

## Performances of Some Estimators of Linear Model with Autocorrelated Error Terms in the Presence of Multicollinearity

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**Abstract:** Assumptions in classical linear regression model that regressors are assumed to be independent and non-stochastic in repeated sampling are often violated by economist and other social scientists. This is because their regressors are generated by stochastic process beyond their control. Consequently, in this study we examine the performances of the Ordinary Least Square (OLS) and four Generalized Least Square (GLS) estimators of linear model with autocorrelated error terms when normally distributed stochastic regressors exhibit multicollinearity. These estimators are compared by examining their finite sampling properties at various levels of autocorrelation and non-validity of the multicollinearity assumption through Monte-Carlo studies. Results show that the Maximum Likelihood (ML) and the Hildreth and LU (HILU) estimators are generally preferable in estimating all the parameters of the model at all the levels of autocorrelation and multicollinearity. Consequently, when the these two forms of correlations can not be ascertained in a data set, it is more preferable to use either the ML or HILU estimator to estimate all parameters of the model.

**Key words:** Stochastic regressors, linear model with autocorrelated error, OLS estimator, feasible GLS estimators, multicollinearity

### INTRODUCTION

Assumption that regressors are assumed fixed (non-stochastic) in repeated sampling under the classical linear regression model is not always satisfied in business, economic and social sciences because their regressors are often generated by a stochastic process beyond their control. Situations and instances where this assumption is violated and its consequences on the Ordinary Least Square (OLS) estimator when used to estimate the model parameters have been discussed by Neter and Wasserman (1974), Fomby *et al.* (1984), Maddala (2002). Graybill (1961), Sampson (1974), Fomby *et al.* (1984) emphasized that if regressors are stochastic and independent of the error terms; the OLS estimator is not only unbiased but has minimum variance even though it is not Best Linear Unbiased Estimator (BLUE). They also pointed out that the traditional hypothesis testing is valid if the error terms are further assumed normal but modification would be required in the area of confidence interval calculated for each sample and the power of the test. The assumption of independence of regressors is also not likely to be achieved because explanatory variables like gross national product and income are most correlated over time. When regressors are dependent (correlated), that is there is multicollinearity, the OLS estimates are still unbiased as long as multicollinearity is not perfect (Johnston, 1984). However when multi-

collinearity is high, only imprecise estimate may be available about the individual true regression coefficients which are often statistically insignificant because of its large standard errors (Charterjee *et al.*, 2000).

When autocorrelation of first-order exist in the OLS residuals, the OLS estimator is unbiased but inefficient. The predicted values are also inefficient and the sampling variances of the autocorrelated error terms underestimated causing the t and F tests to be invalid (Johnston, 1984; Fomby *et al.*, 1984; Charterjee *et al.*, 2000; Maddala, 2002). To compensate for the lost of efficiency, Cochrane and Orcutt (1949) were first to observe that the presence of autocorrelation ( $\rho$ ) in linear model requires some modification of the OLS method. They suggested a transformation that uses the matrix

$$P = \begin{bmatrix} -\rho & 1 & 0 & \dots & 0 & 0 \\ 0 & -\rho & 1 & \dots & 0 & 0 \\ 0 & 0 & -\rho & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \dots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot & \cdot \\ 0 & 0 & 0 & \dots & -\rho & 1 \end{bmatrix}_{(n-1) \times n}$$

which ignores the first observation of the error terms. Paris and Winsten (1954) showed that the appropriate transformation required for the transformation is

$$Q = \begin{bmatrix} (1-\rho^2)^{\frac{1}{2}} & 0 & 0 & \dots & 0 & 0 \\ -\rho & 1 & 0 & \dots & 0 & 0 \\ 0 & -\rho & 1 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \dots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot & \cdot \\ 0 & 0 & 0 & \dots & -\rho & 1 \end{bmatrix}_{n \times n}$$

which retains the first observation. The difference in the usage of Pand  $Q$  can be negligible when  $n$  is large, but in small sample investigation such as in this study, the difference may be major. The process involves the autoregressive form of the error terms to be estimated and the estimate of the autocorrelation used to obtain estimate for the regression coefficients of the linear model via the Generalized Least Square (GLS) estimator given by Aitken (1935) as

$$\hat{\beta} = (X' \Omega^{-1} X)^{-1} X' \Omega^{-1} Y \tag{1}$$

with the variance-covariance matrix

$$V(\hat{\beta}) = \sigma^2 (X' \Omega^{-1} X)^{-1} \tag{2}$$

where

$$\Omega^{-1} = \frac{1}{1-\rho^2} \begin{bmatrix} 1 & -\rho & 0 & \dots & 0 & 0 \\ -\rho & 1+\rho^2 & -\rho & \dots & 0 & 0 \\ 0 & -\rho & 1+\rho^2 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \dots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \dots & 1+\rho^2 & -\rho \\ 0 & 0 & 0 & \dots & -\rho & 1 \\ 0 & 0 & 0 & \dots & -\rho & 1 \end{bmatrix}_{n \times n}$$

