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An Aspect of Test Statistics Based on Order Selection

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Abstract: In this study, we reformulate the test statistic based on order selection criteria as a studentized score test problem whose associated Portmanteau test problem is obtained and exploited computationally to assess the adequacy of the specification of open loop transfer function model. It is shown that the studentized version is better in term of the criteria.

Key words: Studentized score, Portmanteau test, open-loop, order selection, version, statistic

INTRODUCTION

Studentized score test has been widely used as an improvement on the existing score statistic (Koenter, 1981; Cai *et al*., 1998; Yang and Tse, 2008; Onoghojobi, 2009, 2010). The score statistic for a goodness of fit test of a generalized nonlinear model with unit dispersion is the Pearson χ^2 statistic:

$$\chi^2 = \sum_{i=1}^{n} \omega(y_i - \hat{\mu}_i)^2 / V(\hat{\mu}_i) \quad (1)$$

Where, $\hat{\mu}$ is the expected value evaluated at maximum likelihood estimator $\hat{\beta}$. Ξ be the locus of possible value of μ:

$$\Xi = \{\mu(\beta) : \beta \varepsilon \mathbb{R}^p\}$$

A number of methods for the comparism of statistical test for the open loop transfer function in time series are available in literature. These include method of Portmanteau test, originally developed by Box and Pierce (1970) and Pierce (1972) which was subsequently modified by Ljung and Box to assess the adequacy of the model of a chosen specification. We adopt the notation:

$$(1 - \phi_1 B^1 - ... - \phi_p B^P) U_t = (1 - \theta_1 B^1 - ... - \theta_q B^q)\alpha_t \quad (2)$$

For data (U_1,..., U_t) such that U_t are generated by the stationary invertible autoregressive-moving average process of order (p, q) where $B^j U_t = U_{t-j}$ and $\alpha_t \sim (0, \sigma^2)$. Of all these methods, the most commonly used is the for portmanteau test statistic. It is however, uncommon to use the method of studentized goal Portmanteau test because of difficulties inherent in it. In the present study, we attempt to overcome the difficulties associated with Portmanteau test by reformulating the diagnostic test statistics problem as a studentized goal portmanteau problem and solving same by the usual comparism procedure.

MATERIALS AND METHODS

Diagnostic test statistics: Godfrey (1979) contributed to testing the adequacy of a time series model in which he stated that the conventional approach to testing the adequacy of Box and Jenkins (1970) does not require the specification of an alternative hypothesis and is based upon the statistic (Box and Pierce, 1970):

$$\Lambda = \gamma \sum_{k=1}^{m} \hat{\omega}^2 \quad (3)$$

Where:

$$\omega_k = \sum_{t=k+1}^{n} \hat{\alpha}_t \hat{\alpha}_{t-k} / \sum_{t=1}^{n} \hat{\alpha}_t^2 \;\; (k = 1, ...,m)$$

Λ statistic would be asymptotically distributed as χ^2 with (m-p-q) degree of freedom. The unknown value of p constitutes a casualty in autoregressive modeling as attempts to under fit, increases the residuals variance while over fitting results in estimating too many parameter which eventually causes increased unreliability. Several method have be proposed for estimating some of the orders. This method is classified in two: sequential testing procedures and point estimation methods.

Sequential testing procedure were studied by Hannan (1960) and Anderson (1971). The basis for the sequential method is a comparism of AR of order k and AR of order k+r where, k is a natural number and r is a non-zero integer. This procedure is suitable when k and r are known priori if they are not then the procedure need not to applied repeatedly with different values of k and r which will not be too cumbersome. Apart from cumbersome,

Table 1: The correlogram of residual

Residual	ϕ^2	Q	χ^2	Decision
01	0.7002	33.61	30.1	Reject hypothesis/model is adequate
02	08025	38.52	38.9	Reject hypothesis/model is adequate
03	0.5970	28.66	28.9	Do not reject hypothesis/ model is not adequate
04	0.4362	20.94	28.9	Do not reject hypothesis/ model is not adequate
05	0.9342	44.84	30.1	Reject hypothesis/model is not adequate
06	0.6710	32.21	28.9	Reject hypothesis/ model is adequate
07	0.6285	30.17	30.1	Reject hypothesis/ model is adequate
08	0.7233	34.71	28.9	Reject hypothesis/ model is adequate

it is to be recognized not to be to satisfactory. Hence, these point estimation techniques are chosen. The criteria defined by Alkaike as:

$$FPF = \hat{\sigma}_P^2\left(1+\frac{P}{N}\right) \quad (4)$$

The Schwarz's criterion gave a variant of BIC:

$$SIC(P) = N\ln\hat{\sigma}_p + p\ln N \quad (5)$$

Shibata's criterion SN by assuming an infinite order suggested:

$$SN = (N+2P)\hat{\sigma}_P^2 \quad (6)$$

Alkaike Information Criterion is defined as:

$$AIC(P) = N\ln\hat{\sigma}_P + 2p \quad (7)$$

Some power calculation for the Portmanteau and Order selection test were carried out based upon which we fit first and second order autoregressive scheme to the data generated by ARMA (1, 1) model:

$$(1-\pi_1\beta)U_t = (1-\theta_1\beta)\alpha_t \quad (8)$$

The various test statistic were then compared to an appropriate χ^2 value for nominal significance level of 5%. Hence, the results for the Portmanteau test as shown in Table 1.

