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Influence of Environmental Pollution on the Transmission Dynamics of Infection

M.A. Umar
Department of Mathematical Model, Nasarawa State University Keffi, Kefi, Nigeria

Abstract: In this research, we examined the relationship between pollution and infection and proposed pollution/transmission dynamics model for infection, in an age-structured population. Other factors such as population growth in the present of pollution and transmission rate are discussed. It is observed that pollution increases the mortality rate and thus reduce life expectancy of those within the location. However it is independent of infection, but helps to accelerate transmission rate for the infection. The existence of the non-triavial steady states is examined and their stability discussed.

Key words: Infection, pollution, mortality, life expectancy, stability

INTRODUCTION

Pollution has been a major threat to survival of human and animal population in most countries, especially in the developing countries of the world, where poverty, greed and lack of patriotism has been identified as some of the reasons responsible for its it practices. Multinational companies in collusion with national of some of these countries often dumped waste that has serious impact on life of the people of theses countries, Nigeria and a host of other developing countries had experienced this abuse in the past and in recent times, communities in oil producing area in Nigeria have been calling on the United nation and other bodies to come to their aids, as their ecosystem has been completely destroyed by the multination oil companies, through oil spillage and other environmentally hazardous activities carried out by these company without adequate compensation and regard to the laws of the united nation and of these countries. Governments have not helped matters, democracy or otherwise, perhaps this explains reason for agitation for resource control by communities in some of these areas. This study critically examines the long time impact of these hazardous materials/waste, oil spillage on the life of the people in these areas, using factor of nearness to the pollution/waste/oil spillage site. The question then is, whether communities living far from the site have better chances of escaping from its long time impact, than communities living at the site of pollution. Also whether these scenarios influences the transmission dynamics of infection.

MODEL FORMULATION

Let us divide the population into three groups namely, people living at the pollution site I (t, a), who are prone to infection due to their nearness to the waste/pollution. Those living close to the pollution site S (t, a) and those who live far from the pollution site D (t, a) also experiences same, but to a lesser degree. However we assume that infected recovered and return to D (t, a) population compartment. The natural mortality rate is, $\mu(a,k(a)) \in L'([0,\bar{a})x[0,\bar{a}))$ and c is uniform average per capital contact rate with the waste/pollution, $\bar{a}$ is the maximum age and $l \in R_+$.

$$k(a) = \frac{c\eta(a)}{\delta(\alpha_0, \alpha(a))},\ \alpha_0$$

is the point at which pollution is located and $\alpha(a)$ is the point at which an individuals of age a,is located and k(a) age is the natural mortality factor of an individual of age a at time t,

$$\delta(\alpha_0, \alpha(a)) = \inf|\alpha_0 - \alpha(a)|$$

where

$$\inf|\alpha_0 - \alpha(a)|$$

is the shortest distance between the location of pollution site and the location of an individual of age a at time t, c is the average per capital contact rate, with the pollution/waste and $\eta(a)$ is the rate at which an individuals natural mortality is reduced. This depends on the nearness of an individual to the pollution site. We then have the conservation equation in line with the classical Mckendrick-Foerster age structured population model in Castillo-Chavez (2004), Kristina *et al.* (1991), Inaba (1990) and Mats (2002), as

$$(\partial_t + \partial_a)S(t,a) = -\mu(a,k(a))S(t,a) - \alpha S(t,a) - \sigma S(t,a) - c\lambda(t)S(t,a) \quad (1)$$

$$(\partial_t + \partial_a)D(t,a) = -\mu(a,k(a))D(t,a) + \alpha S(t,a) + \gamma I(t,a) \quad (2)$$

$$(\partial_t + \delta_a)I(t,a) = -\mu(a,k(a))I(t,a) + \sigma S(t,a) + c\lambda(t)S(t,a) - \gamma(I(t,a) \quad (3)$$

