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A Genetic Algorithm Approach to Joint Optimization for Product Line and Ordering Quantity

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Abstract: There is a growing consensus among product managers that determining the optimal length of product line is a crucial decision that affects the overall strategy and finance of a company. Product line composition has clear financial implications that can hamper the profitability of an organization. Equally important is the ordering strategy that would determine the quantity of each product type and the timing of each order for replenishing the stocks of items in the product line. This study describes a genetic algorithm model that simultaneously optimizes the product line composition and the ordering quantity so to maximize total profits.

Key words: Genetic algorithms, product line, quantity order, joint optimization

INTRODUCTION

Of the various ways in which a company can vary its marketing strategies to meet the ever changing demands of its target markets, product and service mix followed by price affordability are reported to be the most critical ones. The concept of product line is closely associated with product mix, which also involves price management, communication or promotion and distribution.

Product line involves products that are similar in nature and design, but different in some key attributes (e.g., cost, quality). For instance, a shampoo line may consist of products of different composition, applications and costs. For a retailer, this implies decisions on a particular product category arrangement and the specific characteristics to be offered.

It has been recognized that the product variety or product line decision is one of critical importance to marketing and product managers. It involves determining the variety of brands to be stocked as well as their stocking allocations within available display areas so as to maximize the store's overall profits. This is of extreme relevancy since there is some degree of substitutability between the products that comprise the product line and their individual salability. Introducing new products at the expense of the sales of other products may be detrimental to the firm. Thus, product line decisions need to include analysis of product additions as well as deletions and the degree of complementarily and substitutability among the different items in the line.

An unlimited product variety is clearly not the way to be successful. There needs to be an optimum product range, which would depend on the firm's competitive priorities. Clearly, product variety affects costs and as the number of items that comprise the line becomes larger, the variety related costs will also dramatically increase[1].

Equally important are ordering decisions, which determine the order size and timing of the replenishing of the items that constitute a given product line. Ordering line decisions deal with classical inventory system issues, i.e., how much to order of a given product and when to place such an order. These are relevant issues since they affect the inventory carrying costs, ordering costs, as well as the customer service levels.

Products can also be added and deleted from the product line according to their profitability. Under the present global competitiveness, companies are forced to keep focus on customer needs through a clear understanding of their preferences. Market surveys are continuously employed to keep track of customer likes and dislikes in order to assess product line effectiveness in coping with product diversification.

Customers have greater choices in product variety than ever before. The decision of whether or not to stock a particular item is therefore critical. Retailers have to be aware of the costs involved in the stocking of their products. The expected profits should clearly outweigh stocking or holding costs as well as service and maintenance costs. Furthermore, proper floor space usage is another binding element that should affect retailers' decisions on inventory sizes and product line diversity.

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In this study, the problem of jointly determining product variety and order quantities for different types or brands of a certain item within a particular product line is considered. A floating-point genetic algorithm model was developed to address such a problem. The GA model considers relevant product ordering and stocking considerations in order to maximize the overall profitability of retailers. This incorporates various limiting factors or constraints into the analysis such as floor space utilization, economic order quantity and product substitutability and salability.

Literature background: A considerable amount of articles can be found in the literature on how to solve the optimal product line selection program through measurements that reflect individual consumer values and preferences. There is also a large number of publications entirely devoted to the classical economic order quantity of when to order and how much to order. However, the problem addressed in this study, joint optimization of product line and ordering quantity using evolutionary computing techniques has not yet been addressed.

Product line decisions are difficult because the products in the line are not usually independent. Products cannot be optimized individually and then be added to the line to search for optimum product line decisions; thus, interdependency is the key consideration in product line decision making.

Scholars have proposed preference-based procedures for solving product design problems. Two different approaches can be followed to tackle the problem. The first approach takes into consideration a finite set of candidate items from which a product line can be selected. Preference evaluations for each item are used to select a product line that maximizes a buyer's welfare function or a seller's return criterion[2]. In the second approach, product lines are created directly from part value data. However, if the number of attribute value levels is large and all attribute level combinations define feasible products, it may be computationally infeasible to enumerate all possible utilities associated to the candidate items.

Despite the considerable importance of product line decisions, relatively few published papers have dealt with product line optimization. McBride and Zulfyden[3] developed an optimal integer programming approach to solve the product line selection problem. This approach yields a product line with the greatest value to the seller. Borin *et al.*[4] provided an approach to solve the product line problem while deciding on the shelf space allocation of the products comprising it. This constrained optimization problem was formulated using two decision variables: Assortment and allocation of space of the items in the assortment. Green and Krieger[5] developed a model for determining product line composition. They approached the problem from both the buyer's and seller's perspectives and suggested methods for finding optimal product lines. Moreover, Kohli and Sukumar[6] and Nair *et al.*[7] also developed non-exact techniques to find optimal product lines using maximization of the worth of the items in the line as the main designing criterion. There is only one reported research effort that has examined the problem of product line selection with the simultaneous objective of increased profitability with minimum inventory costs. Vaidynanathan *et al.*[8] developed a non-linear optimization model that, in essence, maximizes the profits for the store and jointly optimizes on the product line and ordering strategies.

