain can simplify the problem. The designer can perform the frequency response analysis on the Finite Element Model (FEM) to determine the transfer function between the load and stress in the structure. This approach requires that the PSD of load is multiplied by the transfer function to obtaining the PSD of stress. The main purpose of this study is to prediction of fatigue life when the component is subjected to statistically define random stresses.

THEORETICAL BASIS

The stress power spectra density (Soong and Grigoriu, 1993; Newland, 1993; Bishop, 1988; Bishop and Sherratt, 2000a, b; MSC/Fatigue, 2005) represents the frequency domain approach input into the fatigue. This is a scalar function that describes how the power of the time signal is distributed among frequencies (Bendat, 1964). Mathematically, this function can be obtained by using a Fourier Transform of the stress time history's auto-correlation function and its area represents the signal's standard deviation. It is clear that the PSD is the most complete and concise representation of a random process. There are many important correlations between the time domain and frequency domain representations (Rahman *et al*., 2005) of a random process. Bendat (1964) presented the theoretical basis for the first frequency response models, so called narrow band solution. The author showed that the Probability Density Function (PDF) of peaks for the narrow band signal tended towards Rayleigh distributions as the bandwidth reduced. Furthermore, for the narrow banded time history Bendat assumed that all the positive peaks in the time history would be followed by corresponding troughs of similar magnitude regardless of whether they actually formed the stress cycles. Using this technique, the PDF of stress range would also tend to the Rayleigh distribution. Bendat used a series of equations derived by Rice (1954) to estimate the expected number of peaks using the moments of area beneath the PSD. Bendat's narrow band solution for the range mean histogram is therefore expressed in Eq. 1. Assume, p(S) is the Rayleigh distribution which is represented by a narrow band process and stress amplitude (S) can be treated as a continuous random variable.

$$\begin{aligned} E[D] &= \sum_i \frac{n_i}{N(S_i)} = \frac{S_t}{k}\int S^b\ p(S)\,dS \\ &= \frac{E[P]\,T}{k}\int S^b\,\Big[\frac{S}{4\,m_0}\ e^{\frac{-S^2}{8m_0}}\Big]\,dS \end{aligned} \tag{1}$$

where, $N(S_i)$ is the number of cycles of stress range (S) occurring in T seconds, n_i is the actual counted number of cycle, S_t is the total number of cycles equals to {E[P]. T}, E[P] is the number of peak per seconds, m_0 is the zero order moment of area of the PSD. Parameters k and b are the materials constant that would be defined in the S-N curve.

The Dirlik solution (Dirlik, 1985) is expressed by Eq. (2) and details the specific literature reported in the Dirlik (1985), Haiba *et al*. (2002), Zhao and Baker (1992), Kim and Kim (1994), Sakai and Okamura (1995) and Fu and Cebon (2000).

$$N(S) = E[P]\ T\ p(S) \tag{2}$$

where, N(S) is the number of stress cycles of range (S) N/mm^2 expected in T seconds. E[P] is the expected number of peaks and p(S) is the probability density function.

$$p(S) = \frac{\frac{D_1}{Q} e^{\frac{-Z}{Q}} + \frac{D_2 Z}{R^2} e^{\frac{-Z^2}{2R^2}} + D_3 Z e^{\frac{-Z^2}{2}}}{2\sqrt{m_0}} \tag{3}$$

$$\begin{aligned} D_1 &= \frac{2(x_m-\gamma^2)}{1+\gamma^2}, \quad D_2 = \frac{1-\gamma-D_1+D_1^2}{1-R}, \\ D_3 &= 1-D_1-D_2, \quad \gamma = \frac{m_2}{\sqrt{m_0\, m_4}} \end{aligned} \tag{4}$$

$$\begin{aligned} Q &= \frac{1.25(\gamma-D_3-D_2\,R)}{D_1}, \quad R = \frac{\gamma-x_m-D_1^2}{1-\gamma-D_1+D_1^2}, \\ x_m &= \frac{m_1}{m_0}\sqrt{\frac{m_2}{m_4}}, \quad Z = \frac{S}{2\sqrt{m_0}} \end{aligned} \tag{5}$$

where, x_m, D_1, D_2, D_3, Q and R are depending on the m_0, m_1, m_2 and m_4; Z is a normalized variable. m_0, m_1, m_2 and m_4 are the zeroth, 1st, 2nd and 4th order spectral moments area, respectively.

