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Ruin Probabilities of Double Compound Poisson Risk Model under Proportional Reinsurance and Interest Force

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Abstract: The researchers introduced interest force and reduced the risk of the insurance company with the proportional reinsurance under double compound Poisson risk model. Differential-Integral equations of ruin probabilities in finite and infinite time were provided. These conclusions have theoretical significance for the insurance company measuring ruin risk.

Key words: Ruin probability, poisson process, interest force, proportional reinsurance

INTRODUCTION

The heart of risk theory is ruin theory. Undoubtedly, study on ruin probability is very significant. It has directive function for insurance company considering financial prewarning system and insurance regulators designing certain monitoring index system. Mathematical risk guidance (Gerber *et al*., 1997) written by Hans U Gerber has been a mathematical classic studying ruin theory (Shixue, 2002). Researchers introduced interest rates to the classical risk model (Na and Mingqing, 2008). Researchers considered effects on survival probability caused by proportional reinsurance factors (Kelin *et al*., 2011). Researchers introduced interest force and reinsurance factors on the basis of double compound Poisson risk model (Baoliang *et al*., 2006). Therefore, researchers established an entirely new risk model and studied ruin probability of the new model.

ESTABLISHMENT OF THE MODEL

Definition: Suppose (Ω, F, P) is a complete probability space. All stochastic processes and stochastic variables in this study are defined in (Ω, F, P). Define the surplus of the insurance company at t is:

$$U(t) = ue^{\delta t} + \alpha \sum_{i=1}^{N_1(t)} X_i e^{\delta(t-s_i)} - \alpha \sum_{i=1}^{N_2(t)} Y_i^{\delta(t-T_i)}, u \geq 0, t \geq 0 \quad (1)$$

- u is the initial capital of the insurance company. $\alpha(0<\alpha<1)$ is proportional reinsurance level of insurance company. Consequently, the amount of claims the reinsurance company paying is $(1-\alpha)Y_i$
- Interest force δ is a constant, $\delta \geq 0$
- $N_1(t)$ a poisson process with parameter λ_1, the number of policies, the insurance company getting from 0-t. $N_2(t)$, a poisson process with parameter λ_2, represent the number of claims of the insurance company
- X_i represents the premiums the insurance company receiving for the ith time. $X = \{X_i, i = 1, 2, 3...\}$ is a random variable sequence of independent identical distribution. The distribution function of X_i is F(x) and F(0) = 0
- $\alpha\Sigma_{i=1}^{N_1(t)} X_i e^{\delta(t-S_i)}$ stand for total premium income at t. S_i represents the time the insurance company receiving premium for the ith time
- Y_i represents the amount of the claim for the ith time. $Y = \{Y_i, i = 1, 2, 3...\}$ is a random variable sequence of independent identical distribution. The distribution function of Y_i is G(x) and G(0) = 0. $\alpha\Sigma_{i=1}^{N_2(t)} Y_i e^{\delta(t-T_i)}$ stand for total claim at t. T_i represents the claim time for the ith time
- $X = \{X_i, i = 1, 2, 3...\}$, $Y = \{Y_i, i = 1, 2, 3...\}$, S_i, T_i, $N_1 = \{N_1(t): t \geq 0\}$ and $N_2 = \{N_2(t): t \geq 0\}$ are mutually independent

Definition: Define the ruin time $T_\delta = \inf\{t, t \geq 0, U(t)<0\}$. The ruin probability in the final with initial surplus u is $\Psi_\delta(u)$. So, the survival probability in the final is $\phi_\delta(u) = 1 - \psi_\delta(u)$. The ruin probability with initial surplus u before t is $\Psi_\delta(u, t)$. So, the survival probability before t is $\Phi_\delta(u,t) = 1 - \Psi_\delta(u, t)$. Where:

$$\Psi_\delta(u) = \Pr\{T < \infty\} = \Pr\{\bigcup_{t \geq 0}(U_\delta(t) < 0)\}$$

DIFFERENTIAL-INTEGRAL EQUATION OF RUIN PROBABILITY

Theorem 1: The survival probability of model (1) in infinite time satisfies the differential-integral equation:

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$$\Phi'_\delta(u) = \frac{\lambda_1+\lambda_2}{u\delta}\Phi_\delta(u) - \frac{\lambda_1}{u\delta}\int_0^\infty \Phi_\delta(u+x)dF(x) - \frac{\lambda_2}{u\delta}\int_0^u \Phi_\delta(u-y)dG(y)$$

Therefore, the ruin probability of model (1) in infinite time satisfies the following differential-integral equation:

$$\Psi'_\delta(u) = \frac{\lambda_1+\lambda_2}{u\delta}\Psi_\delta(u) - \frac{\lambda_1}{u\delta}\int_0^\infty \Psi_\delta(u+x)dF(x) - \frac{\lambda_2}{u\delta}\int_0^u \Psi_\delta(u-y)dG(y) - \frac{\lambda_2}{u\delta}[1-G(u)] \quad (2)$$