However  $\rho$  and hence  $\Omega$  is not always known, it is often estimated by  $\hat{\Omega}$  to have what is known as Feasible GLS estimator. Many consistent estimates of  $\hat{\Omega}$  can be obtained (Fomby *et al.*, 1984). Among those Feasible GLS estimators available in literature are the Cochrane and Orcutt estimator (1949), Hildreth and Lu estimator (1960), Paris and Winsten estimator (1954), Thornton estimator (1982), Durbin estimator (1960), Theil's estimator (1971), the Maximum Likelihood estimator and the Maximum Likelihood Grid estimator (Beach and Mackinnon, 1978). Some of these estimators have now been incorporated into White's SHAZAM program (White, 1978) and the new version of the time series processor (TSP, 2005).

However, all these estimators are known to be asymptotically equivalent but the question on which is to be preferred in small samples is another matter (Fomby *et al.*, 1984). Chipman (1979), Kramer (1980), Kleiber (2001) and many others have observed that the efficiency of these estimators depends on the structure of the regressors that are used. Rao and Griliches (1969) did one of the earliest Monte-Carlo investigation on this study. He examined the performances of some of these estimators with first-order autoregressive stochastic regressor. Their results show that the OLS estimator is only more efficient than any of the GLS estimators considered when; and that the performances of the GLS estimators are not far apart. Park and Mitchell (1980) observed that when regressors are trended, the estimator that retains the first observation (Paris-Winsten) is more efficient than the one that does not (Cochrane-Orcutt) and that the latter is even less efficient than the OLS estimator.

More recently, Nwabuwze (2005a) examined the performances the OLS estimator and four feasible GLS estimators namely: Cochrane-Orcutt (CORC), Hidreth and Lu (HILU), Maximum Likelihood (ML) and Maximum Likelihood Grid (MLGD) estimators with fixed regressor that is normally distributed. The estimators' performances in order of preference are HILU, ML, MLGD, OLS and COCR when the sample size is small and the autocorrelation value is large. Nwabueze (2005b) also examined the performances of these estimators with exponential independent variable. His result, among other things, shows that the OLS estimator compares favorably with the Maximum Likelihood (ML) and Maximum Likelihood Grid (MLGD) estimators for small value of  $\rho$  but it appears to be superior to Cochrane-Orcutt (CORC) and the Hidreth and Lu (HILU) especially when  $\rho$  is large. Some other recent works that are done with different specification of regressors include that of Iyaniwura and Nwabuwze (2005 a,b, c) and Iyaiwura and Olaomi (2006).

Consequently, this study examines the performances of some of these estimators when normally distributed stochastic regressors exhibit different degree of multicollinearity with a view of determining the estimator(s) that can be generally preferable in estimating all the model parameters at all the levels of the two types of correlation.

### MATERIALS AND METHODS

Consider the GLS model with stochastic regressors and AR (1) of the form

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + u_t \tag{3}$$

where,  $u_t = \rho_1 u_{t-1} + \varepsilon_t$ ,  $t = 1, 2, \dots, n$ ,  $\varepsilon_t \sim N(0, \sigma^2 I_n)$  and  $x_2$  is said to have  $\rho_2$  correlation with  $x_1$ ,  $|\rho| < 1$   $i = 1, 2$ .

Now, suppose  $W_i \sim N(\mu_i, \sigma_i^2)$   $i = 1, 2$ . If these variables are correlated, then and can be generated with the equations

$$\begin{aligned} W_1 &= \mu_1 + \sigma_1 Z_1 \\ W_2 &= \mu_2 + \rho \sigma_2 Z_1 + \sigma_2 Z_2 \sqrt{1 - \rho^2} \end{aligned} \quad (4)$$

where  $Z_i \sim N(0,1)$   $i = 1, 2$  and  $|\rho| < 1$  is the value of the correlation between the two variables (Ayinde and Oyejola, 2007).

