Research Journal of Applied Sciences 2 (8): 900-907, 2007
ISSN: 1815-932X

Entropy Generation in Channel Flow for Non-Newtonian Fluids with Viscous Dissipation Effect

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Abstract: The second-law analysis of a non-Newtonian fluid flowing through a channel made of two heated parallel plates is investigated. The flow is assumed to be steady, laminar and fully-developed. The effect of heat generation by viscous dissipation is included. Velocity, temperature and entropy generation profiles are presented. The effects of the flow behaviour index, the Brinkman number and the group parameter on velocity, temperature and entropy generation number are discussed. The results show that velocity profile depends largely on the flow behaviour index. They are flat near the centreline of the channel for pseudoplastic fluids and linear for dilatant fluids. Temperature profiles are higher for higher flow behaviour index and Brinkman number. The entropy generation number increases with Brinkman number and the group parameter because of the heat generated by viscous dissipation effect. For pseudoplastic fluids, the irreversibility is dominated by heat transfer, whereas, for dilatant fluids, irreversibility due to fluid friction is more dominant.

Key words: Channel, entropy generation, laminar flow, non-newtonian, parallel plates, viscous dissipation

INTRODUCTION

Fluid flow and heat transfer characteristics in falling liquid films along inclined plates at different boundary conditions is one of the fundamental researches in engineering. Studies of simpler systems are useful to understand some important features of complex combinations forming processes in many fields of science and technology. These basic geometries are common in many engineering applications as sole units or as a global entity.

Entropy generation is closely associated with thermodynamic irreversibility, which is encountered in all heat transfer processes. Different sources are responsible for the generation of entropy such as heat transfer across finite temperature gradient, characteristic of convective heat transfer, viscous effect etc. Bejan (1982, 1996) focused on the different reasons behind entropy generation in applied thermal engineering. Bejan (1979) presented a simplified analytical expression for entropy generation rate in a circular duct with imposed heat flux at the wall. This analysis is then extended by calculating the optimum Reynolds number as function of the Prandtl number and the duty parameter. Sahin (1998) introduced the second-law analysis to a viscous fluid in a circular duct with isothermal boundary conditions. In another study, Sahin (1999) presented the effect of variable viscosity on the entropy generation rate for a heated circular duct. A comparative study of the entropy generation rate inside duct of different shapes (circular, triangular, square etc.) and the determination of the optimum duct shape subjected to isothermal boundary condition for laminar flow were carried out by Sahin (1998). Narusawa (2001) gave an analytical and numerical analysis of the second-law for flow and heat transfer inside a rectangular duct. In a more recent study, Mahmud and Fraser (2003) applied the second-law analysis to fundamental convective heat transfer problems. They analysed the second-law characteristics of heat transfer and fluid flow due to forced convection of steady-laminar flow of incompressible fluid inside a channel with circular cross-section and channel made of two parallel plates. Different problems are discussed with their entropy generation profiles and heat transfer irreversibility characteristics. In each case, analytical expression for entropy generation number and Bejan number are derived in dimensionless form using velocity and temperature profiles. In another study, Mahmud and Fraser (2002) investigated analytically the first and second law characteristics of fluid flow and heat transfer inside a channel having two parallel plates with finite gap between them. Fully developed forced convection is considered.

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Fluid is assumed non-Newtonian and followed the power law model. Analytical expressions for dimensionless entropy generation number, irreversibility distribution ratio and Bejan number are determined as a function of dimensionless distance, Peclet number, Eckert number, Prandtl number, dimensionless temperature difference and fluid behaviour index. Spatial distribution of entropy generation number, irreversibility ratio and Bejan number are presented graphically. The same authors (Mahmud and Fraser, 2002) reported, in terms of local and average entropy generation, the inherent irreversibility of fluid flow and heat transfer for non-Newtonian fluids in a pipe and a channel made of two parallel plates. They assumed the flow to be fully developed with a uniform heat flux at the duct wall. They applied the first and the second laws of thermodynamics to develop expressions for dimensionless entropy generation number, irreversibility ratio and Bejan number as function of geometric, fluid and flow parameters.

However, in these analyses concerning non-Newtonian fluids, the influence of viscous dissipation is omitted. The present study aims at analysing the mechanism of entropy generation in a non-Newtonian channel flow taking care of the presence of viscous dissipation effect.