RESULTS

Studentised score test: Let $S(\hat{\varphi})$ which can use be to derived a standard score that is by dividing numerator and denominator by a common factor hence, we have:

$$S(\hat{\varphi}) = \frac{\Delta' T\Delta}{\xi}$$

Where;

$$\xi = \Delta' T$$

which can rewritten as;

$$S(\hat{\varphi}) = \Delta'\omega\Delta$$

Where:

$$\omega = \frac{T}{\xi}$$

Using the idea of Cai *et al.* (1998), Koenter (1981), Deng and Perron (2008), Yang and Tse (2008), Lyon and Tsai (1996), Silvias and Cribarri-Neto (2004) and Onoghojobi (2009, 2010). Let:

$$S(\hat{\varphi}) = \Delta'\omega\Delta = S(\hat{\gamma}).2\hat{\sigma}^4 \quad (9)$$

Where:

$$\text{Standard score test} = S(\hat{\gamma})$$

(Lyon and Tsai, 1996). With the following equivalents:

$$\text{Standard score test} = S(\hat{\gamma})$$

$$\Delta' = V'D'$$

$$\omega = \bar{D}'\bar{D}$$

Given all the assumptions in score and studentized score test (Lyon and Tsai, 1996; Koenter, 1981) proposed studentizing the statistic $S(\hat{\gamma})$ by replacing $2\sigma_0^4$ given in Eq. 9 with:

$$\beta = \sum_{i=0}^{n}\left(\hat{e}_0^2 - 2\hat{\sigma}_0^2\right)/n \quad (10)$$

The resulting studentised score test can be obtain by substituting β into Eq. 8 with regard for Eq. 7. Hence:

$$S(\hat{\gamma})_S = 2S(\hat{\gamma})\hat{\sigma}_0^2/\beta \quad (11)$$

Therefor, we have:

$$S(\hat{\gamma})_S = S(\hat{\omega})/\beta \quad (12)$$

Where, $S(\hat{\gamma})_s$ is the studentized score test.

DISCUSSION

Relationship between the method of testing the adequacy of a model and studentized score test: Let Eq. 2 be the portmant test for the adequacy of a model and Eq. 10 be the studentized score test to obtain Eq. 2 in form of Eq. 10, we first reduce Eq. 2 to score test as defined by $S(\hat{\omega})$.

Table 2: Autoregressive model of order p

Criteria	1 $X_t = 0.760529X_{t-1}+\varepsilon_t$	2 $X_t = 0.573967X_{t-1} +0.524202X_{t-2}+\varepsilon_t$	Remarks
FPE	962.16	949.93	Min FPE attained at AR (2)
AIC(P)	330.704	330.88	Min AIC (P) attained at AR (1)
SIC(P)	332.52	334.62	Min SIC (P) attained at AR (1)
SN	47118.5	47405.8	Min SN attained at AR (1)

After studentizing $S(\hat{\omega})$, it is obvious from Eq. 10 that the require studentized portmanteau score test can be rewritten as:

$$S(\hat{\gamma}_s) = 2\Delta'\omega\Delta / \left[\sum_{i=1}^{n} \left(\hat{e}_w^2 - \hat{\sigma}_0^2 \right) / n \right]$$

$$S(\hat{\gamma}_s) = 2\left(\sum_{i=1}^{m} \phi^2 \right)' \omega \left(\sum_{i=1}^{m} \phi^2 \right) / \left[\sum_{i=1}^{n} \left(\hat{e}_w^2 - \hat{\sigma}_0^2 \right) / n \right] \quad (13)$$

However, the example below demonstrates this relationship, thus we consider the problem solved in Table 1. The value of p in Eq. 3-6 is almost invariably unknown. The order with which the minimum criteria value s gives the best order is fitted while the best criteria is the one with the minimum valued as shown in Table 2.

Therefore, it is clear from above that FPF is the best order autocorrelation model to be fitted and it is of order 2. While the rest order criterion shows that the best order autocorrelation model to be fitted is that of order 1.

CONCLUSION

The studentized portmanteau score test provides better solution to testing the adequacy of model than the conventional portmanteau test method.

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