Parameter estimations of model (3) can be done using the OLS and the feasible GLS estimators. Thus, the performances of the OLS estimator and the following feasible GLS estimators are studied: CORC, HILU, ML and the MLGD estimators. The CORC and HILU estimators do not retain the first observation while the ML and MLGD estimators do.

Monte Carlo experiments were performed for  $n = 20$ , a small sample size representative of many time series study (Park and Mitchell, 1980) with two replication (R) levels (R = 80, 120) and nine various degree of autocorrelation and multicollinearity ( $\rho_1$  and  $\rho_2 = -0.99, -0.75, -0.5, \dots, 0.99$ , respectively) utilizing Eq. 3 and 4. At a particular choice of  $\rho_1$ ,  $\rho_2$  and R (a scenario), each replication was first obtained by generating  $\varepsilon_t \sim N(0, 1)$  and hence,  $u_t$ . Assuming the process start from infinite past and continue to operate, the initial value of U (i.e.,  $u_1$ ) was thus drawn from a normal population with mean zero and variance.  $\frac{1}{1 - \rho_1^2}$  Hence

$$u_1 = \frac{\varepsilon_1}{\sqrt{1 - \rho_1^2}} \quad (5)$$

$$u_t = \rho_1 u_{t-1} + \varepsilon_t, t = 2, 3, \dots, 20 \quad (6)$$

Furthermore  $x_{1t} \sim N(0,1)$  and  $x_{2t} \sim N(0,1)$  having  $\rho_2$  correlation were generated using Eq. 4. Hence, we have

$$x_{1t} = z_{2t} \quad (7)$$

$$x_{2t} = \rho_2 x_{1t} + z_{3t} \sqrt{1 - \rho_2^2} \quad (8)$$

The values of in Eq. 3 were also calculated by setting the true regression coefficients as  $\beta_0 = \beta_1 = \beta_2 = 1$ . This process continued until all replications in this scenario were obtained. Another scenario then started until all the scenarios were completed.

Evaluation and comparison of estimators were examined using the finite sampling properties of estimators which include Bias (B), Absolute Bias (AB) and Variance (Var) and the more importantly the Mean Squared Error (MSE) criteria. Mathematically, for any estimator  $\hat{\beta}_1$  of  $\beta_2$  of model (3)

$$\hat{\beta}_1 = \frac{1}{R} \sum_{j=1}^R \hat{\beta}_{1j} \quad (9)$$

$$B(\hat{\beta}_1) = \frac{1}{R} \sum_{j=1}^R (\hat{\beta}_{1j} - \beta_1) = \hat{\beta}_1 - \beta_1 \quad (10)$$

$$AB(\hat{\beta}_1) = \frac{1}{R} \sum_{j=1}^R |\hat{\beta}_{1j} - \beta_1| \quad (11)$$

$$\text{Var}(\hat{\beta}_1) = \frac{1}{R} \sum_{j=1}^R (\hat{\beta}_{1j} - \hat{\beta}_1)^2 \quad (12)$$

$$\text{MSE}(\hat{\beta}_1) = \frac{1}{R} \sum_{j=1}^R (\hat{\beta}_{1j} - \beta_1)^2 = \text{Var}(\hat{\beta}_1) + [B(\hat{\beta}_1)]^2 \quad (13)$$

for  $i = 0, 1, 2$  and  $j = 1, 2, \dots, R$ .

For each of the estimators, a computer program was written using TSP software to estimate all the model parameters and to evaluate the criteria. The effect of autocorrelation and multicollinearity on the performances of the methods (estimators) is examined via the Analysis of Variance (ANOVA) of the criteria of each of the model parameters. Consequently, the highest order significant interaction effect which has method as a factor is discussed. However, since at least one of the estimators (CORC) is biased in small samples (Rao and Griliches, 1969) and that the mean squared error is known to replace the absolute bias (Kruthkoff, 1970) and also comprises variance and bias; therefore a further test using the LSD technique on the highest order significant interaction effect which has method as a factor was performed on the basis of the mean squared error criterion. The LSD test of the estimated marginal mean was done at a particular combination of levels of the two correlations. At a particular combination of levels, estimators were preferred if their estimated marginal means are not significantly different from the most preferred one. An estimator is most preferred if its estimated marginal mean is the smallest. Estimators that are preferred at all the levels of various combinations of autocorrelation and multicollinearity in estimating all the model parameters are simply said to be generally preferable.

Table 1: Summary of the ANOVA TABLE showing the sum of squares of the model parameters based on the criteria.