Where, α is the progression rate from D (t, a) compartment to S (t, a) compartment, σ is the progression rate to active infection from the S(t, a) compartment, γ is the recovery rate from active infection to the D(t, a) compartment and λ(t is the per capital force of infection defined by

$$\lambda(t) = \int_0^\infty \frac{p(t,\bar{a})}{n(t,\bar{a})} I(t,\bar{a})d\bar{a}$$

where $p(t,\bar{a})$ is the transmission rate for an infected individual of age $\bar{a}$, at time t, (Castillo-Chavez, 2004),

$$P(t) = \int_0^\infty n(t,a)da$$

n (t, a) = S (t, a) +D (t, a) + I (t, a). The boundary and initial conditions are,

$$S(0,a) = S_0(a),\ D(0,a) = D_0(a),\quad I(0,a) = I_0(a)$$

$$S(t,0) = \int_0^\infty \beta(a,)S(t,a)da$$

$$D(t,0) = \int_0^\infty \beta(a,)D(t,a)da,\qquad I(t,0) = 0$$

Non-infective birth rate into the population is,

$$b(t) = \int_0^\infty \beta(a,)[D(t,a) + S(t,a)]da.,\quad \beta(a) \in L\infty([0,\bar{a}))$$

The initial age distribution, are non-negative continuous and integral function of a $\in$ (0, ∞). The population mortality rate is influenced by the magnitude of the reproductive factor k (a). In the population compartment D (t, a), k (a) $\rightarrow$ 0 , or δ (α_0, α (a)) >>cη (a). While in the compartment S (t, a), k(a)) is large. This is because the distance between the pollution point a_0 and an individual location point in the area is minimal, hence the rate at which their natural mortality is affected is high. While those in I (t, a) population compartment lives at the pollution site, with δ (α_0, α (a)) $\rightarrow$ 0 and so k(a) is very large compared to the other two. This explains the reason for the occurrence of infection and increase in the level of mortality.

Adding up these equations we get the classical, Mckendrick-Foerster age-structured population model,

$$(\partial_t + \partial_a)n(t,a) = -\mu(a,k(a))n(t,a) \quad (4)$$

$$n(0,a) = n_0(a) \quad (5)$$

$$n(t,0) = \int_0^\infty [\beta(a,)n(t,a)]da \quad (6)$$

If there is no pollution, then every member of the population belongs to the total population density,

$$P(t) = \int_0^\infty n(t,a)da$$

There will be no infection generated and the recruitment rate is,

$$b(t) = n(t,0) = \int_0^\infty \beta(a,)n(t,a)da$$

Where k(t) = 0, their natural mortality rate is not

$$G(a) = \int_0^\infty \beta(a,)n(a)da$$

pollution induced. However, if the
population lives at the pollution site, then α (a) $\rightarrow$ α_0 and δ (α_0, α (a)) $\rightarrow$ 0, their natural mortality is pollution induced. The recruitment rate for new born is then,

$$b(t) = n(t,0) = \int_0^\infty \beta(a)n(t,a)da + \int_0^\infty \beta(a)D(t,a)da$$

consistent with earlier definitions.

POPULATION GROWTH MODEL IN THE PRESENCE OF POLLUTION

Our focus is on the impact of pollution and waste on the population growth; we consider the age-structured population model,

$$(\partial_t + \partial_a)n(t,a) = -\mu(a,k(a))n(t,a) \quad (7)$$

$$n(0,a) = n_0(a) \quad (8)$$

$$n(t,0) = \int_0^\infty [\beta(a,)n(t,a)]da \quad (9)$$

$$k(t) = \frac{c\eta(a)}{\delta(\alpha_0,\alpha(a))},\quad \delta(\alpha_0,\alpha(a)) = \inf|\alpha_0 - \alpha(a)|$$

The steady demographic state of the population, in the presence of pollution, without infections is,

$$n(a) = G(a)e^{-\int_0^a \mu(a,k^*)da}$$

where,

$$G(a) = \int_0^{\infty} \beta(a,)n(a)da \text{ , } k^* = \frac{c\eta(a)}{\delta(\alpha_0, \alpha(a))}$$

