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Effect of Power Law Microwave Heating on the Temperature Profiles of Biological Tissues

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Abstract: The behavior of temperature profile in biological tissue under going power Law microwave heating of the form Q (T) = T^m ; m $\geq$1 is investigated analytically by seeking solution to the coupled Maxwell's -bio-heat equations. An asymptotic series analysis method was introduced in one of the two regions of the simplified model. Adequate matching procedure was invoked. The results agreed with the result in the numerical work of for m = 1.Further insights were given by the results that will promote medical care and preferential heating of the diseased tissues.

Key words: Power law microwave heating temperature profile, biological tissues, asymptotic

INTRODUCTION

The prediction of the thermal state in biological issues due to electromagnetic heating is important in studies of possible biological effects of microwave radiation.

Studies by Guy (1971), Johnson and Guy (1972), Kritikos *et al*. (1981) Kritikos and Schwan (1975, 1979) have discussed heat deposition pattern in tissues. Their works gave good insights to the understanding of the physics of heat deposition mechanism. It's clear from these works that incident microwave energy is absorbed non-uniformly throughout the body. This behaviour to heat deposition can possibly lead to local tissue damage due to overheating despite the body's thermoregulation mechanism activities to dissipate the total added heat burden on the subject.

A number of reseraches have been done in an attempt to predict the temperature rise in tissue planes due to microwave irradiation as reported by Foster *et al* . (1998), Kritikos and Schwan (1979) estimated the temperature rise induced by focused heating in brain tissues. Adebile (1997) investigated the temperature rise in tumor and surrounding normal tissue during microwave heating. He considered the effects the effects of varying thermophy sisal properties on temperature rise and he went ahead to determine the bounds on it for different bio-heat models The criteria for the existence and uniqueness of solution were discussed for their governing equations. In the recent times Marchant and Liu (2001) considered the steady state microwave heating of a finite one-dimensional slab. They assumed the temperature dependency of the electrical conductivity and thermal absorptivity were governed by the Arrhenius law, while both the electrical permittivity and magnetic permeability were considered to be constant.

El-dabe *et al*. (2003) investigated the effects of microwave heating on the thermal states of biological tissues. They used the finite difference numerical scheme to solve their governing equations.

Adebile and Ogunmoyela (2005 a, b) recently studied the effect of spatial and temperature dependent blood perfusion on the temperature field of biological tissue during microwave heating. Their results revealed the possibility of multiple solution when blood perfusion is dependent on temperature but a unique solution when depended on the spatial variable.

Adebile and Koriko (2006) in their work investigated the transient temperature state of biological tissue when irradiated with power law microwave.

In their study they considered the power exponent m; to be one (unity) as in El-dabe *et al*. (2003). They took an assumption that make for the decoupling of the Maxwell's equation from the bio-heat equation.

The aim of his study is to investigate the effect of power-law microwave heating on biological tissue. But the neglected the assumption that provokes decoupling of the Maxwell's equations. El-dabe *et al*. (2003) investigated this situation for the case, when the exponent, m; of the heating coefficient Q (T) = T^m is one (unity) Their method of solution was through a numerical technique, we seek an analytical solution to the problem in this paper for a general m$\geq$1.

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Mathematical formulation: The governing equation is as in El-dabe *et al.* (2003). The Maxwell's equations are solved in conjunction with the bio-heat equation since the heat source arising from microwave irradiation is proportional to the square of the modulus of the electric field intensity (Hill and Pincombe, 1992). These equations are;

$$\frac{\partial H}{\partial x}+\varepsilon\frac{\partial E}{\partial t}+\sigma E=0 \qquad (1)$$

$$\frac{\partial E}{\partial x}+\mu\frac{\partial H}{\partial t}=0 \qquad (2)$$

$$\rho C_p\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)-\omega_b\rho_b C_b(T-T_b)+Q(T)|E|^2 \qquad (3)$$

With the initial and boundary conditions

$$\begin{aligned}&T(x,0)=\frac{T_c}{L}x,\quad T(0,t)=0,\quad T(L,t)=T_c\\ &E(x,0)=\frac{E_o}{L}x,\ E(0,t)=0;\ E(L,t)=E_o\\ &H(x,0)=\frac{Ho}{L}x;\ H(0,t)=0;\ H(L,t)=H_o\end{aligned} \qquad (4)$$

El-dabe *et al.* (2003) solved the above mathematical model numerically for m = 1 assuming that the body heating coefficient has the form;

$$Q(T)=T^m, m\geq 1 \qquad (5)$$

Since Marchant and Liu (2001) in their experiment work revealed that the physical properties of material have power law dependence on temperature.