Parameter	Source	d.f	Type III Sum of squares			
			Bias	Absolute bias	Variance	Mean squared error
$\beta_0$	R2	8	.000	.000	.000	.000
	R1	8	2.868***	2322.860***	184234.265***	184551.437***
	M	4	4.372E-02***	.378***	14.365***	14.250***
	R1*R2	64	.000	.000	.000	.000
	R2*M	32	.000	.000	.000	.000
	R1*M	32	.168***	2.052***	94.450***	93.423***
	R1*R2*M	256	.000	.000	.000	.000
	Error	405	0.213	0.949	41.856	42.188
	Total	809	3.293	2326.238	184384.9	184701.3
$\beta_1$	R2	8	21.881***	224.649***	7780.196***	8251.934***
	R1	8	8.72E-02	17.982***	7764.727***	8110.818***
	M	4	2.57E-02	23.876***	4743.151***	4993.031***
	R1*R2	64	4.896***	17.681***	21820.640***	22846.583***
	R2*M	32	3.822***	22.867***	13268.350***	14004.106***
	R1*M	32	0.172	91.899***	31977.982***	33387.006***
	R1*R2*M	256	7.428***	91.792***	89928.208***	94089.641***
	Error	405	3.998	0.284	1911.193	1334.4
	Total	809	42.311	491.029	179194.4	187017.5
$\beta_2$	R2	8	11.650***	223.493***	7777.521***	8250.190***
	R1	8	2.722***	18.268***	8038.759***	8418.937***
	M	4	2.124***	23.783***	4890.440***	5162.846***
	R1*R2	64	2.607***	17.497***	21786.194***	22816.533***
	R2*M	32	2.035***	22.779***	13253.799***	13992.058***
	R1*M	32	4.129***	94.407***	33129.855***	34672.740***
	R1*R2*M	256	3.955***	90.421***	89786.688***	93968.132***
	Error	405	4.259	0.268	1964.408	1351.296
	Total	809	33.482	490.916	180627.7	188632.7

\* ⇒ Computed F value is significant at  $\alpha = 0.05$ . \*\* Computed F value is significant at  $\alpha = 0.01$ . \*\*\* ⇒ Computed F value is significant at  $\alpha = 0.001$ .  $\rho_1$  ⇒ Autocorrelation levels.  $\rho_2$  ⇒ Multicollinearity levels. M ⇒ Methods

Table 2: Summary of the ANOVA TABLE of the reduced model showing the sum of squares of parameters based on the criteria

Parameter	Source	d.f	Type III Sum of squares			
			Bias	Absolute bias	Variance	Mean squared error
$\beta_0$	R1	8	.319***	258.096***	20470.474***	20505.716***
	M	4	4.858E-03	4.201E-02**	1.596**	1.583*
	R1*M	32	1.862E-02	.228***	10.494***	10.380***
	Error	45	2.367E-02	.105	4.651	4.688
	Total	89	.366	258.471	20487.216	20522.367

## RESULTS AND DISCUSSION

The summary of our findings on the performances of the estimators based on the criteria for each model parameters in the two replication groups is given in Table 1.

From Table 1 it is observed that in estimating,  $\beta_0$  the main effect of multicollinearity and any of its interaction effects are completely insignificant. Thus, multicollinearity does not have effect in estimating  $\beta_0$  in the regression model. The analysis of variance of the reduced model is therefore run in the absence of multicollinearity (i.e., run when  $\rho_2 = 0$ ). The resulting analysis of variance is given in Table 2.

From Table 2 the interaction effect of autocorrelation by method is significant except under bias criterion. Thus, the performances of the estimators are affected by autocorrelation in all the criteria except bias. The

estimated marginal means on the basis of the mean squared error is shown in appendix. From appendix, it is observed that the estimated marginal mean of all the estimators increase as  $\rho_1$  increases. However, the estimated marginal means of the OLS estimator decreases as  $\rho_1$  increases in the interval  $-1 < \rho_1 \leq -0.5$ . Furthermore, as at each level of autocorrelation the most preferred estimate is not significantly different from others except when  $\rho_1 \rightarrow 1$  where the ML and HILU estimators exhibit significant difference from others. Thus, the ML and HILU estimators are preferred in all the levels of autocorrelation.