After examining the reported research efforts in the field, the next logical step would be to investigate whether an evolutionary computing technique, such as genetic algorithms, can be applied to successfully generate optimal solutions for this sort of joint optimization problems with a larger set of constraints and variables.

Mathematical formulation of the problem at hand: To tackle the joint optimization problem for product line, the total profits for the store have to be maximized through an optimum selection of items. There are some specific constraints within which this has to be performed. Hence, the objective function to be maximized is that of the total profits (SP-CP) on account of annual sales (ASj). The mathematical formulation for of such a problem is presented below:

$$\text{Max } z = \sum_{j=1}^{J} (SP_j - CP_j) \times AS_j \quad (1)$$

subject to:

$$Rj = (F_j + \sum_{i=1}^{J} W_{ij}\ (1-K_i) * S_{ij})*K_{ij} \text{⌡} j \quad (2)$$

$$As_j = R_j * \text{Max} \text{⌡} j \quad (3)$$

$$EOQ_j = (2*Q_j*AS_j)1/2/h_j \text{⌡} j \quad (4)$$

$$\sum_J EOQ_j\ [FS \quad (5)$$

$$\sum_J (Cp_j*AS_j+I_j*K_j+(h_j*EOQ_j)/2+(O_j*AS_j)/EOQ_j)[B \quad (6)$$

$$K_j = 0 \text{ or } 1 \quad (7)$$

where:

J denotes the difference types of product brands,
Sp_j is the selling price per unit of brand j,
Cp_j is the cost price per unit of brand j,
As_j is the annual sales in units of brand j,
F_j is the fraction of brand j sales as first choice among customers,
S_{ij} is the fraction of sales for brand i as the first choice and brand j as the second choice among customers,
Max is the total sales in units if all brands are stocked,
R_j is the fraction of Max accounted for by brand j,
O_j is the ordering cost per order for brand j,
h_j is the holding cost per year for brand j,
EOQ is the expected order quantity,
B is the allocated budget,
I is the carrying cost for brand j,
FS is the floor space and
K is a binary decision variable.

Constraints (2) and (3) represent the substitutability of product brand j. Constraint (2) is non-linear in nature. The second part of the constraint introduces the non-linearity into the systems of equations. Constraint (3) accounts for the annual sales in units for brand j. Constraint (4) is the standard economic order quantity constraint. Constraint (5) is a binding constraint on the floor space available to store all the ordered quantities of all brands. Constraint (6) limits the expenses on maintenance, carrying and ordering costs so that they do not exceed the corresponding allocated budgets. Constraint (7) is a binary representation of a decision variable.

As previously noted, this particular problem, being non-linear in nature, has been already addressed by Vaidyanathan, *et al.*[8] using GAMS (Generalized Algebraic Modeling System). It would of interest to apply genetic algorithms to solve this particular problem and then compare the GA results against those reported by Vaidyanathan, *et. al.*[8].

The genetic algorithm procedure: Genetic Algorithms (GA) are search algorithms based on the mechanics of natural genetics. They are different from more conventional optimization and search procedures in various ways. GA's work with the coding of the problem's variables or parameters, not the parameters themselves. Second, they search from a population of points, not a single data value. GA's employ payoff (objective function) information, not derivatives or other auxiliary knowledge. Finally, GA's use probabilistic transition rules and not deterministic ones as it is the case of enumerative and derivative based methods.

A genetic algorithm is initiated with a set of solutions (represented by chromosomes) called the population. Solutions from one population are taken and used to form a new population. This is motivated by a hope that the new population will be better than the previous one. Solutions that are selected to form new solutions (offspring) are chosen according to their fitness-the more suitable they are, the more chances they have to reproduce. This is repeated until some evaluation condition is satisfied.

In the GA model developed to solve the product line problem, it was assumed 5 different brands of a same item to be included in the final product selection. Each input variable was thus coded into a floating-point gene of chromosome length of 35. The population size was taken as 50. The initial population was then randomly generated.

In order to account for the problem constraints, the fitness function was specially coded to accommodate for the limiting circumstances. Constraint (5), which is the control on the number of items to be held in inventory, suggests that there would be a loss of revenue if excess inventory is held. Therefore, in the fitness function, a penalty of maximum cost price ($350) was charged for every item stored beyond the nominal Floor Space (FS) area. Constraint (6), if violated, was penalized with 50 times the cost difference incurred due to maintenance, carrying and ordering costs. Constraint (2) and (3) were also introduced to limit the total number of units that can be sold for a particular brand.

For this particular problem, the GA switch function constant was determined to be 0.25. The floor space and potential total sales were defined annually. The selling prices for all product brands were obtained from a uniform distribution between $350 and $450 while their cost prices from a uniform distribution between $250 and $350. The ordering cost, annual maintenance cost and carrying costs are also drawn from a uniform distribution. Maximum units that can be sold if all brands are stocked were fixed at 4000 units. The floor space was limited to 4000 units. The probability of crossover was set to 0.85 and the probability of mutation to 0.05. These values were finalized after a series of trial runs of the algorithm and after monitoring the behavior of the fitness function over 1000 generations.