LOADING INFORMATION

Loading is one of the major input to the Finite Element (FE) based fatigue analysis. Loading information

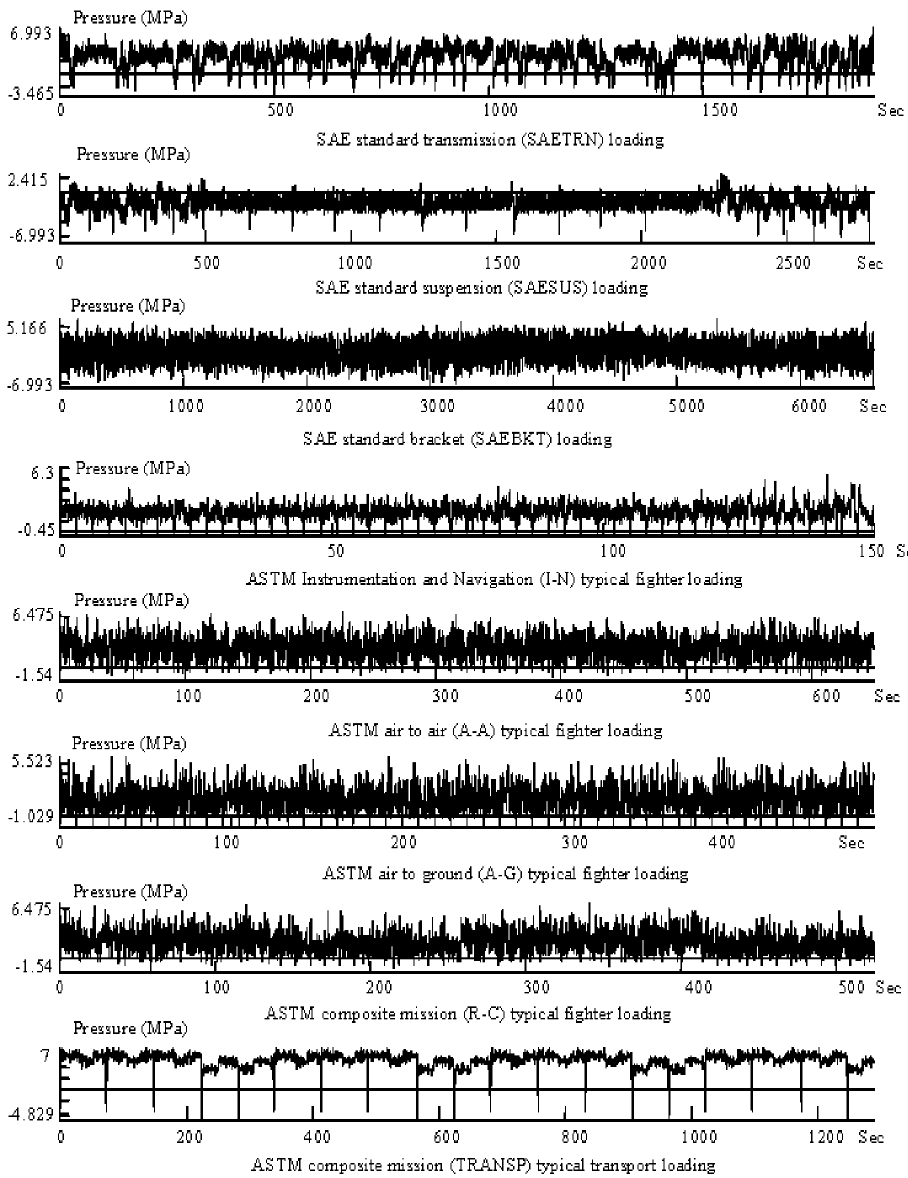

Fig. 1: Different time-loading histories

can be obtained using a number of different methods. The ability to predict component loads analytically means that the physical components are no longer a prerequisite for the fatigue analysis. Thus, the analysis can proceed much earlier in the design cycle. It is important to note that in this context, loading environment is defined as the set of phase-related loading sequences such as time histories that uniquely map the cyclic loads to each external input location on the component. Several types of variable amplitude loading history were selected from the SAE and ASTM profiles for the FE based fatigue analysis. It is important to emphasize that these sequences are not intended to represent standard loading spectra in the same way that Carlos or Falstaf (2005) was performed. However, they do contain many features which are typical of the automotive industries applications and therefore, are useful in the evaluation of the life estimation methods. The detailed information about these histories is given in the literature (Rahman *et al.*, 2005; Tucker and Bussa, 1977). The variable amplitude load-time histories are shown in Fig. 1 and the corresponding power spectral density response