In estimating  $\beta_1$  and  $\beta_2$ , the highest order interaction effect of autocorrelation by multicollinearity by method is significant in all the criteria. Thus, the performances of the estimators are affected by the joint contribution of autocorrelation and multicollinearity. Their estimated marginal means based on the mean squared error of the estimated parameters are also given in appendix. From

appendix, it is observed that the performances of the estimators in estimating  $\beta_2$  are symmetric over the levels of multicollinearity. At each level of multicollinearity, the estimated marginal means of the GLS estimators decrease while that of the OLS estimator increases as autocorrelation increases in its absolute form ( $|\rho_1|$ ). Also at each level of autocorrelation, the estimated marginal means of all the estimators increase as multicollinearity increases in its absolute form ( $|\rho_2|$ ). Furthermore from the appendix, it is observed that all the estimators do not exhibit significant difference from the most preferred one except  $|\rho_1| \rightarrow 1$  and  $|\rho_2| < 1$  and when autocorrelation is negatively high ( $\rho_1 = -0.75$ ) and  $|\rho_1| \rightarrow 1$ . In these

exceptions, only the OLS estimator is significantly different from the most preferred one. Therefore, the GLS estimators are preferred in all the levels of autocorrelation and multicollinearity. However, the OLS estimator competes favourably with the GLS estimators when  $|\rho_1| \leq 0.5$  and  $|\rho_2| \leq 0.5$  and in estimating  $\beta_1$  and  $\beta_2$ .

Summarily, the two GLS methods namely, ML and HILU can therefore be generally preferable to estimate all the parameters of the model at all the levels of autocorrelation and multicollinearity. Consequently, when these two forms of correlations can not be ascertained in a data set, it is more preferable to use either the ML or HILU estimator to estimate all parameters of the model.

Appendix 1: Summary of the results on the LSD test of the estimated marginal means based on mean squared error of the estimated parameter with significant interaction effects

$\rho_1$	M	$\beta_0$	$\rho_2 = -0.99$			$\rho_2 = -0.75$	
			$\beta_1$	$\beta_2$	$\beta_1$	$\beta_0$	
-.99	OLS	0.467	221.834+	216.510+	9.899+	9.848+	
	COCR	1.50E-02	2.23	2.285	9.88E-02	0.104	
	HILU	1.47E-02	2.181	2.236	9.71E-02	0.102	
	ML	1.49E-02	2.174	2.226	9.77E-02	0.101	
	MLGD	1.51E-02	2.135	2.17	0.1	9.87E-02	
-.75	OLS	3.89E-02	8.279+	8.560+	0.33	0.389	
	COCR	1.96E-02	2.699	2.813	0.113	0.128	
	HILU	1.97E-02	2.683	2.8	0.112	0.127	
	ML	1.93E-02	2.616	2.678	0.117	0.122	
	MLGD	1.93E-02	2.599	2.66	0.116	0.121	
-.5	OLS	3.35E-02	4.592	4.774	0.18	0.217	
	COCR	2.67E-02	3.1	3.238	0.129	0.147	
	HILU	2.66E-02	3.115	3.252	0.13	0.148	
	ML	2.56E-02	3.028	3.119	0.13	0.142	
	MLGD	2.55E-02	3.026	3.118	0.129	0.142	
-.25	OLS	3.83E-02	3.379	3.489	0.133	0.159	
	COCR	3.86E-02	3.486	3.621	0.145	0.165	
	HILU	3.85E-02	3.495	3.622	0.146	0.165	
	ML	3.61E-02	3.325	3.449	0.135	0.157	
	MLGD	3.59E-02	3.329	3.449	0.136	0.157	
0	OLS	5.19E-02	2.769	2.841	0.113	0.129	
	COCR	5.86E-02	3.571	3.639	0.158	0.166	
	HILU	5.88E-02	3.57	3.639	0.158	0.166	
	ML	5.30E-02	3.146	3.24	0.135	0.147	
	MLGD	5.31E-02	3.129	3.218	0.135	0.146	
.25	OLS	8.33E-02	2.51	2.568	0.112	0.117	
	COCR	9.56E-02	3.231	3.248	0.148	0.148	
	HILU	9.65E-02	3.262	3.275	0.149	0.149	
	ML	8.51E-02	2.789	2.83	0.126	0.129	
	MLGD	8.52E-02	2.815	2.858	0.127	0.13	
.5	OLS	0.169	2.811	2.849	0.143	0.13	
	COCR	0.204	2.795	2.713	0.132	0.123	
	HILU	0.214	2.819	2.735	0.133	0.124	
	ML	0.169	2.433	2.372	0.116	0.108	
	MLGD	0.168	2.417	2.356	0.115	0.107	
.75	OLS	0.571	4.797	4.764	0.254	0.217	
	COCR	0.936	2.398	2.317	0.113	0.105	
	HILU	0.861	2.382	2.3	0.113	0.105	
	ML	0.549	2.116	2.046	0.102	9.31E-02	
	MLGD	0.546	2.097	2.028	0.101	9.23E-02	
.99	OLS	48.841+	10.076+	10.175+	0.452	0.463	
	COCR	49.897+	2.093	2.008	0.101	9.13E-02	
	HILU	46.888	2.073	1.994	9.87E-02	9.07E-02	
	ML	47.537	1.957	1.888	9.37E-02	8.59E-02	
	MLGD	47.727+	1.95	1.882	9.36E-02	8.56E-02	