The survival probability of model (1) in finite time t satisfies the following differential-integral equation:

$$\frac{\partial\Phi_\delta(u,t)}{\partial t} - u\delta\frac{\partial\Phi_\delta(u,t)}{\partial u} + (\lambda_1+\lambda_2)\Phi_\delta(u,t) = \lambda_1\int_0^\infty \Phi_\delta(u+x,t)dF(x) + \lambda_2\int_0^u \Phi_\delta(u-y,t)dG(y)$$

Therefore, the ruin probability of model (1) in finite time t satisfies the following differential-integral equation:

$$u\delta\frac{\partial\Psi_\delta(u,t)}{\partial u} - \frac{\partial\Psi_\delta(u,t)}{\partial t} = (\lambda_1+\lambda_2)\Psi_\delta(u,t) - \lambda_1\int_0^\infty \Psi_\delta(u+x,t) dF(x) - \lambda_2\int_u^\infty \Psi_\delta(u-y,t)dG(y) - \lambda_2[1-G(u)] \quad (3)$$

Proof: Consider the following four kinds of situations in fully small time interval (0, Δt):

- In (0, Δt), N_1 and N_2 have no jump. In other words, claim does not happen in and the number of the insurance company receiving premium is 0 in (0, Δt). The probability of the situation is $(1-\lambda_1 \Delta t)(1-\lambda_2 \Delta t) + 0(\Delta t)$
- In (0, Δt), N_1 has one jump and N_2 has no jump. In other words, claim does not happen in and the number of the insurance company receiving premium is 1 in (0, Δt). The probability of the situation is $\lambda_1 \Delta t(1-\lambda_2 \Delta t) + 0(\Delta t)$
- In (0, Δt), N_1 has no jump and N_2 has one jump. In other words, the number of claim is 1 and the number of the insurance company receiving premium is 0 in (0, Δt). The probability of the situation is $\lambda_1 \Delta t(1-\lambda_2 \Delta t) + 0(\Delta t)$
- In (0, Δt), $N_1(N_2)$ has either two jump at least or they have jump at the same time. The probability of the situation is 0 (Δt)

Summarizing the above situations, it can be obtain:

$$\Phi_\delta(u) = [(1-\lambda_1\Delta t)(1-\lambda_2\Delta t)+0(\Delta t)]\Phi_\delta(ue^{\delta\Delta t}) + [\lambda_1\Delta t(1-\lambda_2\Delta t)+0(\Delta t)]\int_0^\infty \Phi_\delta(ue^{\delta\Delta t}+\alpha x)dF(x) + [\lambda_2\Delta t(1-\lambda_1\Delta t)+0(\Delta t)]\int_0^{\frac{ue^{\delta\Delta t}}{\alpha}} \Phi_\delta(ue^{\delta\Delta t}-\alpha y)dG(y) + 0(\Delta t) \quad (4)$$

Due to Taylor expansion, the researchers know that:

$$\Phi_\delta(ue^{\delta\Delta t}) = \Phi_\delta(u+ue^{\delta\Delta t}-u) = \Phi_\delta(u) + \Phi'_\delta(u)(ue^{\delta\Delta t}-u) + 0(\Delta t)$$

Substituting the Eq. 4 gives:

$$\Phi_\delta(u) = [(1-\lambda_1\Delta t)(1-\lambda_2\Delta t)+0(\Delta t)][\Phi_\delta(u)+\Phi'_\delta(u)(ue^{\delta\Delta t}-u)+0(\Delta t)] + [\lambda_1\Delta t(1-\lambda_2\Delta t)+0(\Delta t)]\int_0^\infty \Phi_\delta(ue^{\delta\Delta t}+\alpha x)dF(x) + [\lambda_2\Delta t(1-\lambda_1\Delta t)+0(\Delta t)]\int_0^{\frac{ue^{\delta\Delta t}}{\alpha}} \Phi_\delta(ue^{\delta\Delta t}-\alpha y)dG(y) + 0(\Delta t)$$

Using Δt dividing both sides of the above equation, the researchers obtain:

$$(\lambda_1+\lambda_2)\Phi_\delta(u) - \lambda_1\lambda_2\Delta t\Phi_\delta(u) = \frac{\Phi'_\delta(u)(ue^{\delta\Delta t}-u)}{\Delta t} - (\lambda_1+\lambda_2)\Phi'_\delta(u)(ue^{\delta\Delta t}-u) + \lambda_1\lambda_2\Delta t\Phi_\delta(u)(ue^{\delta\Delta t}-u) + \lambda_1(1-\lambda_2\Delta t)\int_0^\infty \Phi_\delta(ue^{\delta\Delta t}+\alpha x)dF(x) + \lambda_2(1-\lambda_1\Delta t)\int_0^{\frac{ue^{\delta\Delta t}}{\alpha}} \Phi_\delta(ue^{\delta\Delta t}-\alpha y)dG(y) + 0(\Delta t)$$

