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An Economic Order Quantity Model for Deteriorating Items with Increasing Time Varying Demand and Cost

[1]Soumendra Kumar Patra, [2]Pradip Kumar Bala and [3]Purna Chandra Ratha
[1]Regional College of Management (Autonomous), Under AICTE,
UGC, Bijupattnaik University of Technology, Bhubaneswar, Orissa, India
[2]Xavier Institute of Management, Bhubaneswar, Orissa, India
[3]Department of Business Administration, Utkal University, Bhubaneswar, Orissa, India

Abstract: In this study, the researchers present a model for economic order quantity model for deteriorating items with increasing time varying demand and cost replacements problems with shortages. The demand rates are increasing with time over in known and finite planning horizon. The demand rate, deterioration rate, holding cost, ordering cost are assumed to be a continuous function of time. Shortages are allowed and are completely backlogged. It is described with the help of numerical example. The effect of changes in the values of different parameters on the decision variable and objective function is studied. A numerical example is given to describe the developed model. Sensitivity analysis also shown for the model.

Key words: Order-level inventory, time dependent deterioration, demand rate, deterioration rate, holding cost, ordering cost, varying demand

INTRODUCTION

An important problem confronting inventory decision makers is the design of efficient replenishment policies to keep the cost of the inventory system as low as possible. In several existing models, it is assumed that products have infinite shelf-life. However, it is known in practice that not all products (medicines, volatile liquids, blood banks, etc.) possess this characteristic. In many inventory systems, deterioration of goods in the form of direct spoilage or gradual physical decay in course of time is a realistic phenomenon and hence, it should be considered in inventory modeling. Hence, the need to study inventory systems with deterioration arises.

In this study, we are interested in finding the optimal replenishment schedule for an inventory system with shortages, in which items are deteriorating at a constant rate. The demand rates are increasing with time over a known and finite planning horizon. For inventory systems, the optimal replenishment policy usually depends on the set-up cost, holding (carrying) cost, the backorder (shortage) cost and the demand pattern. The classical approach in deterministic inventory modeling is to assume a uniform demand rate. Much study has been carried out to extend the EOQ model in order to accommodate time-varying demand patterns. Silver and Meal (1969) gave a heuristic solution procedure for the inventory model with time varying demand Donaldson (1977) exhibited a very complicated solution procedure taking demand to be linear. Ritchie (1980, 1984, 1985) obtained on exact solution for linearly trended demand. Mitra developed a simple procedure for adjusting the economic order quantity model for linearly increasing and decreasing demand.

Dave and Patel (1981) derived a lot size model for constant deterioration of items with time proportional demand. Sachan (1984) allowed shortages in Dave and Patel (1981)'s model. Related study by Bahari Kashani (1989), Deb and Chaudhuri (1987), Mudreshwar (1988), Goyal (1986), Hariga (1994), Xu and Wang (1991), Niketa and Shah (2006), Chung and Ting (1993), Hariga (1994, 1995) and Jalan *et al.* (1996) and their references. Covert and Philip (1973) developed inventory model using a two parameter Weibull distribution for deterioration of units. Philip (1974) formulated inventory model when deterioration start after some time and used a three-parameter Weibull distribution for deterioration of units. Misra (1975) extended Covert and Philip (1973)'s model for finite rate of replenishment. It is assumed in all the earlier models that the holding cost per item/per unit time and the set-up cost per order are known and constant. But the holding cost and the set up cost may not always be constant. In order to generalize EOQ models, various functions describing holding cost were introduced by

Corresponding Author: Soumendra Kumar Patra, Regional College of Management (Autonomous), Under AICTE, UGC, Bijupattnaik University of Technology, Bhubaneswar, Orissa, India

several researchers like Muhlemann and Valtis Spanopulos. In this study, the researchers have developed a generalized EOQ model for deteriorating items where the demand rate, deterioration rate, holding cost and ordering cost are all expressed as linearly increasing functions of time. Shortages in inventory are also allowed and are completely backlogged.

The assumption of time-dependent holding cost and ordering cost is justified when the price index increases with time. Deterioration rate obviously increases with the passage of time.