Appendix continue

$\rho_1$	M	$\rho_2 = -0.5$		$\rho_2 = -0.25$		$\rho_2 = 0$		$\rho_2 = 0.25$	
		$\beta_1$	$\beta_2$	$\beta_1$	$\beta_2$	$\beta_1$	$\beta_2$	$\beta_1$	$\beta_2$
-.99	OLS	5.297+	5.745+	3.862+	4.596+	3.343+	4.309+	3.399+	4.596+
	COCR	6.02E-02	6.06E-02	5.08E-02	4.85E-02	4.99E-02	4.55E-02	5.51E-02	4.85E-02
	HILU	5.95E-02	5.93E-02	5.04E-02	4.75E-02	4.96E-02	4.45E-02	5.48E-02	4.75E-02
	ML	6.01E-02	5.91E-02	5.09E-02	4.73E-02	5.01E-02	4.43E-02	5.52E-02	4.73E-02
-.75	MLGD	6.27E-02	5.76E-02	5.31E-02	4.61E-02	5.18E-02	4.32E-02	5.63E-02	4.61E-02
	OLS	0.189	0.227	0.156	0.182	0.154	0.17	0.175	0.182
	COCR	6.87E-02	7.46E-02	5.91E-02	5.97E-02	5.97E-02	5.60E-02	6.78E-02	5.97E-02
	HILU	6.85E-02	7.43E-02	5.91E-02	5.95E-02	5.99E-02	5.57E-02	6.81E-02	5.95E-02
-.5	ML	7.12E-02	7.11E-02	6.00E-02	5.69E-02	5.90E-02	5.33E-02	6.50E-02	5.69E-02
	MLGD	7.12E-02	7.06E-02	6.01E-02	5.65E-02	5.90E-02	5.29E-02	6.50E-02	5.65E-02
	OLS	0.103	0.127	8.57E-02	0.101	8.57E-02	9.50E-02	9.84E-02	0.101
	COCR	7.93E-02	8.59E-02	6.87E-02	6.87E-02	6.97E-02	6.44E-02	7.92E-02	6.87E-02
-.25	HILU	7.93E-02	8.63E-02	6.85E-02	6.90E-02	6.94E-02	6.47E-02	7.89E-02	6.90E-02
	ML	7.80E-02	8.28E-02	6.57E-02	6.62E-02	6.50E-02	6.21E-02	7.25E-02	6.62E-02
	MLGD	7.73E-02	8.27E-02	6.51E-02	6.62E-02	6.44E-02	6.21E-02	7.20E-02	6.62E-02
	OLS	7.51E-02	9.26E-02	6.12E-02	7.41E-02	6.03E-02	6.94E-02	6.86E-02	7.41E-02
0	COCR	8.75E-02	9.61E-02	7.45E-02	7.69E-02	7.48E-02	7.21E-02	8.47E-02	7.69E-02
	HILU	8.76E-02	9.61E-02	7.43E-02	7.69E-02	7.43E-02	7.21E-02	8.39E-02	7.69E-02
	ML	7.94E-02	9.15E-02	6.66E-02	7.32E-02	6.65E-02	6.86E-02	7.56E-02	7.32E-02
	MLGD	7.96E-02	9.15E-02	6.67E-02	7.32E-02	6.64E-02	6.86E-02	7.53E-02	7.32E-02
.25	OLS	6.46E-02	7.54E-02	5.26E-02	6.03E-02	5.13E-02	5.65E-02	5.75E-02	6.03E-02
	COCR	9.44E-02	9.66E-02	7.83E-02	7.72E-02	7.61E-02	7.24E-02	8.35E-02	7.72E-02
	HILU	9.44E-02	9.66E-02	7.84E-02	7.72E-02	7.62E-02	7.24E-02	8.37E-02	7.72E-02
	ML	8.11E-02	8.60E-02	6.84E-02	6.88E-02	6.77E-02	6.45E-02	7.55E-02	6.88E-02
.5	MLGD	8.10E-02	8.54E-02	6.82E-02	6.83E-02	6.73E-02	6.41E-02	7.50E-02	6.83E-02
	OLS	6.84E-02	6.81E-02	5.76E-02	5.45E-02	5.65E-02	5.11E-02	6.22E-02	5.45E-02
	COCR	8.78E-02	8.62E-02	7.15E-02	6.89E-02	6.79E-02	6.46E-02	7.29E-02	6.89E-02
	HILU	8.84E-02	8.69E-02	7.18E-02	6.95E-02	6.81E-02	6.52E-02	7.30E-02	6.95E-02
.75	ML	7.60E-02	7.51E-02	6.31E-02	6.01E-02	6.11E-02	5.63E-02	6.65E-02	6.01E-02
	MLGD	7.69E-02	7.58E-02	6.39E-02	6.07E-02	6.18E-02	5.69E-02	6.74E-02	6.07E-02
	OLS	9.40E-02	7.56E-02	8.17E-02	6.05E-02	8.02E-02	5.67E-02	8.62E-02	6.05E-02
	COCR	7.35E-02	7.20E-02	5.52E-02	5.76E-02	4.84E-02	5.40E-02	4.88E-02	5.76E-02
.99	HILU	7.40E-02	7.26E-02	5.54E-02	5.81E-02	4.85E-02	5.44E-02	4.88E-02	5.81E-02
	ML	6.65E-02	6.29E-02	5.10E-02	5.03E-02	4.56E-02	4.72E-02	4.64E-02	5.03E-02
	MLGD	6.55E-02	6.25E-02	5.01E-02	5.00E-02	4.46E-02	4.69E-02	4.54E-02	5.00E-02
	OLS	0.164	0.126	0.139	0.101	0.133	9.48E-02	0.139	0.101
-.99	COCR	6.22E-02	6.15E-02	4.60E-02	4.92E-02	3.97E-02	4.61E-02	3.96E-02	4.92E-02
	HILU	6.23E-02	6.10E-02	4.60E-02	4.88E-02	3.98E-02	4.58E-02	3.96E-02	4.88E-02
	ML	5.76E-02	5.43E-02	4.34E-02	4.34E-02	3.80E-02	4.07E-02	3.80E-02	4.34E-02
	MLGD	5.69E-02	5.38E-02	4.29E-02	4.31E-02	3.75E-02	4.04E-02	3.76E-02	4.31E-02
-.75	OLS	0.267	0.27	0.218	0.216	0.208	0.202	0.226	0.216
	COCR	5.55E-02	5.33E-02	4.07E-02	4.26E-02	3.47E-02	4.00E-02	3.40E-02	4.26E-02
	HILU	5.42E-02	5.29E-02	3.97E-02	4.23E-02	3.40E-02	3.97E-02	3.35E-02	4.23E-02
	ML	5.22E-02	5.01E-02	3.89E-02	4.01E-02	3.37E-02	3.76E-02	3.36E-02	4.01E-02
-.5	MLGD	5.23E-02	4.99E-02	3.90E-02	4.00E-02	3.39E-02	3.75E-02	3.38E-02	4.00E-02