Using the following dimensionless variables as in El-dabe *et al.* (2003)

$$\begin{aligned}&\tau=\frac{t\upsilon}{L^2},\eta=\frac{X}{L},\theta=\frac{T}{T_b},C_1=\frac{C_b}{C_p}\\ &\overline{E}=\frac{E}{E_o},\overline{H}=\frac{H}{H_o},T_1=\frac{\rho_b}{P},\lambda_1=\frac{\upsilon\varepsilon E_o}{LH_o}\\ &\lambda_2=\frac{L\sigma E_o}{H_o};\lambda_3=\frac{\mu H_o\upsilon}{LE_o};\lambda=\frac{L^2 T_b{}^{m-1}|E_o|^2}{\upsilon\rho C_p}\end{aligned} \qquad (6)$$

The dimensionless equations corresponding to Eq. (1-4) are;

$$\frac{\partial H}{\partial \eta}+\lambda_1\frac{\partial E}{\partial \tau}+\lambda_2 E \qquad (7)$$

$$\frac{\partial E}{\partial \eta}+\lambda_3\frac{\partial H}{\partial \tau}=0 \qquad (8)$$

$$\frac{\partial \theta}{\partial \tau}=\frac{1}{Pr}\frac{\partial^2\theta}{\partial \eta^2}-w_1\rho_1 C_1(\theta-1)+\lambda Q|E|^2 \qquad (9)$$

Subject to the following dimensionless initial and boundary conditions.

$$\begin{aligned}&\theta(\eta,0)=\frac{T_c}{T_b}\eta,\ \theta(0,\tau)=0,\ \theta(1,\tau)=\frac{T_c}{T_b}\\ &E(\eta,0)=\eta,\ E(0,\tau)=0,\ E(1,\tau)=1\\ &H(\eta,0)=\eta,\ H(0,\tau)=0,\ H(1,\tau)=1\end{aligned}$$

METHOD OF SOLUTION

The equations for the study state are;

$$\frac{dH}{d\eta}+\lambda_2 E=0 \qquad (10)$$

$$\frac{dE}{d\eta}=0 \qquad (11)$$

$$\Omega\frac{d^2\theta}{d\eta^2}-\alpha(\theta-1)+\lambda|E|^2\theta^m=0 \qquad (12)$$

with the boundary conditions

$$\left.\begin{aligned}&\theta(0)=0,\ \theta(1)=\Omega_1\\ &E(0)=0,\ E(1)=1\\ &H(0)=0,\ H(1)=1\end{aligned}\right\} \qquad (13)$$

The solution to (10), (11) and (13) b, c are

$$E(\eta)=H(\eta-a)=\begin{cases}0,\eta<a\\ 1,\eta\geq a\end{cases} \qquad (14)$$

$$H(\eta)=\eta H(\eta-a)=\begin{cases}0,\eta<a\\ \eta\eta\geq a\end{cases} \qquad (15)$$

The energy Eq. in (12) now becomes

$$\Omega\frac{d^2\theta}{d\eta^2}-\alpha(\theta-1)+\lambda|H(\eta-a)|^2\theta^m=0 \qquad (16)$$

$$\theta(0)=0,\theta(1)=\Omega \qquad (17)$$

where,

$$\Omega = Pr^{-1}, \Omega_1 = T_c T_b^{-1}, \alpha = w_1 T_1 C_1$$

h(η-A) is the heaviside function.
The solution to E(η) in (14) provokes the splitting of the tissues volume into two region (Region I and II).
The relevant equation for the Region I and II are;

$$\Omega \frac{d^2\theta_{11}}{d\eta^2} - \alpha(\theta_{11} - 1) = 0 \quad (18)$$

