Journal of Modern Mathematics and Statistics 5 (1): 1-2, 2011
ISSN: 1994-5388

Flow of a Viscoelastic Fluid over a Stretching Sheet Using Method of Weighted Residuals

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Abstract: In this study, the method of weighted residuals is used to solve the problem of flow of a viscoelastic fluid over a stretching sheet. The solution procedure is simple and the obtained result is accurate.

Key words: Method of weighted residuals, viscoelastic, residual function, integral method, stretching sheet, Nigeria

INTRODUCTION

In this study, the researchers consider the solution of the flow of a second-order fluid governed by the following nonlinear ordinary differential equation:

$$f'(\eta)^2 - f(\eta)f''(\eta) = f''' - k_1[2f'(\eta)f'''(\eta) - (f''(\eta))^2 - f(\eta)f^{iv}(\eta)], \eta \in (0,\infty) \tag{1}$$

with boundary conditions:

$$f'(0) = 1,\ f(0) = 0,\ f'(\infty) = 0 \tag{2}$$

where, a prime denotes differentiation with respect to η, k_1 is a small parameter defined by:

$$k_1 = \frac{kc}{\mu}$$

Rajagopal *et al*. (1984) solved the equation by using a power series expansion cum Runge-Kutta method. In this study, we will use the method of weighted residuals to solve the problem. The obtained results show that the method is very effective and simple.

METHODS OF SOLUTION

The idea of method of weighted residuals (Table 1) is to seek an approximate solution to the differential Eq. 1 in the form:

$$f = \sum_{j=0}^{n} c_j e^{-j\eta} \tag{3}$$

where, c_j are constants to be determined. For n = 2, substitution of Eq. 3 into boundary conditions Eq. 2 and simplifying we have:

Table 1: Comparison of f' (η) solutions by method of weighted residuals (mwr) with the Runge-Kutta (RK) method for k_1 = 0.05

η	mwr	RK
0.28	0.7499394845	0.7487
0.64	0.5194001659	0.5164
1.11	0.3225629433	0.3205
2.47	0.08213315571	0.0795
4.70	0.008809909266	0.0080
6.32	0.001743053805	0.0015
8.38	0.0002221477190	0.0001

$$f = 1 + c_2 + (-1 - 2c_2)e^{-\eta} + c_2 e^{-2\eta} \tag{4}$$

We then substitute Eq. 4 into Eq. 1 to have the residual function R. The idea is to make this residual function as small as possible. This is integrated over the apposite domain. The system of these equations are then solved to determine the parameters c_j, f is then considered as the approximate solution (Aregbesola, 1996, 2003; Odejide and Aregbesola, 2006). For the ongoing problem, the residual function R is given by:

$$\begin{aligned} R = {} & (-(-1-2c_2)e^{-\eta} - 2c_2 e^{-2\eta})^2 - \\ & (1 + c_2 + (-1-2c_2)e^{-\eta} + c_2 e^{-2\eta}) \\ & ((-1-2c_2)e^{-\eta} + 4c_2 e^{-2\eta}) + \\ & (-1-2c_2)e^{-\eta} + 8c_2 e^{-2\eta}) + \\ & k_1(2(-(-1-2c_2)e^{-\eta} - 2c_2 e^{-2\eta}) \\ & (-(-1-2c_2)e^{-\eta} - 8c_2 e^{-2\eta}) - ((-1-2c_2)e^{-\eta} + \\ & 4c_2 e^{-2\eta})^2 - (1 + c_2 + (-1-2c_2)e^{-\eta} + \\ & c_2 e^{-2\eta})((-1-2c_2)e^{-\eta} + 16c_2 e^{-2\eta}))) \end{aligned} \tag{5}$$

Integrating this in the interval (0, ∞) and set the result equal to zero and solve for the constant c_2, we have:

$$c_2 = \frac{-(5k_1 - 5 + (49k_1^2 - 56k_1 + 25)^{1/2})}{2(4k_1 - 1)} \tag{6}$$

Hence, we have:

$$f = 1 - \frac{-(5k_1 - 5 + (49k_1^2 - 56k_1 + 25)^{1/2})}{2(4k_1 - 1)} + (-1 + \frac{(5k_1 - 5 + (49k_1^2 - 56k_1 + 25)^{1/2})}{(4k_1 - 1)})e^{-\eta} - \frac{(5k_1 - 5 + (49k_1^2 - 56k_1 + 25)^{1/2})}{2(4k_1 - 1)}e^{-2\eta} \quad (7)$$

The shear stress τ at the wall which is given by $\tau = (1 - k_1) f''(0)$ then becomes:

$$\tau = (1 - k_1)(-1 - \frac{(5k_1 - 5 + (49k_1^2 - 56k_1 + 25)^{1/2})}{(4k_1 - 1)} \quad (8)$$

CONCLUSION

In this study, the method of weighted residuals is used to solve the viscoelastic problem arising from a second-order fluid which is proven to be very simple but effective. The solution obtained are valid in the whole solution domain and are very accurate.

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