  

$\rho_1$	M	$\rho_2 = 0.5$		$\rho_2 = 0.75$		$\rho_2 = 0.99$	
		$\beta_1$	$\beta_2$	$\beta_1$	$\beta_2$	$\beta_1$	$\beta_2$
q-.99	OLS	4.262+	5.745+	7.867+	9.848+	209.255+	216.510+
	COCR	6.99E-02	6.06E-02	1.18E-01	1.04E-01	2.35E+00	2.29E+00
	HILU	6.94E-02	5.93E-02	1.17E-01	1.02E-01	2.30E+00	2.24E+00
	ML	6.97E-02	5.91E-02	1.17E-01	1.01E-01	2.29E+00	2.23E+00
-.75	MLGD	6.97E-02	5.76E-02	1.14E-01	9.87E-02	2.22E+00	2.17E+00
	OLS	0.233	0.227	0.416	0.389	8.808+	8.560+
	COCR	8.81E-02	7.46E-02	1.51E-01	1.28E-01	2.93E+00	2.81E+00
	HILU	8.85E-02	7.43E-02	1.51E-01	1.27E-01	2.93E+00	2.80E+00
-.5	ML	8.23E-02	7.11E-02	1.38E-01	1.22E-01	2.75E+00	2.68E+00
	MLGD	8.21E-02	7.06E-02	1.38E-01	1.21E-01	2.73E+00	2.66E+00
	OLS	0.132	0.127	2.36E-01	0.217	4.94E+00	4.77E+00
	COCR	1.03E-01	8.59E-02	1.76E-01	1.47E-01	3.39E+00	3.24E+00
-.25	HILU	1.03E-01	8.63E-02	1.76E-01	1.48E-01	3.40E+00	3.25E+00
	ML	9.33E-02	8.28E-02	1.60E-01	1.42E-01	3.22E+00	3.12E+00
	MLGD	9.28E-02	8.27E-02	1.59E-01	1.42E-01	3.21E+00	3.12E+00
	OLS	9.17E-02	9.26E-02	1.66E-01	1.59E-01	3.58E+00	3.49E+00
-.99	COCR	1.10E-01	9.61E-02	1.90E-01	1.65E-01	3.76E+00	3.62E+00
	HILU	1.09E-01	9.61E-02	1.88E-01	1.65E-01	3.75E+00	3.62E+00