Figure 1 shows the convergence of the developed algorithm towards an optimal solution for the previously discussed problem. As it can be seen from the figure, there is a rapid convergence due to the tight penalties on the constraint violation. Also, the overall population starts to converge towards a single individual point as it moves towards the 1000th generation. The average fitness grows very slowly since the constraints' violation causes the fitness to fluctuate with a high variance.

Table 1: GA suggested product line

Variables	A	B	C	E	Total
Annual sales(no.)	825	465	1105	1305	3700
Selling price per item($)	449.95	449.97	449.93	449.25	
Cost price per item ($)	250.24	250.02	250.01	250.78	
Total profit ($)	164760.75	92976.75	220911.6	259003.4	737652.5
EOQ (no.)	36.14	28.81	42.51	52.5	
Carrying cost/year ($)	56.7	56.7	56.7	56.7	
Ordering cost/order ($)	44.94	50.62	46.37	72	
Maintenance cost ($)	17702.67	17857.56	11761.96	14411.86	

Fig. 1: GA convergence behavior

In order to verify that the genetic algorithm model was capable of yielding sound solutions to the product line problem, the algorithm was tested on a simpler problem involving 5 brands with equal probabilities of customer selection. It was thus assumed that Fj was 0.20 for each one of the brands. The model was expected to suggest equal sales amounts for each brand since potential customers equally desire each of them. The algorithm was then run on this set of inputs, selecting three final brands with equal net sales. Since there were no cost differentiation between brands, any three brands out of five gave maximum profits.

Once verified the performance of the genetic algorithm, the problem at hand was solved using the developed model. According to the problem, 20% of the potential customers identify brand A as their first choice, while 11% identify brand B, 27% identify brand C, 10% brand D and 32% identify brand E. A switch function constant of 0.25 was assumed. It took fifty seconds for the GA model to converge to an optimal solution, which proved to be more logical and justifiable than that reported by Vaidyanathan, *et al.*[8].

Table 1 shows the best solution obtained by the genetic algorithms. The available budget was limited to $1,000,000 for this scenario. In this solution, the optimal set included brands A, B, C and E from the potential set of five brands (A, B, C, D, E). The GA model recommended selling 825 units of brand A, 465 of brand B, 1105 of brand C and 1305 units of brand E. In additions, the constraints on ordering quantity (EOQ <Floor Space) and on budget allocated were satisfactorily met by this solution.

The solution provided by the genetic algorithm seems to be better than that given by Vaidyanathan, *et al.*[8]. The solution suggests stocking A, B, C and E brands in order to maximize on the store's profits, while Vaidyanathan, *et al.*[8] recommends that brands B, D and E are to be stocked instead. Looking at the customer preferences for the various brands, it clear that the recommendation provided by the genetic algorithm is more consistent with the customers' liking. By stocking A, B, C and E, 90% of all customers' first choices are met, while by stocking B, D and E, only 43% of the potential customers' first choices are satisfied.

The total number of units to be stocked is then 3,700, almost 550 units more than suggested by Vaidyanathan, *et al.*[8]. Accordingly, the expected profits are significantly higher than what the previous research had suggested.

CONCLUSION

From this study, it can be concluded that the developed genetic algorithm is capable of yielding sound solutions to the product line joint optimization problem. It was demonstrated that genetic algorithm can be successfully applied to handle the complexity involved in constrained problems such as the joint optimization of product selection and ordering quantity. Due to their robustness and efficiency in reaching a final solution, genetic algorithms seem to be more proficient than conventional optimization and search procedures when applied to partially non-linear solution spaces.

REFERENCES

1. Mather, H., 1992. Optimize your product variety, Production and Inventory Management, 2: 38-42.
2. Alexouda, G. and K. Paparrizos, 2001. A genetic algorithm approach to the product line design problem using the seller's return criterion: An extensive comparative computational study, European J. Operational Res., 134: 165-178.
3. McBride, R.D. and F. Zufryden, 1988. An integer programming approach to the optimal product line selection, Marketing Sci., 7: 126-139.
4. Borin, N., P. Farris and J. Freeland, 1994. A model for determining retail product category assortment and shelf space allocation, Decision Sci., 25: 359-384.
5. Green, P. and A. Krieger, 1985. Models and heuristics for product line selection, Marketing Sci., 4: 1-19.
6. Kohli, R. and R. Sukumar, 1990. Heuristics for product-line design using conjoint analysis, Management Sci., 36: 1464-1478.
7. Nair, S., L. Thakur and K. Wen, 1995. Near optimal solutions for product line design and selection: Bean search heuristics, Management Sci., 41: 767-785.
8. Vaidyanathan, J., R. Srivastava and W.C. Benton, 1998. A joint optimization of product variety and ordering approach, Computers and Operations Res., 25: 557-566.