$$\theta_{11}(0) = 0; \theta_{11}(a) = \Theta; \ 0 < \eta \le a \quad (19)$$

For Region I,

$$\Omega \frac{d^2\theta_{12}}{d\eta^2} - \alpha(\theta_{12} - 1) + \lambda^2 \theta_{12}{}^m = 0 \quad (20)$$

$$\theta_{12}(a) = \Theta; \theta_{12}(1) = \Omega_1 \quad (21)$$

For region II,
The solution to Eq. 18 and 19 is:

$$\theta_{11}(\eta) = 1 - \text{Cosh}\, m_1\eta + \frac{\text{Sinh}\, m_1\eta}{\text{Sinh}\, m_1 a} \left\{(\Theta - 1) + \text{Cosh}\, m_1 a\right\}, \ 0 \le \eta \le a \quad (22)$$

where $m_1 = \sqrt{\frac{\alpha}{\Omega}}$

We seek for the solution to θ_{12} for Region II using the asymptotic series

$$\theta_{12}(\eta) = \theta^o{}_{12} + \lambda\theta_{12}{}^1 + \lambda^2\theta_{12}{}^2 + \quad + ... + \text{h.o.t} \quad (23)$$

Where h.o.t. means higher order terms. λ is as defined in Eq. (6) and λ<<< 1.

Substituting (23) into (20) and (21) and collecting the coefficient of orders λ^0, λ^1; we have the equations.
Order λ^0:

$$\Omega \frac{d^2\theta_{12}{}^{(0)}}{d\eta^2} - \alpha\theta_{12}{}^{(0)} = -\alpha \quad (24)$$

$$\theta_{12}{}^{(0)}(a) = H; \quad \theta_{12}{}^{(0)}(1) = \Omega_1 \quad (25)$$

Order λ^1:

$$\Omega \frac{d^2\theta_{12}{}^{(1)}}{d\eta^2} - \alpha\theta_{12}{}^{(1)} = -\left(\theta_{12}{}^{(0)}\right)^m \quad (25)$$

The solution to Eq. 24 is

$$\theta_{12}{}^{(0)} = A_1 \text{Coshm}_1\eta + B_1 \text{Sinhm}_1\eta + 1 \quad (26)$$

Where

$$A_1 = \frac{(\Theta - 1)\text{Sinh}\, m_2 - (\Omega_1 - 1)\text{Sinh}\, m_1 a}{\text{Sinh}(1-a)m_1}$$

$$B_1 = \frac{(\Theta - 1) - A_1 \text{Cosh}\, m_1 a}{\text{Sinh}\, m_1 a} \quad (27)$$

Consider that

$$\left(\theta_{12}{}^{(0)}\right)^m = \psi e^{m m_1 \eta}$$

where

$$\psi = \left(\chi_1 + \delta + \chi_2\delta^2\right)$$

$$\chi_1 = \frac{A_1 + B_1}{2}, \quad \chi_2 = \frac{(A_1 - B_1)}{2} \quad (28)$$

$$\delta \approx e^{m_1\eta} \cong \frac{1 + e^{-m_1}}{2}; \qquad 0 \le \eta \le 1$$

The Eq 25 after some simplification becomes;

$$\Omega \frac{d^2\theta_{12}{}^{(1)}}{d\eta^2} - \alpha\theta_{12}{}^{(1)} = -\psi e^{m_1 m\eta} \quad (29)$$

$$\theta_{12}{}^{(1)}(a) = 0, \quad \theta_{12}{}^{(1)}(1) = 0$$

The solution to (29) is given by

$$\theta_{12}{}^{(1)} = A_3 \text{Cosh}\, m_1\eta + B_3 \text{Sinh}\, m_1\eta - \frac{e^{m_1\eta}}{2m_1\Omega}\eta \quad (30)$$

for ;m = 1

$$\theta_{12}{}^{(1)} = A_2 \text{Cosh}\, m_1\eta + B_2 \text{Sinh}\, m_1\eta - \frac{\psi e^{m_1 m\eta}}{\left(\Omega m_1^2 m - \alpha\right)} \quad \text{for}\ m \ge 2 \quad (31)$$