GOVERNING EQUATIONS

The physical configuration is illustrated schematically in Fig. 1. A non-Newtonian fluid flows thought a channel of height 2L made with two parallel plates. The fluid is considered laminar and fully developed. The non-Newtonian fluid considered in this study is the power-law model (Ostwald-de Waele fluid). Such fluids are characterized by the following rheological law:

$$\tau = K\left(\frac{\partial u(y)}{\partial y}\right)^n \quad (1)$$

Where n is the flow behaviour index and k is the consistency of the fluid. A fluid is pseudoplastic fluid when n<1; a Newtonian fluid when n = 1and a dilatant fluid when n<1.

Neglecting the inertia terms in the momentum equation compared with the body force term, the momentum equation reduces to the following form:

$$K\frac{\partial}{\partial y}\left(\frac{\partial u(y)}{\partial y}\right)^n - \frac{\partial P}{\partial x} = 0 \quad (2)$$

The associated boundary conditions are:

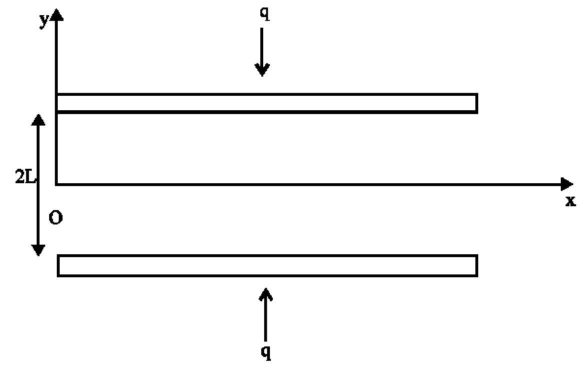

Fig. 1: Schematic diagram of the problem

No-slip condition $u(-L) = 0$ (3a)

Symmetry at the centreline $\frac{\partial u(0)}{\partial y} = 0$ (3b)

The velocity distribution is obtained by integrating Eq. 2 and using the boundary conditions given by Eq. 3. It may be written:

$$u(y) = u_m\left(1 - \left(\frac{y}{L}\right)^{\frac{n+1}{n}}\right) \quad (4)$$

Where u_m is the velocity at the centreline of the channel:

The governing energy equation is:

$$u(y)\frac{\partial T(x,y)}{\partial x} = a\frac{\partial^2 T(x,y)}{\partial y^2} + \frac{K}{\rho C_P}\left(\frac{\partial u(y)}{\partial y}\right)^{n+1} \quad (5)$$

Subject to the following boundary conditions:

Inlet condition $T(0,y) = T_0$ (6a)

Wall heat flux $-\lambda\frac{\partial T(x,-L)}{\partial y} = q$ (6b)

Symmetry at the centreline $\frac{\partial T(x,0)}{\partial y} = 0$ (6c)

The equation of energy can be transformed into a dimensionless form by introducing the following dimensionless variables:

$$X = \frac{ax}{u_m L^2}, Y = \frac{y}{L} \quad (7)$$

$$U(Y)=\frac{u(y)}{u_m},\ \Theta(X,Y)=\frac{T(x,y)-T_0}{\frac{qL}{\lambda}} \tag{8}$$

The transformation yields:

$$U(Y)\frac{\partial\Theta(X,Y)}{\partial X}=\frac{\partial^2\Theta(X,Y)}{\partial Y^2}+Br\left(\frac{\partial U(Y)}{\partial Y}\right)^{n+1} \tag{9}$$

The transformed boundary conditions are:

$$\Theta(0,Y)=0 \tag{10a}$$

$$\frac{\partial\Theta(X,-1)}{\partial Y}=-1 \tag{10b}$$

$$\frac{\partial\Theta(X,0)}{\partial Y}=0 \tag{10c}$$

To get a solution of Eq. 9, a separation of variables solution is assumed in the following form (Arpaci and Larsen, 1984):

$$\Theta(X,Y)=\Theta_1(X)\Theta_2(Y)+\Theta_1(X)+\Theta_2(Y) \tag{11}$$

The first term in the right-hand side of Eq. 11 is significant for decaying initial transition and entrance effect, the second term is significant for axial temperature rise due to accumulated wall heat flux and the third term is significant for transverse temperature variation to wall heat flux into fluid. Neglecting entrance effect and assuming that the system already passed the decaying initial transition. Then the first term at the right-hand side of Eq. 11 will disappear (Mahmud and Fraser, 2002, 2003). Combination of Eq. 9 and 11 leaves two separated ordinary equations connected by a scalar constant α:

$$\frac{\partial\Theta_1(X)}{\partial X}=\alpha \tag{12}$$

$$\frac{\partial^2\Theta_2(Y)}{\partial Y^2}=\alpha\left(1-Y^{\frac{n+1}{n}}\right)-\left(\frac{n+1}{n}\right)^{n+1}BrY^{\frac{n+1}{n}} \tag{13}$$

