Coefficient Bounds for Certain Classes of Analytic Functions

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Abstract: In the present study we examine the coefficient bounds for certain classes of analytic functions with negative coefficient.

Key words: Coefficient bounds, certain classes, analytic function

INTRODUCTION

Let $T(n)$ denote the class of function $f(z)$ of the form

$$f(z) = z - \sum_{k=2}^{\infty} a_k z^k \qquad (1)$$

which are analytic in the unit disk $E = \{z : |z| < 1\}$

Also, let P denote the class of functions $p(z)$ of the form

$$p(z) = 1 + p_1 z + p_2 z^2 + \dots \qquad (2)$$

which are analytic in E and satisfy the inequality Re $(p(z)) > \alpha$, $(0 \leq \alpha < 1, z \in E)$ (Srivastava and Owa, 1992; Janteng *et al.*, 2006).

A function $f(z)$ in $T(n)$ is said to be in the class $T_n(\lambda, \alpha)$ if it satisfies the condition

$$Re\left\{\frac{zf'(z)}{\lambda zf'(z) + (1-\lambda)f(z)}\right\} > \alpha \qquad (3)$$

for some $\alpha(0 \leq \alpha < 1)$, $\lambda(0 \leq \lambda < 1)$ and all $z \in E$.

Let $C_n(\lambda, \alpha)$ denote the subclass of $T(n)$ of all functions $f(z)$ satisfying the inequality

$$Re\left\{\frac{f'(z) + zf''(z)}{f'(z) + \lambda zf''(z)}\right\} > \alpha \qquad (4)$$

for some $\alpha(0 \leq \alpha < 1)$, $\lambda(0 \leq \lambda < 1)$ and all $z \in E$ (Darwish, 2006; Altintes and Owa, 1988).

Note here that $f(z) \in C_n(\lambda, \alpha)$ if and only if $zf'(z) \in T_n(\lambda, \alpha)$.

The aim of the present study is to derive the coefficient bounds for the functions $f(z)$ satisfying (3) and (4), respectively.

RESULTS

Theorem 1: Let $f(z) \in T_n(\lambda, \alpha)$. Then

$$|a_2| \leq \frac{2(1-\alpha)}{\lambda - 1} \qquad (5)$$

$$|a_3| \leq \frac{1-\alpha}{\lambda - 1} + \frac{2(\lambda+1)(1-\alpha)(\alpha-1)}{(\lambda-1)^2}$$

$$|a_4| \leq \frac{2(4\lambda+3)(\alpha-1)(1-\alpha)}{3(\lambda-1)^2} +$$
$$\frac{4(2\lambda+1)(\lambda+1)(1-\alpha)(\alpha-1)^2}{3(\lambda-1)^3} + \frac{2(1-\alpha)}{3(\lambda-1)}$$

and

$$|a_2 a_4 - a_3^2| \leq \frac{4(1-\alpha)^2}{3(\lambda-1)^2}\left[1 + \frac{(\alpha-1)(4\lambda+3)}{2(\lambda-1)}\right] -$$
$$\frac{2(1-\alpha)^2}{(\lambda-1)^2}\left[1 + \frac{4(\lambda+2)(\lambda+1)(\alpha-1)^2}{(\lambda-1)^2}\right]$$

for some $\alpha(0 \leq \alpha < 1)$, $\lambda(0 \leq \lambda < 1)$.

Proof: Let us define $p(z)$ by

$$\frac{zf'(z)}{\lambda zf'(z) + (1-\lambda)f(z)} = \alpha + (1-\alpha)p(z) \qquad (6)$$

On comparing the coefficient in (6) the results follow immediately.

Theorem 2: Let $f(z) \in C_n(\lambda, \alpha)$. Then,

$$|a_2| \leq \frac{1-\alpha}{\lambda - 1} \qquad (7)$$

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$$|a_3| \le \frac{1-\alpha}{3(\lambda-1)} + \frac{2(\lambda+1)(\alpha-1)(1-\alpha)}{3(\lambda-1)^2}$$

$$|a_4| \le \frac{(4\lambda+3)(\alpha-1)(1-\alpha)}{6(\lambda-1)^2} +$$

$$\frac{(2\lambda+1)(\lambda+1)(1-\alpha)(\alpha-1)^2}{3(\lambda-1)^3} + \frac{1-\alpha}{6(\lambda-1)}$$

$$|a_2a_4 - a_3^2| \le \frac{(1-\alpha)}{18(\lambda-1)^2} + \frac{(\alpha-1)(1-\alpha)^2}{3(\lambda-1)^3}$$

$$\left[\frac{4\lambda+3}{2} + \frac{(2\lambda+1)(\lambda+1)(\alpha-1)}{3(\lambda-1)} \right]$$

for some $\alpha(0 \le \alpha < 1)$, $\lambda(0 \le \lambda < 1)$ and $z \in E$

Proof: Let us define p(z) by

$$\frac{f'(z)+zf''(z)}{f'(z)+\lambda zf''(z)} = \alpha + (1-\alpha)p(z) \qquad (8)$$

On comparing the coefficient in (8) the result follows immediately.

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