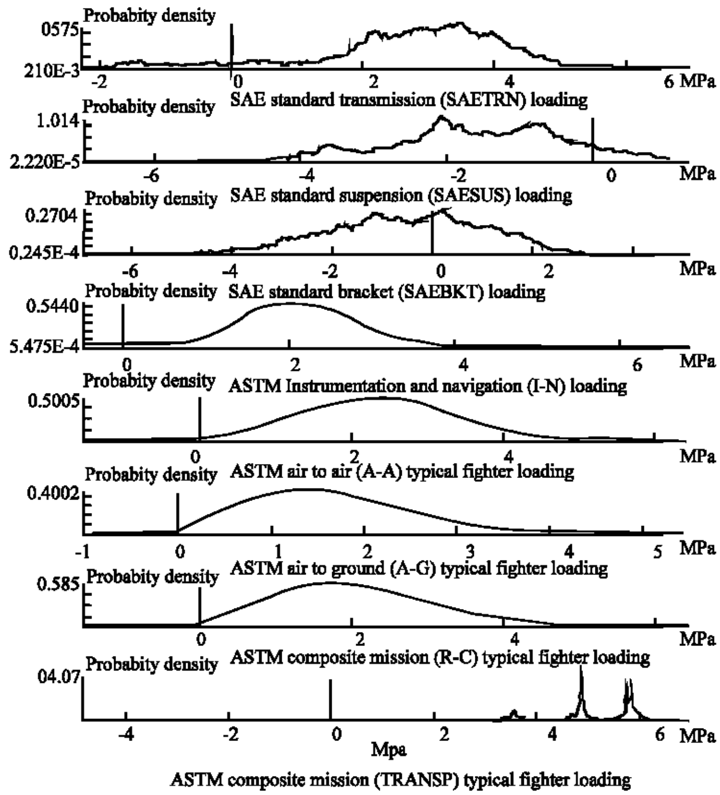

Fig. 2: Power Spectral Densities (PSD) responses

are shown in Fig. 2. The terms of SAETRN, SAESUS and SAEBRAKT represent the load-time history for the transmission, suspension and bracket, respectively. The considered load-time histories are based on the SAE's profile. In addition, I-N, A-A, A-G, R-C and TRANSP are represent the ASTM instrumentation and navigation typical fighter, ASTM air to air typical fighter, ASTM air to ground typical fighter, ASTM composite mission typical fighter and ASTM composite mission typical transport loading history, respectively (Rahman *et al.*, 2005). The abscissa is the time in seconds. The realistic loading histories are often difficult to obtain. A measurement of these is required during typical and extreme operating conditions. This is normally carried out by a dedicated measurement device such as transducers and data acquisition equipment. The finite element based fatigue analysis helps to eliminate unnecessary tests by allowing the designer to checkout the fatigue performance analytically and to optimize the selection of the material, manufacturing processes and geometry within the constraints of total cost and loading environment. Carrying out this optimization analytically, rather than by testing, can shorten the time to market and reduce costs significantly.

NUMERICAL EXAMPLE

Finite Element Modeling (FEM): There are a number of safety-critical component of the free piston engine. The cylinder block is the one of the important and safety-critical components of the free piston engine. A geometric model of the cylinder block of the free piston engine is considered as an example parts in this study. There are several contact areas including the cylinder head, gasket and hole for bolt. Therefore, constraints are employed for the following purposes: to specify the prescribed enforce displacements, to simulate the continuous behavior of displacement in the interface area and to enforce rest condition in the specified directions at grid points of reaction.

Three-dimensional model of the free piston linear engine cylinder block was developed using CATIA® software. A 10 nodes tetrahedral element was used for the solid mesh. Sensitivity analysis was performed to obtain the optimum element size. These analyses were performed iteratively at different element lengths until the solution obtained appropriate accuracy. Convergence of the stresses was observed, as the mesh size was successively refined. The element size of 0.20 mm was finally considered. A total of 35415 elements and 66209 nodes were generated with 0.20 mm element length. A pressure of 7.0 MPa was applied on the surface of the combustion chamber generating a compressive load. A pressure of 0.3 MPa was applied on the bolt-hole surface generating a preload. This preload is obtained according to the RBandW recommendations (Shigley and Mischke, 2002). In addition, 0.3 MPa pressure was applied on the gasket surface.

The Multi-point Constraints (MPCs) were applied on the bolt-hole surface for all six degree of freedom. Multi-point constraints (Rahman *et al*., 2005) were used to connect the parts thru the interface nodes. These MPCs were acting as an artificial bolt and nut that connect each parts of the structure. Each MPC's will be connected using a Rigid Body Element (RBE) that indicates the independent and dependent nodes. The configuration of the engine is constrained by bolting between the cylinder head and cylinder block. In the condition with no loading configuration, the RBE element with six-degrees of freedom were assigned to the bolts and the hole on the cylinder block. The independent node was created on the cylinder head hole. Due to the complexity of the geometry and loading on the cylinder block, a three-dimensional FEM was adopted as shown in Fig. 3. The loading and constraints on the cylinder block are also shown in Fig. 4.

Finite Element Analysis (FEA)

Modal analysis: The modal analysis is usually used to determine the natural frequencies and mode shapes of a component. It can also be used as the starting point for the frequency response analysis. The finite element analysis codes usually used several mode extraction methods. The Lanczos mode extraction method is used in