where

$$A_2 = \frac{1}{\text{Sinh}(1-a)m_1}\left\{\frac{\psi e^{m_1 m a}\,\text{Sinh}\, m_1}{(\Omega m_1{}^2 m^2 - \alpha)} - \frac{\psi e^{m_1 a}\text{Sinh}\, m_1 a}{(\Omega m_1{}^2 m^2 - \alpha)}\right\} \quad (32)$$

$$B_2 = \left\{ \frac{\psi e^{m_1 m}}{\left(\Omega m_1^2 m^2 - \alpha\right)} - A_2 \operatorname{Cosh} m_1 \right\} \frac{1}{\operatorname{Suh} m_1}$$

and

$$A_3 = \frac{N_1 \operatorname{Suh} m_1 - N_2 \operatorname{Suh} m_1 a}{\operatorname{Suh} m_1 (1 - a)}$$

$$B_3 = \frac{N_2 \operatorname{Cosh} m_1 a - N_2 \operatorname{Cosh} m_1}{\operatorname{Suh} m_1 (1 - a)} \tag{33}$$

$$N_1 = \frac{a}{2\Omega m_1}, \quad N_2 = \frac{1}{2\Omega m_1}$$

RESULTS AND DISCUSSION

In Fig. 1a and b the graph of temperature against space coordinate is displayed for different values of L.

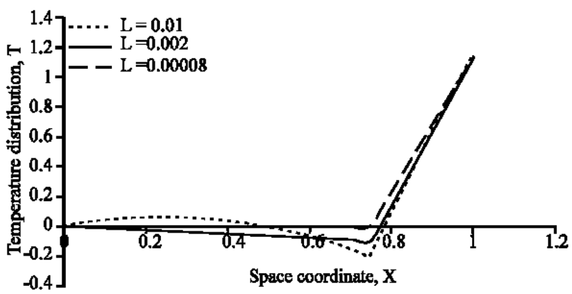

Fig. 1a: Temperature distribution against the space coordinate for different skin tissue thicknesses, L, where a = 0.75, m = 2, k = 0.24. E0 = 2, wb = 0.00125, cb = 3770, cp = 3590, gb = 1060, g = 1050

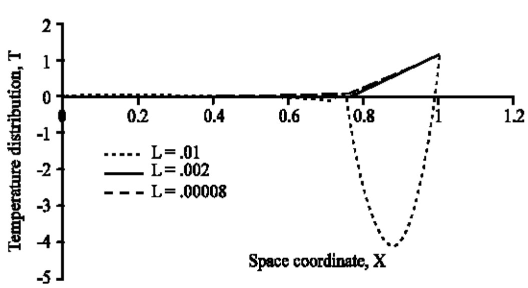

Fig. 1b: Temperature distribution against Space coordinate for different skin tissue thicknesses, L; where m = 3, k = 0.24, E0 = 2, a = 0.75, wb = 0.00125, cb = 3770, cp = 3590, g = 1050 and gb = 1060

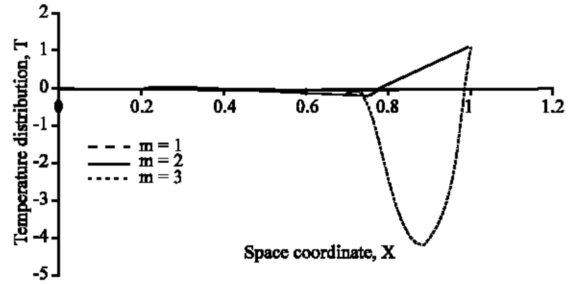

Fig. 1c: Temperature distribution against Space coordinate for different values of m, where L = 0.01, a = 0.75, k = 0.24, E0 = 2, g = 1050, gb = 1060, cb = 3770 and cp = 3590

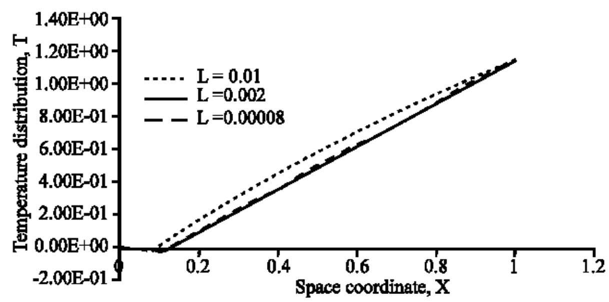

Fig. 2: Temperature distribution against the space coordinate for different skin tissue thicknesses, L; where m = 2, k = 0.24, a = 0.1, E 0 = 2, wb = 0.00125, cb = 3770, cp = 3590, gb = 1060 and g = 1050