Model formulation

Assumptions: The model contains the following fundamental assumptions:

- An Inventory of a single item operates for a prescribed time-horizon of length 'H'
- The demand rate R (t) = a + bt, a>0, b>0, a>>b is increasing function of time
- Shortages are allowed and are completely backordered. Shortages are not allowed in the last replenishment cycle. The shortage cost is C_1 per unit short per unit of time
- C is the purchase cost per unit of the item in inventory
- The holding cost, h (t) per item per unit of time, unit is time dependent and its functional form is assumed as $h(t) = h + \alpha t$, $h>0$, $\alpha \geq 0$
- The ordering cost, K (t) depends on the total time elapsed up to the beginning of each cycle and is taken as $K(t) = K + \beta t$, $K>0$, $\beta \geq 0$
- The units in an inventory deteriorate per unit time during the period H. i.e., $\theta(t) = \theta_0+\gamma t$, $0\leq\theta(t)<1$ and $\gamma \geq 0$
- The inventory level at the end of the time-horizon H is zero
- Replenishment rate is infinite and lead time is zero
- S_j is the time point at which the inventory level in the jth-replenishment cycle drops to be zero, j = 1, 2, 3 ….., (m-1). For the last replenishment cycle, $t_m = H$
- $t_j = j(H/n)$ is the total time elapsed up to and including the jth replenishment cycle, j = 0, 1, 2, … m

Let $Q_j(t)$ be the inventory level at any time t in the jth replenishment cycle (j = 1, 2, 3, ….. m) (Fig. 1) then the instantaneous states of $Q_j(t)$ are described by the following differential equations. (This inventory is depleted by the combined effect of demand and deterioration). So $Q_j(t)$ is given by:

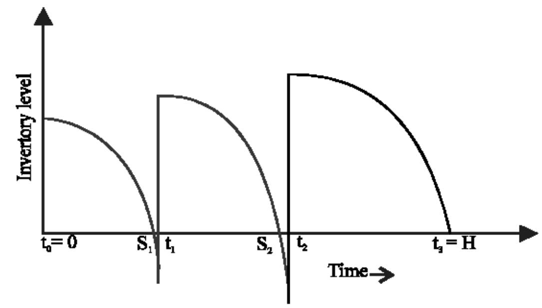

Fig. 1: Inventory level variation with time (for the case of n = 3)

$$\frac{dQ_{1j}(t)}{dt} + (\theta_0 + \gamma(t)) \times Q_{1j}(t) = -(a+bt), t_{j-1} \leq t \leq s_j,$$
$$j = 1, 2, 3, \ldots, m \tag{1}$$

With the boundary condition, $Q_{1j}(S_j) = 0$ and

$$\frac{dQ_{2j}(t)}{dt} = -(a+bt), S_j \leq t \leq t_j, j = 1, 2, 3, \ldots (m-1) \tag{2}$$

From Eq. 1, we get:

$$Q_{1j}(t) = -e^{(\theta_{0t} + \frac{\gamma^{t2}}{2})}\begin{bmatrix} a(S_j - t) + (S_j^2 - t^2)(a\theta_0 + b) + \frac{1}{3} \\ (S_j^3 - t^3)(\frac{a\gamma}{2} + b) + \frac{b\gamma}{8}(S_j^4 - t^4) \end{bmatrix} \tag{3}$$

And the general solution for Eq. 2 is shown by:

$$Q_{2j}(t) = a(S_j - t) + \frac{b}{2}(S_j^2 - t^2) \tag{4}$$

The holding cost for the jth replenishment cycle is:

$$H_j = \int_{t_{j-1}}^{s_j} h(t).Q_{1j}(t)dt, j = 1, 2, 3, \ldots m$$

$$= -h\begin{bmatrix} a\left(S_j^2 - S_j t - \frac{t^2}{2}\right) + \frac{1}{2}\left(\frac{2S_j^3}{3} - S_j^2 t + \frac{t^3}{3}\right)(a\theta_0 + b) + \\ \frac{S_j^4}{4}\left(\frac{a\gamma}{2} + b\right) + \frac{b\gamma}{8}\left(\frac{4S_j^5}{5} - S_j^4 t + \frac{t^5}{5}\right) \end{bmatrix} +$$

$$(\theta_0 h - \alpha)\begin{bmatrix} a\left(\frac{S_j^3}{6} - \frac{S_j t^2}{2} + \frac{t^3}{3}\right) + \frac{1}{2}\left(\frac{S_j^4}{4} - \frac{S_j^2 t^2}{2} + \frac{t^4}{4}\right)(a\theta_0 + b) + \\ \frac{1}{2}\left(\frac{3S_j^5}{10} - \frac{S_j^3 t^2}{2} + \frac{t^5}{5}\right)\left(\frac{a\gamma}{2} + b\right) + \frac{b\gamma}{8}\left(\frac{S_j^6}{3} - \frac{S_j^4 t^2}{2} + \frac{t^6}{6}\right) \end{bmatrix} +$$