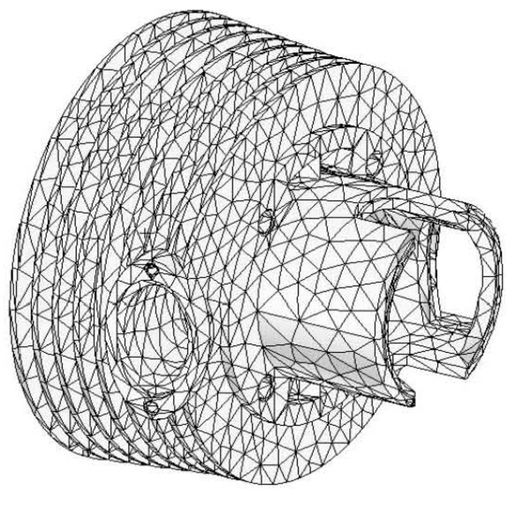

Fig. 3: Three dimensional finite element model

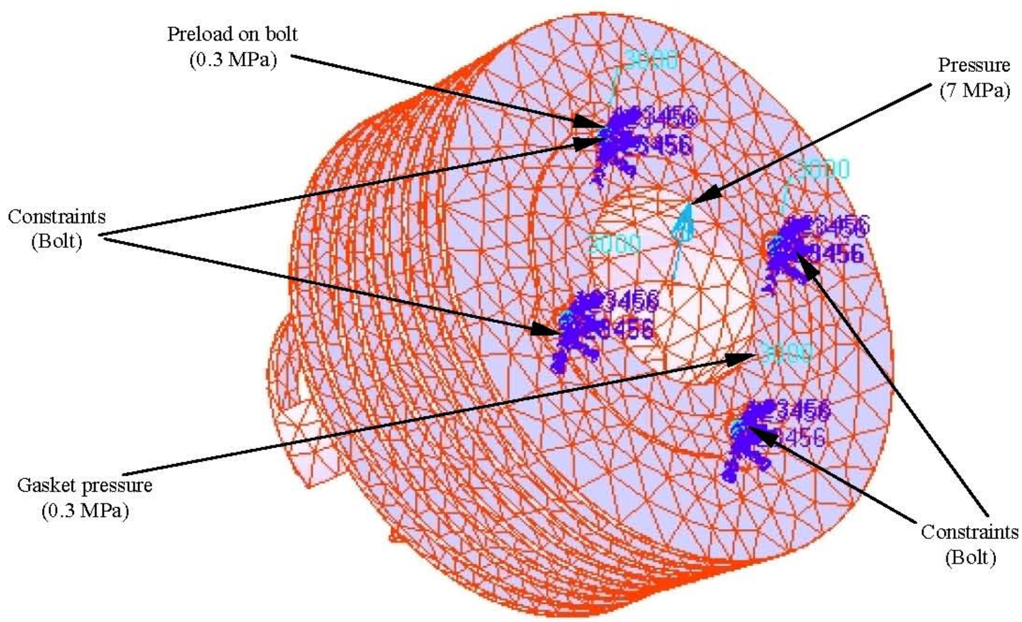

Fig. 4: Loading, constraints and boundary conditions

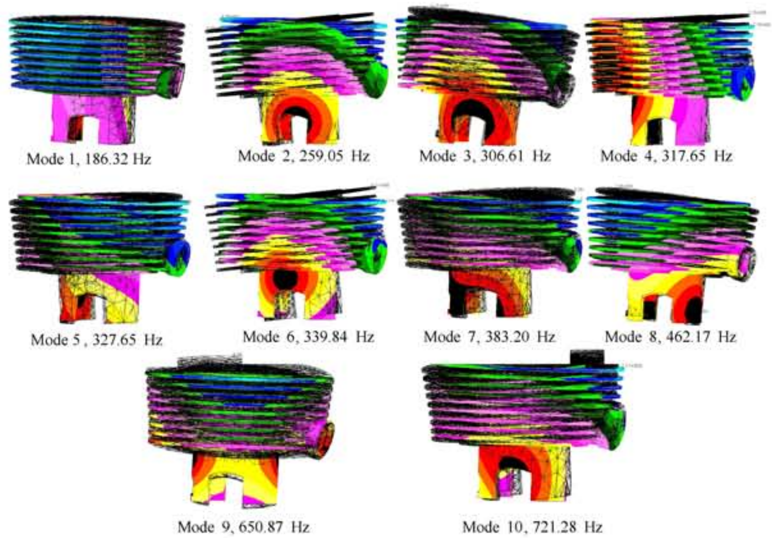

Fig. 5: The mode shapes of the cylinder block

Table 1: The results of the modal analysis

Mode No.	Natural frequency (Hz)
1	186.32
2	259.05
3	306.61
4	317.65
5	327.65
6	339.84
7	382.20
8	462.17
9	650.87
10	721.28

this study due to Lanczos is the recommended method for the medium to large models. In addition to its reliability and efficiency, the Lanczos method supports sparse matrix methods that significantly increase computational speed and reduce the storage space. This method also computes precisely the eigenvalues and eigenvectors. The number of modes to be extracted and used to obtain the cylinder block stress histories, which is the most important factor in this analysis. Using this method to obtain the first 10 modes of the cylinder block, which are presented in Table 1 and the shape of the mode are shown in Fig. 5. It can be seen that the working frequency (50 Hz) is far away from the natural frequency (186.32 Hz) of the first mode.