Integrating Eq. 12 and 13 and applying boundary conditions described in Eq. 10, the expression for the dimensionless temperature is obtained in the following form:

$$\Theta(X,Y)=\alpha X+\frac{\alpha}{2}Y-\frac{n^2\alpha}{(2n+1)(3n+1)}Y^{\frac{3n+1}{n}}-\frac{n^2\left(\frac{n+1}{n}\right)^{\frac{n+1}{n}}Br}{(2n+1)(3n+1)}Y^{\frac{3n+1}{n}}+C \tag{14}$$

Where $$\alpha=\frac{2n+n\left(\frac{n+1}{n}\right)^{\frac{n+1}{n}}Br+1}{n+1}$$

To obtain the constant of integration C, the mean bulk temperature is used:

$$\Theta_b(X)=\frac{1}{A}\int_A\Theta(X,Y)\,dA=\int_0^1\Theta(X,Y)\,dY \tag{15}$$

Since Eq. 10a requires $\Theta_b(0)=0$, the constant of integration is:

$$C=-\frac{\alpha}{6}+\frac{n^3\alpha}{(2n+1)(3n+1)(4n+1)}+\frac{n^3\left(\frac{n+1}{n}\right)^{\frac{n+1}{n}}Br}{(2n+1)(3n+1)(4n+1)} \tag{16}$$

ENTROPY GENERATION RATE

The entropy generation rate according to Mahmud and Fraser (2002) is:

$$S_G=\frac{\lambda}{T_0^2}\left[\left(\frac{\partial T(x,y)}{\partial x}\right)^2+\left(\frac{\partial T(x,y)}{\partial y}\right)^2\right]+\frac{K}{T_0}\left(\frac{\partial u(y)}{\partial y}\right)^{n+1} \tag{17}$$

The entropy generation number may be defined as:

$$N_S=\frac{\lambda T_0^2}{q^2}S_G \tag{18}$$

Using the definitions of dimensionless velocity and temperature, the following expression is obtained for the entropy generation number:

$$N_S = \frac{1}{Pe^2}\left(\frac{\partial\Theta(X,Y)}{\partial X}\right)^2 + \left(\frac{\partial\Theta(X,Y)}{\partial Y}\right)^2 + \frac{Br}{\Omega}\left(\frac{\partial U(Y)}{\partial Y}\right)^{n+1} = N_C + N_Y + N_F \qquad (19)$$

In the above equation, P_e is the Peclet number, which determines the relative importance between convection and diffusion B_r is the Brinkman number, which determines the relative importance between dissipation effects and fluid conduction. Ω is the dimensionless temperature difference.

On the right-hand side of Eq. 25, the first term represents the entropy generation by heat transfer due to axial conduction, the second term accounts for entropy generation due to the transverse direction and the third is the part of the entropy generation due to the fluid friction.

RESULTS AND DISCUSSION

Dimensionless axial velocity profiles are plotted as function of dimensionless transverse distance in Fig. 2 for five different values of the flow behaviour index. For pseudoplastic fluids ($n < 1$), velocity profiles remain flat near the centreline of the channel and this flatness decreases with the increase of the low behaviour index. For Newtonian fluids ($n = 1$), the dimensionless axial velocity shows the usual parabolic shape. For dilatant fluids ($n > 1$), velocity profiles approach a linear shape as the flow behaviour index increases.

Dimensionless temperature profiles are plotted in Fig. 3 for the same range of the flow behaviour index. For present boundary condition, temperature is maximum at the wall where a heat flux is imposed and minimum at centreline of the channel whatever the value of the flow behaviour index is. For a particular transverse distance, the temperature is higher for a higher flow behaviour index. This means that dilatant fluids heat more easily than pseudoplastic fluids.