$$\left(\frac{\gamma h}{2}+\alpha\theta_0\right)\left[a\left(\frac{S_j^4}{12}-\frac{S_jt^3}{3}+\frac{t^4}{4}\right)+\frac{1}{2}\left(\frac{2S_j^5}{5}-\frac{S_j^2t^3}{3}+\frac{t^5}{5}\right)(a\theta_0+b)+\frac{1}{3}\left(\frac{S_j^6}{6}-\frac{S_j^3t^3}{3}+\frac{t^6}{6}\right)\left(\frac{a\gamma}{2}+b\right)+\frac{b\gamma}{8}\left(\frac{4S_j^7}{21}-\frac{S_j^4t^3}{3}+\frac{t^7}{7}\right)\right]+ \\ \frac{\alpha\gamma}{2}\left[a\left(\frac{S_j^5}{20}-\frac{S_jt^4}{4}+\frac{t^5}{5}\right)+\frac{1}{2}\left(\frac{S_j^6}{12}-\frac{S_j^2t^4}{4}+\frac{t^6}{6}\right)(a\theta_0+b)+\frac{1}{3}\left(\frac{3S_j^7}{28}-\frac{S_j^3t^4}{4}+\frac{t^7}{7}\right)\left(\frac{\alpha\gamma}{2}+b\right)+\frac{b\gamma}{8}\left(\frac{S_j^8}{8}-\frac{S_j^4t^4}{4}+\frac{t^8}{8}\right)\right] \quad (5)$$

Now, the number of items deteriorated during the jth replenishment cycle is:

$$D_j = Q_{1j}(t_{j-1}) - \int_{t_{j-1}}^{s_j} D(t)dt,\ j = 1, 2, 3, \ldots\ldots m,\ D_j = Q_{1j}(t_{j-1}) - a\left(S_j - t\right) + \frac{b}{2}\left(t - S_j^{\,2}\right) \quad (6)$$

Putting the value of $Q_{1j}(t)$ in D_j in the Eq. 6, we get:

$$D_j = -e^{(\theta_0 t+\frac{\gamma t^2}{2})}\left[a(S_j - t) + (S_j^{\,2} - t^2)(a\theta_0 + b) + \frac{1}{3}(S_j^{\,3} - t^3)(\frac{a\gamma}{2} + b) + \frac{b\gamma}{8}(S_j^{\,4} - t^4)\right] - a\left(S_j - t\right) + \frac{b}{2}\left(t - S_j^{\,2}\right) \quad (7)$$

Now, the ordering cost for the jth interval is:

$$K_j = K + \beta t_{j-1}, j{=}1, 2, 3, \ldots..m$$

The total shortage over the jth replenishment cycle is:

$$S_j = \int_{s_j}^{t_j}\left(\int_{s_j}^{t}(a + bu)du\right)dt,\ j = 1, 2, 3, \ldots.(m-1),\ S_j = \left(\frac{at^2}{2} + \frac{bt^3}{6} - as_jt - \frac{bs_j^{\,2}t}{2} + \frac{aS_j^{\,2}}{2} + \frac{bs_j^{\,3}}{3}\right) \quad (8)$$

Therefore, the Total Cost (TC) of the inventory system over the time H is shown by:

$$TC = \sum_{j=1}^{n}\left(O_j + H_j + C.D_j\right) + c_1\sum_{j=1}^{n-1}s_j \quad (9)$$