With solution via characteristics lines,

$$n(t,a) = n_0(a)e^{-\int_0^a \mu(r,k(a))dr} = n_0(a\text{-}t)e^{-\int_{a-t}^a \mu(r,k(a))da}, \text{ for } a > t$$

$$= n(t\text{-}a,0)e^{-\int_0^a \mu(r,k(a))da}, \quad \text{for} \quad a < t$$

The recruitment rate for newborn, reduces to,

$$b(t) = n(t,0) = \int_0^{\infty} \beta(a,)\tau(a)n_0(a-t)da, +$$

$$\int_0^{\infty} \beta(a,)B(a)n(t\text{-}a,0)da. \quad B(a) = e^{-\int_0^a \mu(r,k(a))dr}$$

$$\tau(a) = e^{-\int_{a-t}^a \mu(r,k(a))da}$$

The expected number of offspring of an individual of the population R_0 is,

$$R_0 = \int_0^{\infty} \beta(a)B(a)da, \quad B(a) = e^{-\int_0^a \mu(a,k(a))da}$$

is the survival function and β (a) is fertility or maternity function and the life expectancy of the population is

$$\int_0^{\bar{a}} B(a)da$$

in line with Inaba (1990). Thus, it is pollution depended. However, without pollution and infections we would have a steady demographic population as,

$$n(a) = G(a)e^{-\int_0^a \mu(a,k^*)da},$$

$$\text{where } G(a) = \int_0^{\infty} \beta(a,)n(a)da, \; k^* = \frac{c\eta(a)}{\delta(\alpha_0, \alpha(a))} \to 0.$$

The solution of Eq. (7-9) is obtained via characteristics lines as,

$$n(t,a) = n_0(a)e^{-\int_0^a \mu(r)dr} = n_0(a-t)e^{-\int_{a-t}^a \mu(r)dr}, \quad a > t$$

$$= n(t\text{-}a, 0)e^{-\int_0^a \mu(r))dr}, \quad a < t$$

Recruitment rate for newborn is,

$$b(t) = n(t,0) = \int_0^{\infty} \beta(a)B(a)n_0(a-t)da, +$$

$$\int_0^{\infty} \beta(a)B(a)n(t\text{-}a,0)da. \quad B(a) = e^{-\int_0^a \mu(r)dr}$$

$$\tau(a) = e^{-\int_{a-t}^a \mu(r)dr}$$

The expected number of offspring of an individual in the population is,

$$\int_0^{\infty} \beta(a)B(a)da, \quad B(a) = e^{-\int_0^a \mu(r)dr}$$

while life expectancy is,

$$\int_0^{\bar{a}} B(a)da$$

Since k (a) → 0, the natural mortality rate is reduced, thus life expectancy is increased. Hence the population density is increased..

TRANSMISSION DYNAMICS MODEL IN THE PRESENCE OF INFECTION

If an infection is noticed in the population, then Eq. 1-3 simply represent its transmission dynamics. Introducing the following population fractions,

$$d(t,a) = \frac{D(t,a)}{n(t,a)}, \; s(t,a) = \frac{S(t,a)}{n(t,a)}, \; i(t,a) = \frac{I(t,a)}{n(t,a)}$$

into theses equation gives,

$$(\partial_t + \partial_a)s(t,a) = -\eta(t)s(t,a)$$

$$(\partial_t + \partial_a)d(t,a) = \alpha s(t,a) + \gamma i(t,a)$$

$$(\partial_t + \partial_a)i(t,a) = \alpha s(t,a) + \eta(t)s(t,a) - \gamma i(t,a)$$

$$s(0,a) = s_0(a), \; d_0(0,a) = d_0(a), \; i(0,a) = i_0(a),$$
$$\eta(t) = \alpha + \sigma + c\lambda(t).$$

$$s(t,0) = \int_0^{\infty} \beta(a)s(t,a)da, \; d(t,0) = \int_0^{\infty} \beta(a)d(t,a)da, \; i(t,0) = 0$$

$$b(t) = \int_0^{\infty} \beta(a)[d(t,a) + s(t,a)]da.$$

$$\lambda(t) = \int_0^{\infty} p(t,\bar{a})i(t,\bar{a})d\bar{a}$$

Where $s(t,a) + d(t,a) + i(t,a)€€ = 1$

EXISTENCE AND STABILITY OF NON-TRIVIAL STEADY STATES

Using above equation we examine the transmission dynamics of epidemics coursed by pollution in the population compartments along the line of Doma (2004)

and Inaba (1990). Let d*(a), s*(a), i *(a) and λ* be the steady demographic states. Then we have the following Equations,

$$s^*(a) = s^*(0)G(a)e^{-c\lambda^*\int_0^a p(a)da},\ G(a) = e^{-(\alpha+\sigma)a}$$