Let $\Delta t \to 0$, therefore:

$$(\lambda_1+\lambda_2)\Phi_\delta(u) = u\delta\Phi_\delta^{'}(u) - \lambda_1\int_0^\infty \Phi_\delta(u+\alpha x)dF(x) + \lambda_2\int_0^{\frac{u}{\alpha}}\Phi_\delta(u-\alpha y)dG(y)$$

Where:

$$\Phi_\delta^{'}(u) = \frac{\lambda_1+\lambda_2}{u\delta}\Phi_\delta(u) - \frac{\lambda_1}{u\delta}\int_0^\infty \Phi_\delta(u+\alpha x)dF(x) - \frac{\lambda_2}{u\delta}\int_0^{\frac{u}{\alpha}}\Phi_\delta(u-\alpha y)dG(y) \tag{5}$$

$\Phi'_\delta(u) = -\Psi'_\delta(u)$ due to $\Phi_\delta(u) = 1-\Psi_\delta(u)$. Applying $\Phi'_\delta(u) = -\Psi'_\delta(u)$ to Eq. 5, the researchers obtain:

$$-\Psi_\delta^{'}(u) = \frac{\lambda_1+\lambda_2}{u\delta}[1-\Psi_\delta(u)] - \frac{\lambda_1}{u\delta}\int_0^\infty [1-\Psi_\delta(u+\alpha x)]dF(x) - \frac{\lambda_2}{u\delta}\int_0^{\frac{u}{\alpha}}[1-\Psi_\delta(u-\alpha y)]dG(y)$$

Therefore:

$$\Psi_\delta^{'}(u) = \frac{\lambda_1+\lambda_2}{u\delta}\Psi_\delta(u) - \frac{\lambda_1}{u\delta}\int_0^\infty \Psi_\delta(u+\alpha x)dF(x) - \frac{\lambda_2}{u\delta}\int_0^{\frac{u}{\alpha}}\Psi_\delta(u-\alpha y)dG(y) - \frac{\lambda_2}{u\delta}\int_{\frac{u}{\alpha}}^\infty dG(y)$$

Where:

$$\Psi_\delta^{'}(u) = \frac{\lambda_1+\lambda_2}{u\delta}\Psi_\delta(u) - \frac{\lambda_1}{u\delta}\int_0^\infty \Psi_\delta(u+\alpha x)dF(x) - \frac{\lambda_2}{u\delta}\int_0^{\frac{u}{\alpha}}\Psi_\delta(u-\alpha y)dG(y) - \frac{\lambda_2}{u\delta}[1-G(\frac{u}{\alpha})]$$

Therefore, the consequence of Eq. 2 is right. And then researchers prove the sequence of Eq. 3.

$$\begin{aligned}\Phi_\delta(u,t) &= [(1-\lambda_1\Delta t)(1-\lambda_2\Delta t)+0(\Delta t)]\Phi_\delta(ue^{\delta\Delta t},t-\Delta t) + [\lambda_1\Delta t(1-\lambda_2\Delta t)+0(\Delta t)]\cdot\int_0^\infty \Phi_\delta(ue^{\delta\Delta t}+\alpha x,t-\Delta t)\\ &dF(x) + [\lambda_2\Delta t(1-\lambda_1\Delta t)+0(\Delta t)]\int_0^{\frac{ue^{\delta\Delta t}}{\alpha}}\Phi_\delta(ue^{\delta\Delta t}-\alpha y,t-\Delta t)dG(y)+0(\Delta t) = \Phi_\delta(ue^{\delta\Delta t},t-\Delta t)-(\lambda_1+\lambda_2)\\ &\Delta t\Phi_\delta(ue^{\delta\Delta t},t-\Delta t)+\lambda_1\lambda_2\Delta t^2\Phi_\delta(ue^{\delta\Delta t},t-\Delta t)+[\lambda_1\Delta t(1-\lambda_2\Delta t)+0(\Delta t)]\cdot\int_0^\infty\Phi_\delta(ue^{\delta\Delta t}+\alpha x,t-\Delta t)dF(x)+\\ &[\lambda_2\Delta t(1-\lambda_1\Delta t)+0(\Delta t)]\int_0^{\frac{ue^{\delta\Delta t}}{\alpha}}\Phi_\delta(ue^{\delta\Delta t}-\alpha y,t-\Delta t)dG(y)+0(\Delta t) = \Phi_\delta(ue^{\delta\Delta t},t-\Delta t)-(\lambda_1+\lambda_2)\Delta t\Phi_\delta\\ &(ue^{\delta\Delta t},t-\Delta t)+\lambda_1\Delta t\int_0^\infty\Phi_\delta(ue^{\delta\Delta t}+\alpha x,t-\Delta t)dF(x)+\lambda_2\Delta t\int_0^{\frac{ue^{\delta\Delta t}}{\alpha}}\Phi_\delta(ue^{\delta\Delta t}-\alpha y,t-\Delta t)dG(y)+0(\Delta t)\end{aligned}$$