Frequency response stress analysis: The frequency response analyses were performed using the finite element analysis code. The frequency response analysis used the damping ratio of 5% of critical. The damping ratio is the ratio of the actual damping in the system to the critical damping. Most of the experimental modal reported that the modal damping in terms of non-dimensional critical damping ratio expressed as a percentage (Formenti, 1999; Gade, 2002). In fact, most structures have critical damping values in the range of 0 to 10%, with values of 1-5% as the typical range (User's Guide, 2002). Zero damping ratio indicates that the mode is undamped. Damping ratio of one represents the critically damped mode. The results of the Pseudo-static and the frequency response finite element analysis with zero Hz i.e. the maximum principal stresses distribution of the cylinder block are presented in Fig. 6 and 7, respectively. From the results, the maximum and minimum principal stresses of 380.0 and -77.5 MPa for the Pseudo static analysis and 380.0 and -78.3 MPa for the frequency response analysis for zero Hz were obtained, respectively. These two maximum principal stresses contour plots are almost identical.

The variation of the maximum principal stresses with the frequency is shown in Fig. 8. It can be seen that the maximum principal stress varies with the higher frequencies. This variation is due to the dynamic influences of the first mode shape. It is also observed that the maximum principal stress occurs at a frequency of 32 Hz. The maximum principal stresses of the cylinder block for 32 Hz is presented in Fig. 9. From the results, the maximum and minimum principal stresses of 561.0 and -207.0 MPa were obtained at node 49360 and 47782, respectively.

Vibration fatigue life prediction analysis: The results of the fatigue life contour for the SAETRN loading histories at most critical locations for the Pseudo-static analysis

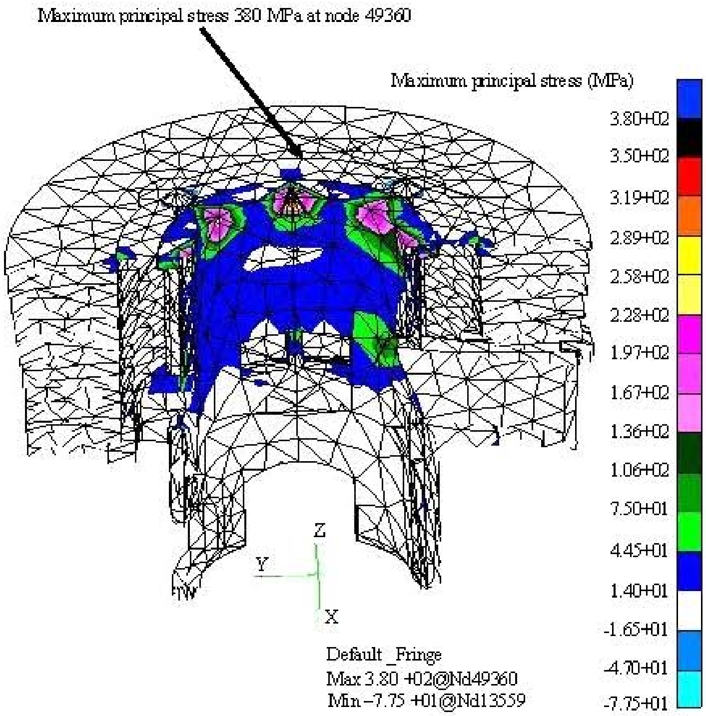

Fig. 6: Maximum principal stresses distribution for the Pseudo-static linear analysis

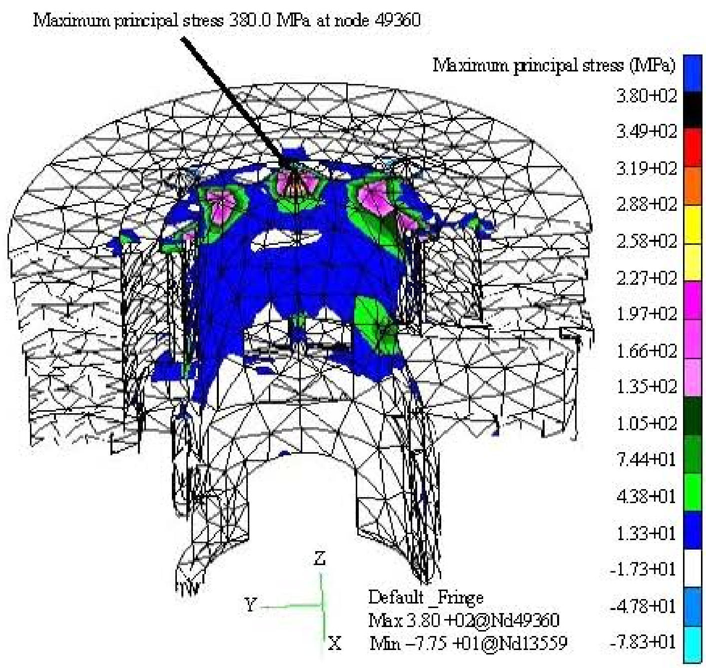

Fig. 7: Maximum principal stresses distribution for the frequency response analysis at 0 Hz

and frequency response approach with 32 Hz are shown in Fig. 10 and 11, respectively. The minimum life prediction for Pseudo-static analysis and frequency response approach with 32 Hz are obtained $10^{7.67}$ and $10^{9.44}$ sec, respectively. It would be expected that the condition of lower stress would correspond to longer life and vice versa. However, the results indicates the opposite because the frequency resolution of the transfer function are selected the small value then the result show the non-conservative prediction. From Fig. 10 and 11, it can be seen that the fatigue life contours are different and most damage was found at frequency of 32 Hz.