$$i^*(a) = e^{-\gamma a}((\frac{\sigma}{\gamma})e^{\gamma a} - 1) + \lambda^* ce^{-\gamma a}\int_0^a p(a)e^{\gamma a}da$$

$$d^*(a) = d(0) + \gamma\int_0^\infty i^*(a) - \alpha\int_0^\infty s^*(a)da$$

$$\lambda^* = \int_0^\infty p(a,\bar{a})i(a)d\bar{a}$$

The disease-free steady state exist if, $\lambda^* = 0$ and

$$\frac{\sigma}{\gamma}e^{\gamma a} = 1$$

However if $\lambda^* \neq 0$ and

$$\frac{\sigma}{\gamma}e^{\gamma a} > 1$$

then we would have an endemic state. Thus, the local stability of the infection-free non-uniform steady state distribution is tested using the perturbation,

$$d(t,a) = \bar{d}(a)e^{\varepsilon t} + d^*(a)$$

$$s(t,a) = \bar{s}(a)e^{\varepsilon t} + s^*(a)$$

$$i(t,a) = \bar{i}(a)e^{\varepsilon t} + i^*(a)$$

$$\eta(t) = \eta_0 e^{\varepsilon t},\ \eta_0 = \alpha + \sigma + c\lambda_0,\ \lambda_0 = \int_0^\infty p(a,\bar{a})\bar{i}(\bar{a})d\bar{a}$$

Linearization about the steady state as in Castillo-Chavez (2004) and Inaba (1990), gives the following eigenvalue problem,

$$\frac{d\bar{s}(a)}{da} = -(\varepsilon + \eta_0)\overline{s(a)}$$

$$\frac{d\bar{d}(a)}{da} + \varepsilon\bar{d}(a) = \alpha\bar{s}(a) + \gamma\bar{i}(a)$$

$$\frac{d\bar{i}(a)}{da} + (\varepsilon + \gamma)\bar{i}(a) = (\alpha + \eta_0)\bar{s}(a)$$

Where $\bar{s}(a) = \bar{s}(0)e^{-(\varepsilon+\eta_0)a}$, $\bar{i}(a) = \frac{(\alpha+\eta_0)}{\gamma-\eta_0}\bar{s}(0)e^{-(\eta_0+\varepsilon)}$

$+[\bar{i}(0) - (\alpha+\eta_0)\bar{s}(0)e^{-(\varepsilon+\gamma)a}$

$$\bar{d}(a) = (\bar{d}(0) + B(a))e^{-\varepsilon a}$$

The solution of the ODE, associated with the lineralized problem decay away with age, indicative of a stable steady states. Thus the disease-free steady state exists and is locally asymptotically stable, if

$$\lambda^* = 0 \quad \text{and} \quad \frac{\sigma}{\gamma}e^{\gamma a} = 1$$

If

$$\frac{\sigma}{\gamma}e^{\gamma a} > 1 \quad \text{and} \quad \lambda^* \neq 0$$

then the endemic steady state exists and is locally asymptotically stable.

Equations for the population compartment are obtained via characteristic lines as,

$$s(t,a) = s_0(a)e^{-\eta(t)a}$$

$$i(t,a) = e^{-\gamma a}G(a), G(a) = \int_0^a s_0(a)(\alpha + \eta(t))e^{-(\eta(t)-\gamma)}\,da$$

$$d(t,a) = d(a) + \int_0^a(\alpha s_0(a)e^{-\eta(t)} + e^{-\gamma a}G(a))da$$

CONCLUSION

This study has shown the influence of environment on the transmission dynamics of infection and the overall effect on the population, mortality rate and life expectancy of the population. It is observed that the population living close to environmentally polluted area is prone to higher mortality rate, low life expectancy and hence low population growth rate as demonstrated by the influence of the pollution factor, k (t) on mortality rate, μ (a, k (t)). Also to obtain a stable diseases-free steady state, in such a polluted environment we must have

$$\lambda = 0 \text{ and } \frac{\sigma}{\gamma} = e^{-\gamma a}.$$

Meaning that with increase in age of an individual in the population, we would have $\sigma = \gamma$. That is the progression rate to infection should equal to recovery rate.

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