$$TC = \sum_{j=1}^{n}\left[\begin{array}{l} K + \beta t_{j-1} - h\left[a\left(S_j^{\,2} - S_jt - \frac{t^2}{2}\right) + \frac{1}{2}\left(\frac{2S_j^{\,3}}{3} - S_j^{\,2}t + \frac{t^3}{3}\right)(a\theta_0 + b) + \frac{S_j^{\,4}}{4}\left(\frac{a\gamma}{2} + b\right) + \frac{b\gamma}{8}\left(\frac{4S_j^{\,5}}{5} - S_j^{\,4}t + \frac{t^5}{5}\right)\right] + \\ (\theta_0 h - \alpha)\left[a\left(\frac{S_j^{\,3}}{6} - \frac{S_jt^2}{2} + \frac{t^3}{3}\right) + \frac{1}{2}\left(\frac{S_j^{\,4}}{4} - \frac{S_j^{\,2}t^2}{2} + \frac{t^4}{4}\right)(a\theta_0 + b) + \frac{1}{2}\left(\frac{3S_j^{\,5}}{10} - \frac{S_j^{\,3}t^2}{2} + \frac{t^5}{5}\right)\left(\frac{a\gamma}{2} + b\right) + \frac{b\gamma}{8}\left(\frac{S_j^{\,6}}{3} - \frac{S_j^{\,4}t^2}{2} + \frac{t^6}{6}\right)\right] + \\ \left(\frac{\gamma h}{2} + \alpha\theta_0\right)\left[a\left(\frac{S_j^{\,4}}{12} - \frac{S_jt^3}{3} + \frac{t^4}{4}\right) + \frac{1}{2}\left(\frac{2S_j^{\,5}}{5} - \frac{S_j^{\,2}t^3}{3} + \frac{t^5}{5}\right)(a\theta_0 + b) + \frac{1}{3}\left(\frac{S_j^{\,6}}{6} - \frac{S_j^{\,3}t^3}{3} + \frac{t^6}{6}\right)\left(\frac{a\gamma}{2} + b\right) + \frac{b\gamma}{8}\left(\frac{4S_j^{\,7}}{21} - \frac{S_j^{\,4}t^3}{3} + \frac{t^7}{7}\right)\right] + \\ \frac{\alpha\gamma}{2}\left[a\left(\frac{S_j^5}{20} - \frac{S_jt^4}{4} + \frac{t^5}{5}\right) + \frac{1}{2}\left(\frac{S_j^6}{12} - \frac{S_j^{\,2}t^4}{4} + \frac{t^6}{6}\right)(a\theta_0 + b) + \frac{1}{3}\left(\frac{3S_j^7}{28} - \frac{S_j^{\,3}t^4}{4} + \frac{t^7}{7}\right)\left(\frac{a\gamma}{2} + b\right) + \frac{b\gamma}{8}\left(\frac{S_j^8}{8} - \frac{S_j^{\,4}t^4}{4} + \frac{t^8}{8}\right)\right] + \\ C\left[-e^{(\theta_0 t+\frac{\gamma t^2}{2})}\left[a(S_j - t) + (S_j^{\,2} - t^2)(a\theta_0 + b) + \frac{1}{3}(S_j^{\,3} - t^3)(\frac{a\gamma}{2} + b) + \frac{b\gamma}{8}(S_j^{\,4} - t^4)\right] - a\left(S_j - t\right) + \frac{b}{2}\left(t - S_j^{\,2}\right)\right] \end{array}\right] + \\ c_1\sum_{j=1}^{n-1}\left(\frac{at^2}{2} + \frac{bt^3}{6} - as_jt - \frac{bs_j^{\,2}t}{2} + \frac{aS_j^{\,2}}{2} + \frac{bs_j^{\,3}}{3}\right)$$

Taking;

$$\frac{dTC}{dS_J} = 0$$

$$
\begin{aligned}
&-h\left[a\left(2S_j - t_{j-1}\right) + \left(S_j - S_j t_{j-1}\right)(a\theta_0 + b) + S_j^3\left(\frac{a\gamma}{2} + b\right) + \frac{b\gamma}{2}\left(S_j^4 - S_j^3 t_{j-1}\right)\right] \\
&+(\theta_0 h - \alpha)\left[\frac{a}{2}\left(S_j^2 - t_{j-1}^2\right) + \frac{1}{2}\left(S_j^3 - S_j t_{j-1}^2\right)(a\theta_0 + b) + \frac{1}{2}\left(S_j^4 - S_j^2 t_{j-1}^2\right)\left(\frac{a\gamma}{2} + b\right) + \frac{b\gamma}{4}\left(S_j^5 - S_j^3 t_{j-1}^2\right)\right] \\
&+\left(\frac{\lambda h}{2} + \alpha\theta_0\right)\left[\frac{a}{3}\left(S_j^3 - t_{j-1}^3\right) + \left(S_j^4 - S_j \frac{t_{j-1}^3}{3}\right)(a\theta_0 + b) + \frac{1}{3}\left(S_j^5 - S_j^2 t_{j-1}^3\right)\left(\frac{a\gamma}{2} + b\right) + \frac{b\gamma}{6}\left(S_j^6 - S_j^3 t_{j-1}^3\right)\right] \\
&+\frac{\alpha\gamma}{2}\left[\frac{a}{4}\left(S_j^4 - t_{j-1}^4\right) + \frac{1}{4}\left(S_j^5 - S_j t_{j-1}^4\right)(a\theta_0 + b) + \frac{1}{4}\left(S_j^6 - S_j^2 t_{j-1}^4\right)\left(\frac{a\gamma}{2} + b\right) + \frac{b\gamma}{8}\left(S_j^7 - S_j^3 t_{j-1}^4\right)\right] \\
&-C\left[e^{-(\theta_0 t + \frac{\gamma t^2}{2})}\left\{a + S_j(a\theta_0 + b) + S_j^2\left(\frac{a\gamma}{2} + b\right) + \frac{b\gamma}{2}S_j^3 - a - b\right\}\right] \\
&-C_1\left[at_j + bS_j t_j - aS_j - bS_j^2\right] = 0
\end{aligned}
$$

