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Constrained Controllability of Infinite Dimensional Systems with Single Point Delay in Control

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Abstract: In this research, we study the semi linear infinite dimensional abstract control system with single constant delay in the control given by x (t) = Ax (t) + F (x (t)) + B_0u (t) + B_1u (t−h) with zero initial conditions x (0) = 0 , u (t) = 0 for t ∈ [-h,0) where the state x (t) takes values in a real Banach space X and the control u (t) is in another real Banach space U.

Key words: Constrained controllability, infinite dimensional systems, single point, control

INTRODUCTION

Originally, a dynamical system was understood as an isolated mechanical system which motion is described by the Newtonian differential equations and which is characterized by a finite set of generalized coordinates and velocities. In this research, we associate any time dependent process with the motion of a dynamical system. This is a qualitative property of dynamical control systems and is of particular importance in control theory. In recent years, various control problems for different types of nonlinear dynamical systems have been considered in many publications and monographs. However, it should be stressed that the most literature in this direction has been mainly concerned with controllability problems for finite dimensional nonlinear dynamical systems with unconstrained controls and without delays (Robinson, 1876) or for linear infinite dimensional dynamical systems with constrained controls and without delays (Klamka, 1996; Nse, 2006; Robinson, 1876).

In this study, we shall consider constrained global relative controllability problems for infinite dimensional stationary semi-linear dynamical systems with single point delay in the control described by ordinary differential state equations. Let us recall that semi-linear dynamical control systems contain linear and pure nonlinear parts in the differential state equation. More precisely we shall formulate and prove sufficient conditions for constrained global relative controllability in a prescribed time interval for semi-linear dynamical systems with single point delay in the control which nonlinear term is continuously differentiable near the origin.

It will be proved that, under suitable assumptions, constrained global relative controllability of linear associated, approximated dynamical systems implies constrained local relative controllability near the origin of the original semi-linear abstract dynamical system.

SYSTEM DESCRIPTION

In this study, we study the semi-linear stationary finite dimensional dynamical systems with single delay in the control defined by the following ordinary differential state equation:

$$x'(t) = Ax(t) + F(x(t)) + B_0 u(t) + B_1 u(t-h) \quad (1)$$

for t ∈ [0, T] , T>h with zero initial conditions

$$x(0) = 0 ,\ u(t) = 0 \text{ for } t \in [-h, 0) \quad (2)$$

Where the state x (t) ∈ R^n = X and the control u (t) ∈ R^m= U. A is n×n dimensional constant matrix. B_j, j = 0,1 are nxm dimensional constant matrices.

Moreover, let us assume that the nonlinear mapping F: X → X is continuously differentiable near the origin and such that F (0) = 0. We assume also that the set of values of controls $U_c \subset U$ is a given closed and convex cone with non-empty interior and vertex at zero. Then the set of admissible controls for the dynamical control system (1) has the following form:

$$U_{ad} = L_\infty([0, T], U_c)$$

Thus for a given admissible control u (t), there exists a unique solution x(t,u) for t ϵ [0,T] of the state Eq. 1 with zero initial conditions (2) described by the integral formula:

$$x(t,u) = \int_0^t S(t-s)(F(x(s,u))) + B_0 u(t) + B_1 u(t-h) ds \quad (3)$$

Where the semi group S (t) = exp (At) is nxn transition matrix for the linear part of the semi-linear control system (1).

For the semi-linear dynamical system with single point delays in the control, it is possible to define many concepts of control. In the sequel, we shall focus our attention on the so-called constrained global relative control ability in the time int6erval [0, T]. In order to do this, we first introduce the notion of the attainable set at time T >0 from zero initial conditions (2) denoted by $K_T(U_c)$ and defined as follows:

$$K_T(U_c) = \{x \in X: x = x(T, u), u(t) \in U_c \text{ for a.e. } t \in [0, T]\} \quad (4)$$

Where x (t, u), t>0 is the unique solution of Eq. 1 with initial conditions (2).

Now using the concept of the attainable set, let us recall the well-known definitions of constrained relative controllability in [0, T] for the dynamical system (1).