These graph revealed that as the thickness L; of the tissue increases, the temperature decreases for $m \geq 1$. In Fig. 1c it is observed that the temperature increases as m increases for given thickness of tissue but at a given value of tissue thickness the temperature decreases as it's evident for m = 3 when L = 0.01.

In Fig. 2, the activation of the electric field at a = 0.1 changes the behaviour of the temperature profiles. The temperature increase as the tissue thickness increases.

The graph of temperature against the space coordinate for various perfusion w; is shown in Fig. 3a, b. As the blood perfusion increases the temperature increases. The temperature gradient with respect to x; is decreasing as the blood perfusion increases in the region $a \geq 0.75$, although this is dependent on m and L.

In Fig. 4a, b, the temperature decreases as the thermal conductivity k increases, but increases as the conductivity k increases for a given value of m and L.

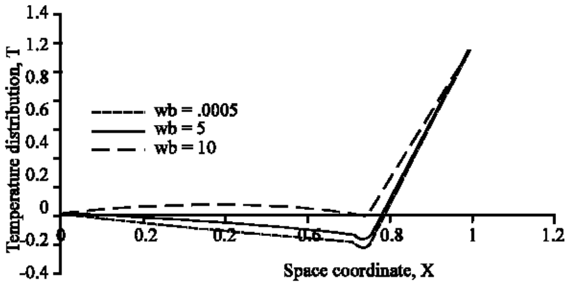

Fig. 3a: Temperature distribution against Space coordinate for blood perfusion rates, wb, where m = 3, E0 = 2, L = 0.00008, a = 0.75, k = 0.24, g = 1050, gb = 1060, cb = 3770 and cp = 3590

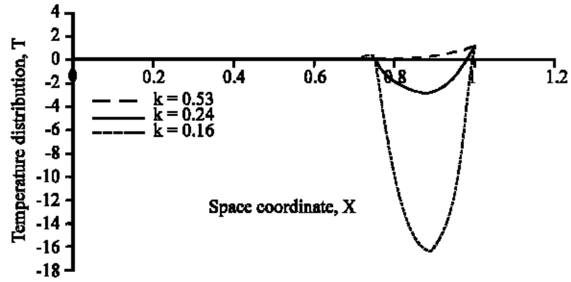

Fig. 4b: Temperature distribution plotted against the Space Coordinate for different values of the thermal conductivity, k. Where m = 3, a = 0.75, h = -0.1, g = 1050, gb = 1060, cb = 3770 and cp = 3590

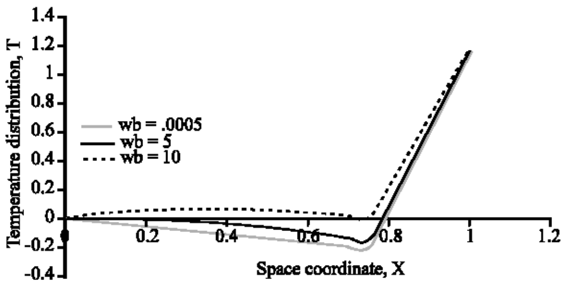

Fig. 3b: Temperature distribution against the space coordinate for different rates of blood perfusion, wb, where m = 3, E0 = 2, L = 0.00008, a = 0.75, k = 0.24, g = 1050, gb = 1060, cb = 3770 and cp = 3590

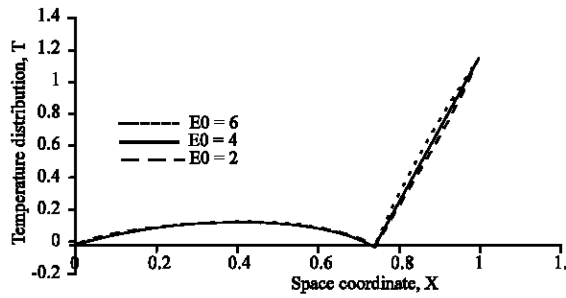

Fig. 5a: Temperature distribution plotted against the space coordinate for different values of E0. Where m = 2, a =.75, L = 0.01, k = 0.24, wb = 0.00125, g = 1050, gb = 1060, cb = 3770 and cp = 3590