The axial variations of the dimensionless temperature profiles are plotted in Fig. 4 and 5 for pseudoplastic fluids ($n = 0.2$) and dilatant fluids ($n = 5.0$). In all cases, the temperature increases in the axial direction because of the continuous heating of the wall.

The effect of the Brinkman number on the temperature is illustrated in Fig. 6 and 7 for pseudoplastic fluids ($n = 0.2$) and for dilatant fluids ($n = 5.0$). The temperature increases as the Brinkman number increases either for pseudoplastic fluids or dilatant fluids. As the Brinkman number which determines the relative

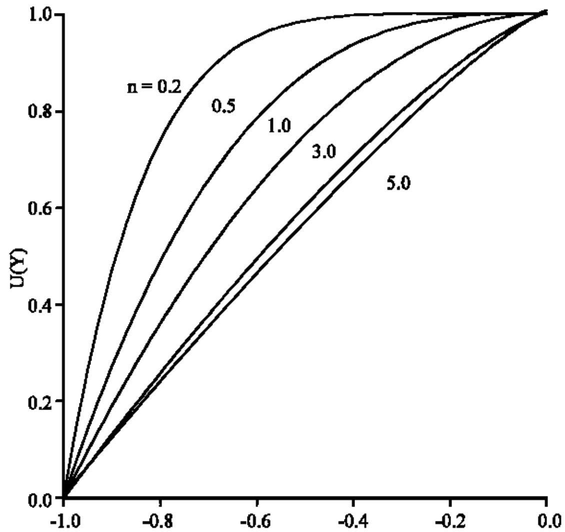

Fig. 2: Dimensionless velocity distribution

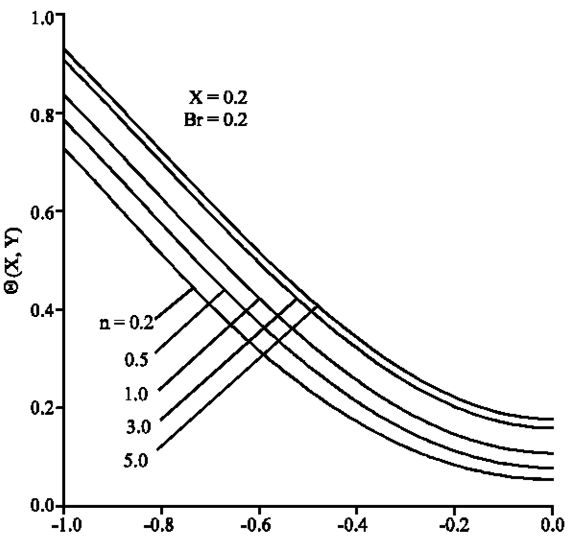

Fig. 3: Dimensionless temperature distribution

importance between viscous dissipation effects and fluid conduction increases, more heat is generated by viscous dissipation effect in the fluid. This generated heat by viscous dissipation effect results in higher temperature profiles.

In Fig. 8 and 9, the entropy generation number is plotted as function of the dimensionless transverse distance for different values of the Brinkman number for pseudoplastic fluids ($n = 0.2$) and dilatant fluids ($n = 5.0$). In all cases, no entropy is generated at the centerline of the channel where both velocity and temperature are maximum (or minimum) which cause zero velocity and temperature gradients leaving no contribution to the entropy generation number (second and third term of Eq. 24.