For the purpose of validation, the predicted life listed below will be taken to be the definitive answers. Table 2 shows the comparison between the frequency response analysis method and the conventional time domain Pseudo-static approach. It can be seen that there is s good agreement between the Pseudo-static and frequency response analysis approaches. The frequency response using narrow band approach for various loading conditions of the cylinder block made of AA6061-T6 material is considered for the validation. The load time-histories and corresponding power spectral density responses are presented in Fig. 1 and 2. The predicted fatigue life on log-log coordinates using the Pseudo-static and frequency response analysis is presented in Fig. 12. The dotted straight line in Fig. 12 represents the perfect correlation between the pseudo-static and frequency response analysis results i.e., one to one correspondence if the vibration fatigue predicted life exactly equivalent to the pseudo-static predicted life. The 2 straight solid lines represent a three times factor indicating a goodness band. Data points that fall above the solid line represent non-conservative estimates, while points below the solid line represent conservative predictions in comparison to the pseudo-static time domain results. It can be seen that the correlation between the 2 approaches results are well

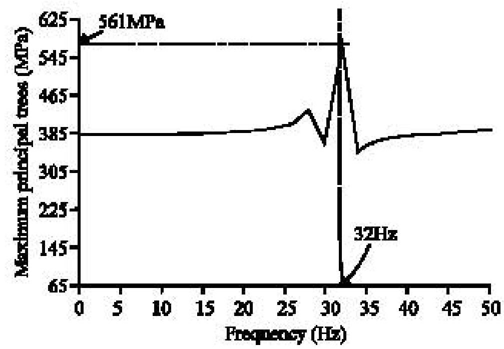

Fig. 8: Maximum principal stresses plotted against frequency

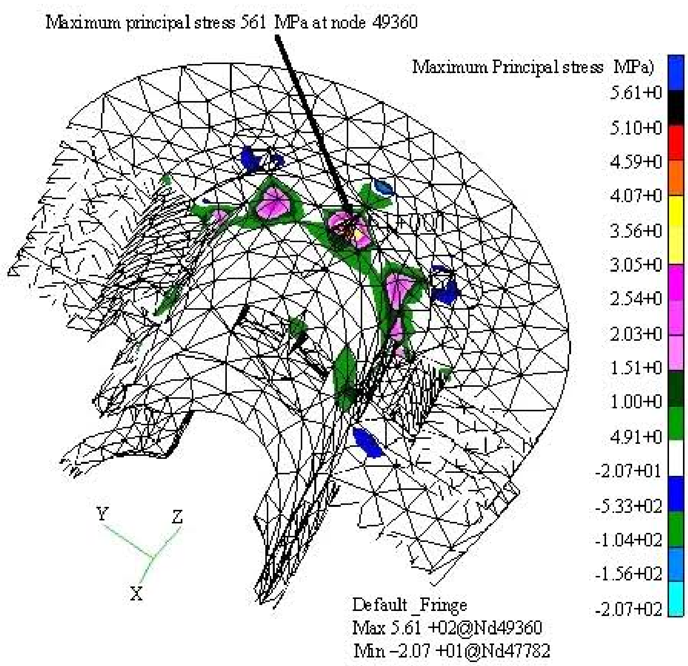

Fig. 9: Maximum principal stresses contour for the frequency response analysis at 32 Hz

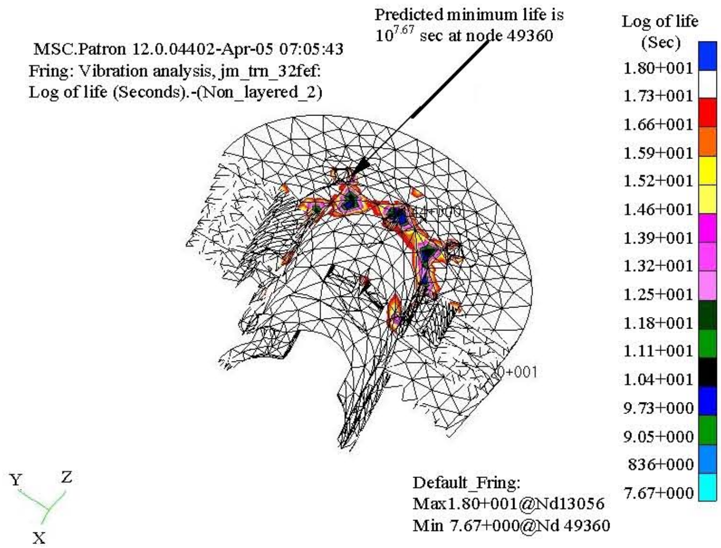

Fig. 10: Predicted fatigue life contours plotted for Pseudo-static linear analysis