PRELIMINARIES

Definition 1: The dynamical system (1) is said to be U_c-exactly locally relatively controllable in [0, T] if the attainable set $K_T(U_c)$ contains neighborhoods of zero in the space X.

Definition 2: The dynamical system (1) is said to be Uc-exactly globally relatively controllable in [0, T] if $K_T(U_c)$ = X.

Lemma 1: Let X, U, U_c and Ω be as described above. Let g: Ω→X be a nonlinear mapping and suppose that on Ω, the nonlinear mapping has derivative Dg which is continuous at zero. Moreover, suppose that g (0) = 0 and assume that linear map Dg (0) maps U_c onto the whole space. Then there exists neighborhoods $N_0 \subset X$ about 0 εX and $M_0 \subset \Omega$ about 0 ε U such that the nonlinear equation x = g (u) has for each x ε N0, at least one solution u ε $M_0 \cap U_c$, where $M_0 \cap U_c$ is a set called conical neighborhood of zero in the space U.

Lemma 2: Let $D_u x$ denotes derivative of x with respect to u . Moreover, if x (t; u) is continuously differentiable with respect to its u argument, we have for u ε: L_∞ ([0, T], U) $D_u x$ (t; u) (v) = z (t, u, v) where the mapping t→z (t, u, v) is the solution of the linear ordinary equation:

$$z(t) = Az(t) + D_x(F(x; u)) z(t) + B_0 u + B_1 u(t-h) \quad (5)$$

with zero initial conditions z (0, u, v) = 0 and u (t)= 0 for t ε [−h, 0).

Proof: (Klamka, 1991).

CONTROLLABILITY CONDITIONS

In this research, we shall study constrained global relative controllability in [0, T] for semi-linear dynamical system (1) using the associated linear dynamical system with multiple point delays in the control given by

$$z \{t) = Cz(t) + B_0 u(t) + B_1 u(t-h) \quad (6)$$

for t ϵ [0,T]} with zero initial conditions z (0) = 0, u (t) = 0 for t ϵ [−h, 0) where

$$C = A + D_x F(0) \quad (7)$$

The main result is the following sufficient condition for constrained local relative controllability of the semi-linear dynamical system (1).

Theorem 1: Suppose that:

- F (0) = 0
- $U_c \subset U$ is a closed and convex cone with vertex at zero.
- The associated linear control system with multiple point delays in the control is U_c−exactly locally relatively controllable in [0, T].

Then, the semi-linear dynamical control system with multiple point delays in the control is Uc−exactly globally relatively controllable in [0, T].

Proof: Let us define for the nonlinear dynamical system (5) a nonlinear map

$$g: L_\infty([0, T], U_c) \to X$$

by g (u) = x(T, u)

Similarly for the associated linear dynamical system (10), we define a linear map

$$H: L_\infty([0, T], U_c) \rightarrow X$$

by Hv = x (T, v)

By the assumption (iii) the linear dynamical system (1) is Uc-globally relatively controllable in [0, T]. Therefore, by definition 2, the linear operator H is surjective, that is it maps cones of admissible controls U_{ad} onto the whole space X. Furthermore, by lemma 2, we have that Dg (0) = H.

Since U_c is a closed and convex cone, then the cone of admissible controls.

$U_{ad} = L_\infty([0, T], U_c)$ is also a closed and convex cone in the function space $L_\infty([0, T], U_c)$. Therefore the nonlinear map g satisfies all the assumption of the generalized open mapping theorem stated in lemma 1. Therefore, the nonlinear map g transforms a conical neighborhood of zero in the cone of admissible controls U_{ad} onto some neighborhood of zero in the whole space X. This is by definition 1 is equivalent to the U_c-local relative controllability in [0, T] of the semi linear dynamical control system (1). Hence our theorem follows.

CONCLUSION

In the present study, sufficient conditions for constrained relative controllability for semi linear infinite dimensional stationary dynamical systems with single point delay in the control have been formulated and proved. In the proof of the main result, generalized open mapping theorem (Robinson, 1876) has been extensively used. The relative controllability conditions given in this papers extend to the case of constrained relative controllability of finite dynamical semi linear stationary control systems and also for unconstrained nonlinear stationary control systems.

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