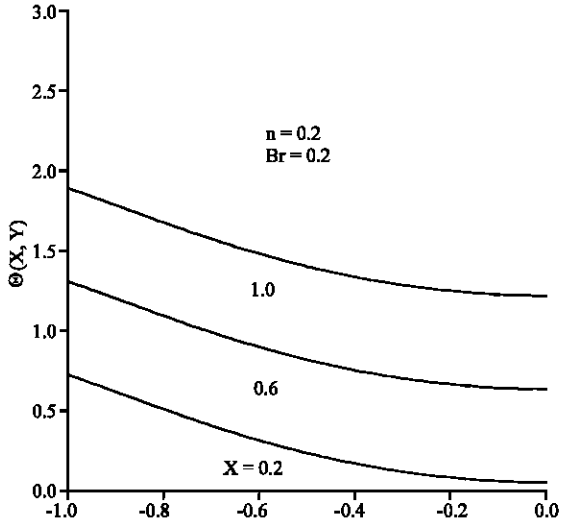

Fig. 4: Axial variations of dimensionless temperature for n = 0.2

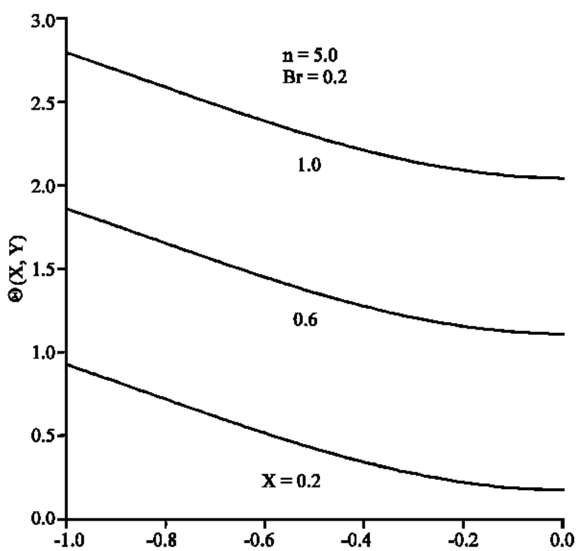

Fig. 5: Axial variations of dimensionless temperature for n = 5.0

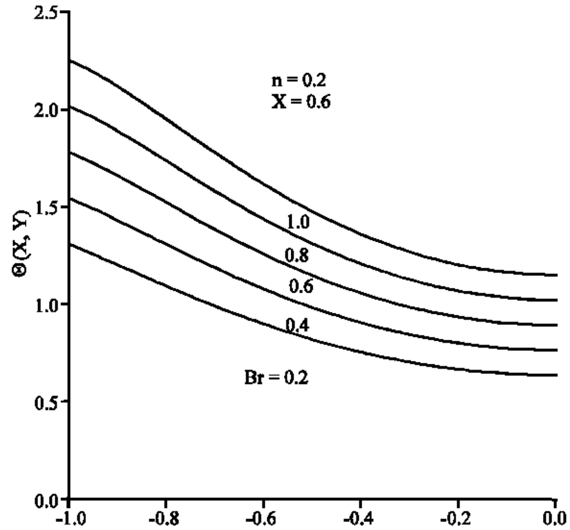

Fig. 6: Axial variations of dimensionless temperature for n = 0.2 at different Brinkman number

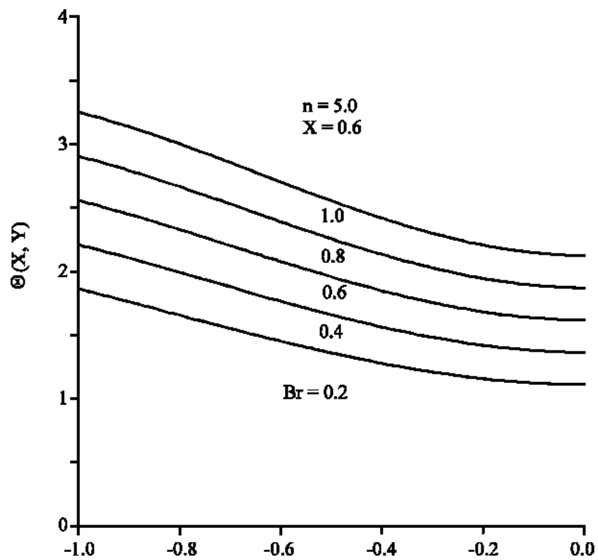

Fig. 7: Axial variations of dimensionless temperature for n = 5.0 at different Brinkman number

For a particular transverse distance, the magnitude of the entropy generation number is higher for higher Brinkman because of the heat generated by viscous dissipation effect. C In the case of pseudoplastic fluids (n = 0.2), the entropy generation number decreases along the transverse distance to reach zero at the centreline of the channel. This can be explained by the fact, that for pseudoplastic fluids, where the velocity profile is flat near the centreline of the channel leaving no contribution of fluid friction on entropy generation. Therefore, the irreversibility is mainly dominated by heat transfer. For dilatant fluids (n = 5.0), for a particular transverse distance, the entropy generation number shows maximum near the wall as the Brinkman number increases. According to Fig. 2, the velocity profile is nearly linear (high velocity gradient), this means that the contribution of fluid friction on entropy generation number augments. Thus, for dilatant fluids, the irreversibility is dominated by fluid friction.