Using Δt dividing both sides of the above equation, the researchers obtain:

$$\begin{aligned}\frac{\Phi_\delta(u,t)}{\Delta t} - \frac{\Phi_\delta(ue^{\delta\Delta t},t-\Delta t)}{\Delta t} &= -(\lambda_1+\lambda_2)\Phi_\delta(ue^{\delta\Delta t},t-\Delta t)+\lambda_1\int_0^\infty\Phi_\delta(ue^{\delta\Delta t}+\alpha x,t-\Delta t)dF(x)+\\ &\lambda_2\int_0^{\frac{ue^{\delta\Delta t}}{\alpha}}\Phi_\delta(ue^{\delta\Delta t}-\alpha y,t-\Delta t)dG(y)+0(\Delta t)\end{aligned}$$

Therefore:

$$\begin{aligned}\frac{\Phi_\delta(u,t)-\Phi_\delta(u,t-\Delta t)}{\Delta t}+\frac{\Phi_\delta(u,t-\Delta t)-\Phi_\delta(ue^{\delta\Delta t},t-\Delta t)}{\Delta t} &= -(\lambda_1+\lambda_2)\Phi_\delta(ue^{\delta\Delta t},t-\Delta t)+\lambda_1\int_0^\infty\Phi_\delta(ue^{\delta\Delta t}+\alpha x,t-\Delta t)dF(x)+\\ &\lambda_2\int_0^{\frac{ue^{\delta\Delta t}}{\alpha}}\Phi_\delta(ue^{\delta\Delta t}-\alpha y,t-\Delta t)dG(y)+0(\Delta t)\end{aligned}$$

Let $\Delta t \to 0$, therefore:

$$\frac{\partial\Phi_\delta(u,t)}{\partial t} - u\delta\frac{\partial\Phi_\delta(u,t)}{\partial u} + (\lambda_1+\lambda_2)\Phi_\delta(u,t) = \lambda_1\int_0^\infty\Phi_\delta(u+\alpha x,t)dF(x)+\lambda_2\int_0^{\frac{u}{\alpha}}\Phi_\delta(u-\alpha y,t)dG(y)$$

The researchers obtain $\frac{\partial\Phi_\delta(u,t)}{\partial t} = -\frac{\partial\Psi_\delta(u,t)}{\partial t}$ and $\frac{\partial\Phi_\delta(u,t)}{\partial u} = -\frac{\partial\Psi_\delta(u,t)}{\partial u}$ due to $\Phi_\delta(u,t) = 1-\Psi_\delta(u,t)$.

Therefore:

$$-\frac{\partial\Psi_{\delta}(u,t)}{\partial t}+u\delta\frac{\partial\Psi_{\delta}(u,t)}{\partial u}+(\lambda_1+\lambda_2)[1-\Psi_{\delta}(u,t)]=\lambda_1\int_0^{\infty}[1-\Psi_{\delta}(u+\alpha x,t)]dF(x)+\lambda_2\int_0^{\frac{u}{\alpha}}[1-\Psi_{\delta}(u-\alpha y,t)]dG(y)$$

Where:

$$u\delta\frac{\partial\Psi_{\delta}(u,t)}{\partial u}-\frac{\partial\Psi_{\delta}(u,t)}{\partial t}=(\lambda_1+\lambda_2)\Psi_{\delta}(u,t)-\lambda_1\int_0^{\infty}\Psi_{\delta}(u+\alpha x,t)dF(x)-\lambda_2\int_{\frac{u}{\alpha}}^{\infty}\Psi_{\delta}(u-\alpha y,t)dG(y)-\lambda_2[1-G(\frac{u}{\alpha})]$$

Therefore, the consequence of Eq. 3 is right.

CONCLUSION

In this research, the researchers considered interest force on the basis of double compound Poisson Risk Model. The researchers reduced the ruin risk of the insurance company with the proportional reinsurance. Double compound Poisson Risk Model under proportional reinsurance and interest force, a more practical model was presented.

Finally the researchers derived the Differential-Integral equation satisfied by the ruin probability of the new model. It has theoretical significance for the insurance company measuring ruin risk in complex economic environment.

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