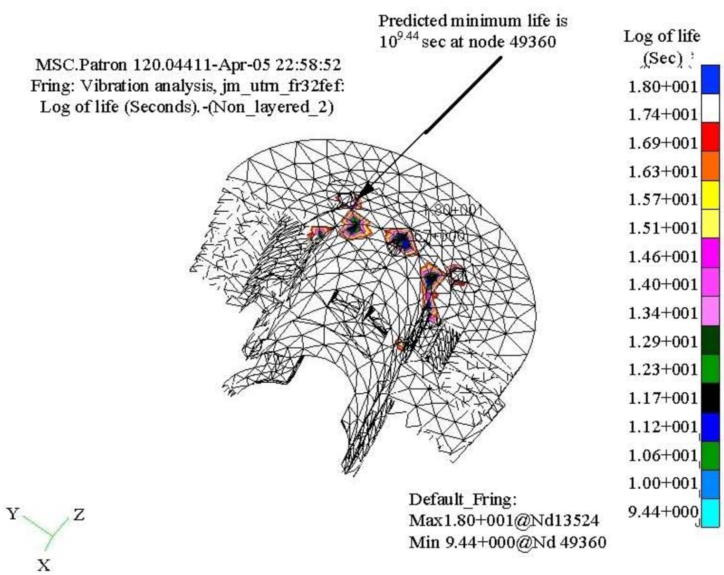

Fig. 11: Predicted fatigue life contours plotted for the frequency response analysis at 32 Hz

Table 2: Predicted life in seconds between two approaches at critical location (node49360)

Loading conditions	Pseudo-static approach	Vibration fatigue
SAETRN	1.14×10^{7}	2.10×10^{7}
SAESUS	6.34×10^{9}	8.74×10^{9}
SAEBKT	7.56×10^{7}	1.06×10^{8}
I-N	2.13×10^{8}	2.30×10^{8}
A-A	1.39×10^{7}	3.93×10^{7}
A-G	6.72×10^{6}	8.23×10^{6}
R-C	1.59×10^{7}	1.83×10^{7}
TRANSP	8.47×10^{8}	2.27×10^{9}

correlated within the expected scatter band. It can be also seen that the frequency response analysis method tended to be non-conservative due to the data points concentrates on the upper band i.e. with the dotted line and the upper dotted line. The full set of the comparison results for the untreated polished cylinder block at critical location (node 49360) is given in Table 3 with various loading conditions. The narrow band solution is considered in this study. It is observed that from Table 3,

Table 3: Predicted life in seconds at critical location (node 49360) for the various loading conditions and materials

	Predicted vibration fatigue life in seconds					
	2024-HV-T6			6061-T6-80-HF		
Loading conditions	No (zero) mean	Goodman	Gerber	No (zero) mean	Goodman	Gerber
SAETRN	2.75×10^{9}	2.53×10^{9}	2.74×10^{9}	2.10×10^{7}	2.02×10^{7}	2.09×10^{7}
SAESUS	1.53×10^{12}	1.46×10^{12}	1.52×10^{12}	8.74×10^{9}	7.97×10^{9}	8.62×10^{9}
SAEBKT	3.36×10^{10}	3.15×10^{10}	3.35×10^{10}	1.06×10^{8}	8.86×10^{7}	1.03×10^{8}
I-N	1.47×10^{10}	1.39×10^{10}	1.46×10^{10}	2.30×10^{8}	2.21×10^{8}	2.28×10^{8}
A-A	3.60×10^{9}	3.37×10^{9}	3.59×10^{9}	3.93×10^{7}	3.79×10^{7}	3.89×10^{7}
A-G	9.14×10^{8}	8.47×10^{8}	9.13×10^{8}	8.23×10^{6}	8.17×10^{6}	8.20×10^{6}
R-C	1.96×10^{9}	1.82×10^{9}	1.95×10^{9}	1.83×10^{7}	1.75×10^{7}	1.81×10^{7}
TRANSP	1.51×10^{11}	1.44×10^{11}	1.50×10^{11}	2.27×10^{9}	2.12×10^{9}	2.24×10^{9}

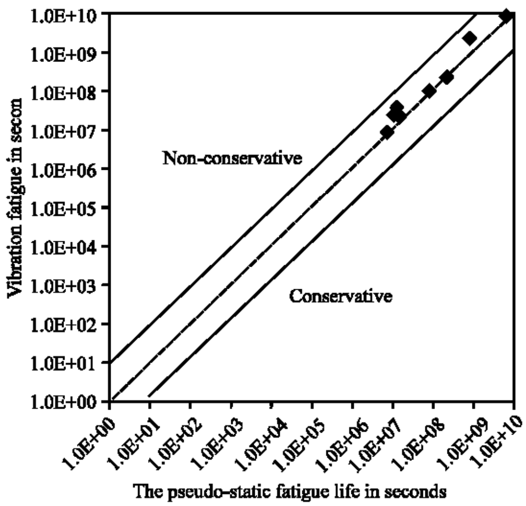

Fig. 12: Comparison between the fatigue life of the Pseudo-static and the frequency response analysis

the SAESUS loading condition gives the longest life for all materials due to the load histories are predominantly compressive mean. The compressive mean loading beneficial for fatigue. In addition, the A-G loading condition gives the lowest lives for all materials.