Figure 10 and 11 show the distribution of the entropy generation number as function of the transverse distance at different values of group parameter ranging 0.2-1. No

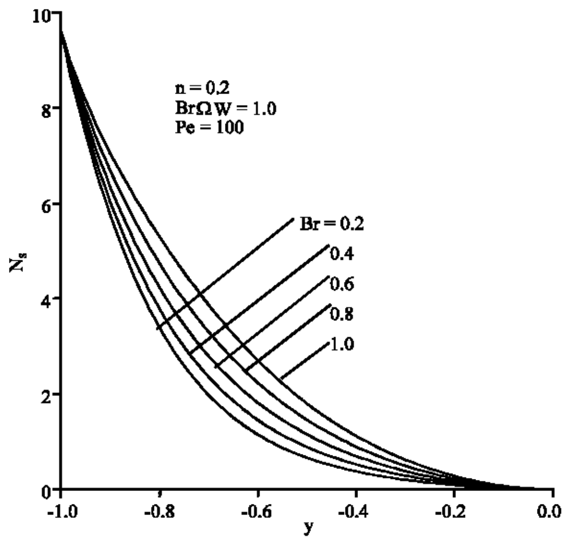

Fig. 8: Entropy generation number at different Brinkman number for n = 0.2

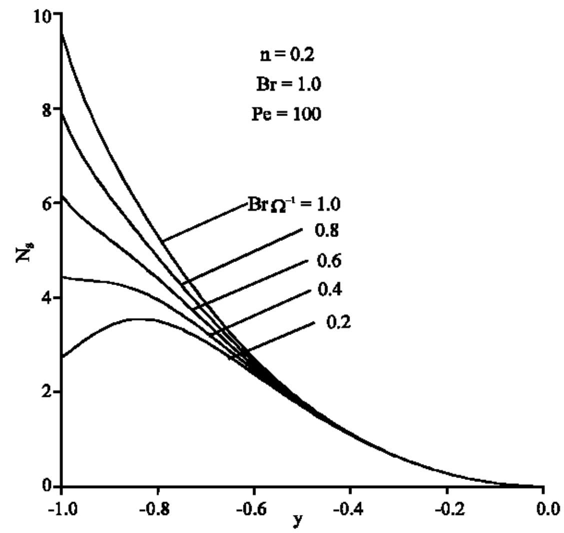

Fig. 10: Entropy generation number at different group parameter for n 0.2

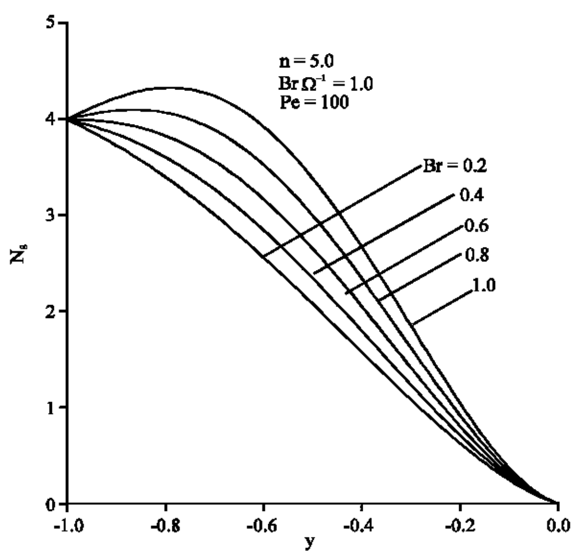

Fig. 9: Entropy generation number at different Brinkman number for n = 5.0

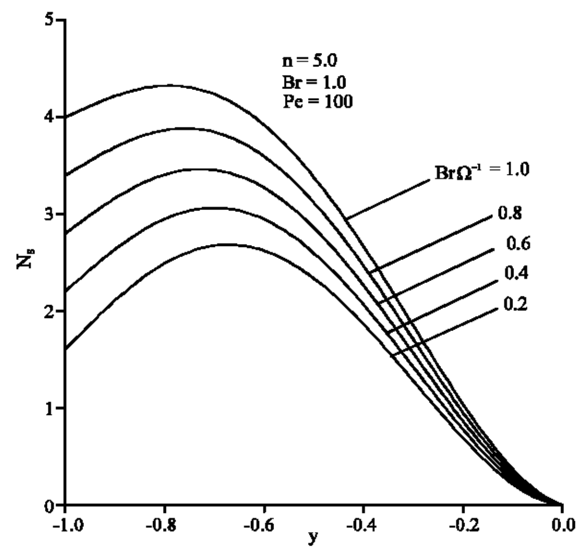

Fig. 11: Entropy generation number at different group parameter for n = 5.0

entropy generates at the centreline of the channel where both velocity and temperature are maximum (or minimum) which cause zero velocity and temperature gradients leaving no contribution to the entropy generation number (second and third term of Eq. (24)) for all values of group parameter. For a particular transverse distance, the entropy generation number is higher for higher group parameter. For pseudoplastic fluids (n = 0.2), the entropy generation number decreases with the transverse distance and do not show maxima except for the case where ($Br\Omega^{-1}$ = 0.2), this means that the irreversibility is dominated by heat transfer and the wall acts as a strong concentrator of irreversibility. For dilatant fluids (n = 0.5), the contribution of fluid friction on entropy generation number is dominant, the entropy generation number show maxima near the wall. Comparing the magnitude of entropy generation number for pseudoplastic and dilatant fluids, the results show that irreversibility is more pronounced for pseudoplastic fluids.

CONCLUSION

The second-law analysis is applied to a laminar non-Newtonian liquid film flowing through a channel. The heat generation by viscous dissipation is included in the analysis. Analytical expressions for velocity and temperature within the film are provided as function of the flow behaviour index and the Brinkman number. The effects of the flow behaviour index, the Brinkman number and the group parameter on entropy generation number are discussed. From the results the following conclusions could be drawn:

- Velocity profile depends largely on the flow behaviour index. They are flat near the centreline of the channel for pseudoplastic fluids and linear for dilatant fluids.