Appendix continue

	ML	9.94E-02	9.15E-02	1.74E-01	1.57E-01	3.57E+00	3.45E+00
	MLGD	9.90E-02	9.15E-02	1.74E-01	1.57E-01	3.57E+00	3.45E+00
	OLS	7.57E-02	7.54E-02	1.35E-01	1.29E-01	2.90E+00	2.84E+00
	COCR	1.06E-01	9.66E-02	1.81E-01	1.66E-01	3.71E+00	3.64E+00
0	HILU	1.06E-01	9.66E-02	1.81E-01	1.66E-01	3.72E+00	3.64E+00
	ML	9.72E-02	8.60E-02	1.66E-01	1.47E-01	3.34E+00	3.24E+00
	MLGD	9.63E-02	8.54E-02	1.65E-01	1.46E-01	3.32E+00	3.22E+00
	OLS	7.87E-02	6.81E-02	1.32E-01	1.17E-01	2.64E+00	2.57E+00
	COCR	9.10E-02	8.62E-02	1.54E-01	1.48E-01	3.27E+00	3.25E+00
.25	HILU	9.12E-02	8.69E-02	1.55E-01	1.49E-01	3.30E+00	3.28E+00
	ML	8.37E-02	7.51E-02	1.41E-01	1.29E-01	2.88E+00	2.83E+00
	MLGD	8.47E-02	7.58E-02	1.43E-01	1.30E-01	2.91E+00	2.86E+00
	OLS	1.04E-01	7.56E-02	1.63E-01	1.30E-01	2.94E+00	2.85E+00
	COCR	5.92E-02	7.20E-02	1.04E-01	1.23E-01	2.62E+00	2.71E+00
.5	HILU	5.92E-02	7.26E-02	1.04E-01	1.24E-01	2.64E+00	2.74E+00
	ML	5.61E-02	6.29E-02	9.60E-02	1.08E-01	2.31E+00	2.37E+00
	MLGD	5.50E-02	6.25E-02	9.46E-02	1.07E-01	2.29E+00	2.36E+00
	OLS	0.165	0.126	0.255	0.217	4.806	4.76E+00
	COCR	4.79E-02	6.15E-02	8.49E-02	1.05E-01	2.22E+00	2.32E+00
.75	HILU	4.78E-02	6.10E-02	8.44E-02	1.05E-01	2.21E+00	2.30E+00
	ML	4.55E-02	5.43E-02	7.85E-02	9.31E-02	1.97E+00	2.05E+00
	MLGD	4.51E-02	5.38E-02	7.78E-02	9.23E-02	1.95E+00	2.03E+00
	OLS	0.285	0.27	0.486	0.463	10.285+	10.175+
	COCR	4.05E-02	5.33E-02	7.13E-02	9.13E-02	1.91E+00	2.01E+00
.99	HILU	4.02E-02	5.29E-02	7.13E-02	9.07E-02	1.90E+00	1.99E+00
	ML	4.03E-02	5.01E-02	7.04E-02	8.59E-02	1.81E+00	1.89E+00
	MLGD	4.05E-02	4.99E-02	7.06E-02	8.56E-02	1.81E+00	1.88E+00

CONCLUSION

Although, the OLS estimator competes favourably with the GLS estimators when the levels of autocorrelation and multicollinearity are moderately low, however, this study has revealed that the two GLS estimators namely, ML and HILU are generally preferable in estimating all the parameters of the model in all the levels of autocorrelation and multicollinearity. Thus when these correlations are present in a data set but can not be ascertained, it is more preferable to use either the ML or HILU estimator in estimating the model parameters.

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