Effect of the frequency resolution: Frequency resolution of the transfer function is significant to capture the input PSD. The significances of the frequency resolutions of SAETRN loading histories are also shown in Fig. 13 and 14. Two types of Fast Fourier Transform (FFT) buffer size width namely 8192:0.06104 Hz and 16384:0.03052 Hz were used in these figures. The FFT buffer size defines the resolution of the power spectrum. The buffer must be a power of 2 and of course the longer the buffer, the higher the resolution of the spectral lines. To calculate the resolution divide the Nyquist frequency by half the FFT buffer size. If the Nyquist frequency is

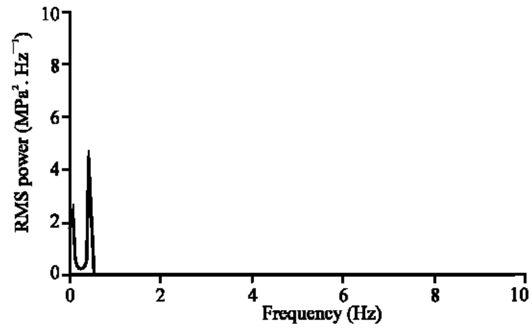

Fig. 13: Power spectral density at FFT buffer size of 8192:0.06104 Hz width

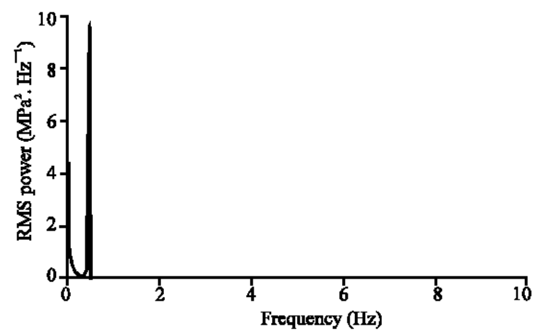

Fig. 14: Power spectral density at FFT buffer size of 16384:0.03052 Hz width

250 Hz and the FFT buffer size selected is 8192, then the spectral lines are 250/(8192/2) = 0.06104 Hz apart. Another use of a smaller buffer size is for short data files as these cannot be adequately analyzed with a big buffer, since there may not be enough data to give a good spectral average. Using a smaller buffer size could give a better spectral average at the expense of spectral line width. The total area under each input PSD curve is determined to be identical. However, the 16384:0.03052 Hz width has twice as many points compares to the 8192:0.06104 Hz.

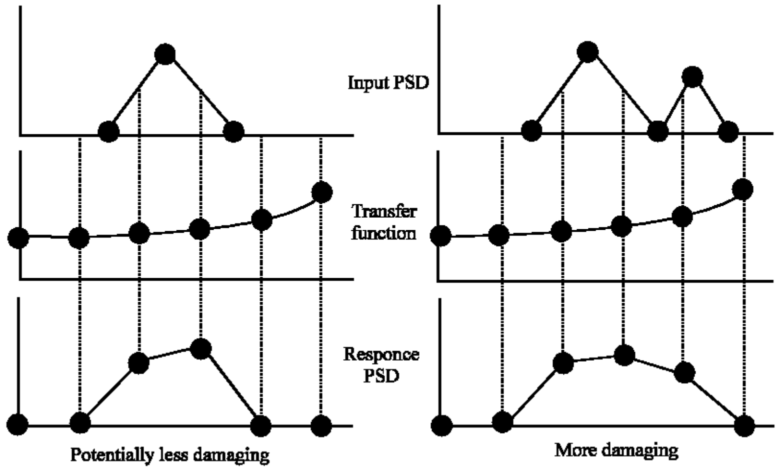

Fig. 15: Effect of the frequency resolution

The frequency resolution of the transfer function in the important areas of the input PSD is the dominant factor. This is shown in Fig. 15 for 2 different cases. Figure 15 shows that utilizing many points in the input PSD can identify the damage more accurately. However, the approach is not suitable for accurately identifying damage with a large spike occurred between two frequencies in the transfer function. For the worst case scenario the technique can entirely miss the damage.

CONCLUSION

The concept of vibration fatigue analysis has been presented, where the random loading and response are categorized using PSD functions. A state of art of the vibration fatigue techniques has been presented. Frequency response fatigue analysis has been applied to a typical cylinder block of the free piston engine. From the results, it can be concluded that Goodman mean stress correction method gives the most conservative prediction for all loading conditions and materials. The results clearly indicate that the AA2024-HV-T6 is a superior material for all the mean stress correction methods. The life predicted from the vibration fatigue analysis is consistently higher except for the bracket loading condition. In addition, the vibration fatigue analysis can improve the understanding of the system behaviors in terms of frequency characteristics of the structures, loads and their couplings.

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