- Temperature profiles shift to higher temperatures with an increasing flow behaviour index.
- Either for pseudoplastic fluids or dilatant fluids, temperature profiles increase with the axial distance because of the continuous heating of the wall.
- As the Brinkman number increases, the temperature profile increases because of the heat generated by viscous dissipation effect.
- The entropy generation number increases with the Brinkman number and the group parameter. This is due to the heat generated by viscous dissipation effect.
- For pseudoplastic fluids, the irreversibility is dominated by heat transfer, whereas, for dilatant fluids, irreversibility due to fluid friction is more dominant.

Nevertheless, it is necessary to carry out further analyses and calculations for different geometries and non-Newtonian fluids other than those obeying the power-law model.

Nomenclature:

α Thermal diffusivity, $=\lambda/\rho C_P$ ………………… $m^2.s^{-1}$
A Area……………………………………………….m^2
B_r Brinkman number, $=Ku^2m/\lambda\Delta T$
C_P Specificheat……………………..………………….$J.kg^{-1}.K^{-1}$
K Consistency of the fluid ……………………… $Pa.s^n$
L Half width of the channel…..……………………… m
n Flow behaviour index
N_C dimensionless entropy generation number, conduction, $=Pe^{-2}(\partial\Theta/\partial X)^2$
N_F Dimensionless entropy generation number, friction, $= Br\Omega^{-1}(\partial U/\partial Y)^{n+1}$
N_S Dimensionless entropy generation number, total, = $(\partial\Theta/\partial Y)^2)$
N_Y Dimensionless entropy generation number, transverse, =Pa
P_e Pressure……………………………………………Pa
P_e Peclet number, $=u_mL/a$
q all heat flux……………………….…...$W.m^{-2}$
S_G Tropygenerationrate……………………..…. $W.m^{-3}.K^{-1}$
T Temperature……………………………………….K
u Axial velocity…………………………...$m.s^{-1}$
U Dimensionless axial velocity, $=U/U_M$
X Axial distance………………………………………m
X Dimensionless axial distance, $=ax/u_mL2$
y Transverse distance…………………...m
Y Dimensionless transverse distance, =y/L

Greek symbols

α Scalar constant
ΔT Reference temperature difference, $=qL/\lambda$
Θ Dimensionless temperature, $=(T - T_0)/\Delta T$
λthermal conductivity …………….……. $W.m^{-1}.K^{-1}$
Ω Dimensionless temperature difference, $=\Delta T/T_0$
ρ Densityofthefluid…………………………….....$kg.m^{-3}$
τShear stress…………………………………..Pa

Subscripts

b Bulk value
m Maximum value
0 Reference value

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