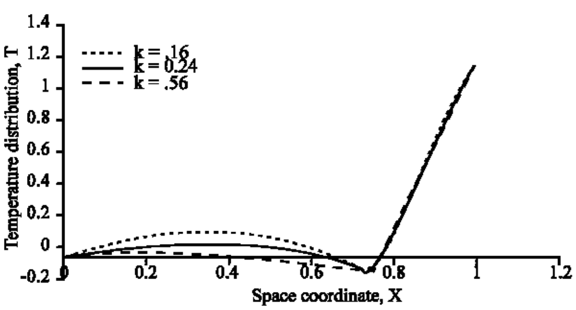

Fig. 4a: Temperature distribution plotted against the space coordinate for different values of thermal conductivity, k where m = 2, L = 0.01, a = 0.75, h = -0.1, wb = 0.00125, E0 = 2, cb = 3770, cp = 3590, gb = 1060 and g = 1050

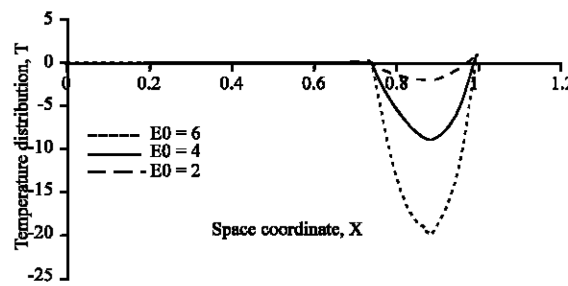

Fig. 5b: Temperature distribution plotted against the space coordinate for different values of E0. where m = 3, a = .75, k = 0.24, L = 0.01, wb = 0.00125, g = 1050, gb = 1060, cb = 3770 and cp = 3590

From Fig. 5a and b, it is clear that as the Electric field of the free space E_0 increases the temperature increases, although for a selected m and L the temperature decreases as E_0 increases.

Furthermore it is discovered in Fig. 6a and b that the location a; of the activation of the electric field in the tissue volume will provoke a higher or lower temperature if the point a is nearer to the surface or the core of the tissue, respectively.

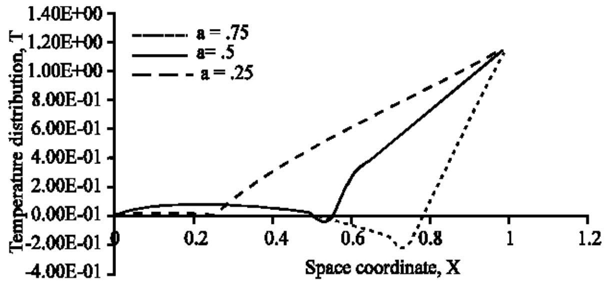

Fig. 6a: Temperature distribution plotted against the Space coordinate for different skin depths, a; Where L = 0.01, m = 2, k = 0.24, E0 = 2, wb = 0.00125, cb = 3770, cp = 3590, gb = 1060 and g = 1050

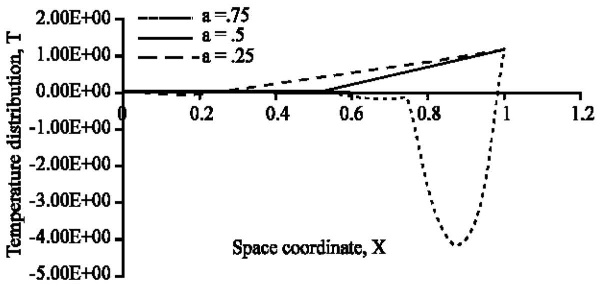

Fig. 6b: Temperature distribution plotted against the space Coordinate for different values of a where m = 3, L = 0.01, k = 0.24, wb = .00125, cb = 3770, cp = 3590, gb = 1060 and g = 1050

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