COMPUTATIONAL ALGORITHIM

Researchers find optimal solution numerically using the following algorithm:

Step 1: a, b, C, C1, K, h, θ, α, β, γ and H. Set m = 2 and TC (1) =0.

Step 2: Set t_{j-1} = (j-1) H/m for j = 1, 2, 3… m.

Step 3: For j = 1, 2, 3… m-1, find S_j using Eq. 10.

Step 4: Calculate the Total Cost (TC (m)) of the inventory system using Eq. 9.

Step 5: If TC (m)<TC (m-1), then set m = m + 1 and go to step 2, otherwise go to step 6.

Step 6: Set m* = m-1, S_j * = Sj (m-1), j = 1, 2, 3, …, m*-1 and TC* = TC(m-1).

Step 7: Stop

If $C_1 \to \infty$ (i.e., shortages are not allowed) then $S_j = t_j$ for j = 1, 2, 3… m. Then m* and TC* can be determined easily following the algorithm as stated above.

NUMERICAL EXAMPLE

Consider the parameters of the inventory system as:

K = 90 h = 4 a = 10 b = 2 C = 0.5 Cl = 1.0
θ = 0.1 α = 0.1 β = 0.15 H = 10 γ = 0.001

$$0.0000125S^7_j+0.003215S^6_j+0.06675S^5_j+0.69S^4_j-7.50829S^3_j-21.844S^2_j-149.912S^2_j+81.2749 = 0 \quad (10)$$

One root of the equation i.e., S_j = 0.4998 (Table 1) shows the effect of changes in various parameters on number of orders to be placed and total cost TC of an inventory system.

- An increase in a and decrease in m and increase in j gives TC less after two points it becomes negative

Table 1: The effect of changes in verious parameters on numbers of orders to be placed and total cost TC of an inventory system

Changes in	m	j	TC
a			
7	14	1	171.59090
8	12	2	24.70674
9	10	3	-369.40642
8	18	1	194.59340
9	16	2	82.63867
10	14	3	-175.38040
b			
2	8	1	240.59840
3	12	2	31.91960
4	14	3	-172.38610
C			
0.5	8	1	240.59830
1.0	9	2	-60.37460
1.5	10	3	-412.51880
C_1			
1	8	1	240.59830
2	9	2	-60.51790
3	10	3	-410.83680
α			
0.10	8	1	240.59840
0.20	9	2	-60.55850
0.30	10	3	-411.30056
h			
2	12	1	240.54920
2.5	11	2	107.01350
3	10	3	-203.05930
4	9	4	-1090.30060
θ			
0.10	8	1	240.54920
0.11	9	2	110.63100
0.12	10	3	-23.22530
0.13	11	4	-151.19780

- An increase in b, m and j, TC decreases and it becomes negative after two points also
- An increase in C, m and j, TC decreases and it becomes negative after one point
- An increase in C_1, m and j, TC decreases and it becomes negative after one point
- An increase in α, m and j, TC decreases and it becomes negative after one point
- An Increase in h, decrease in m and increase in j, TC decreases and it becomes negative after second point
- An increase in θ, m and j, TC decreases and it becomes negative after second point

CONCLUSION

In the present study, we have formulated an EOQ model for deteriorating items over a finite time horizon H. The concluding remarks of the model are.

The deterministic demand is decreasing with time. It is usually observed in the market of electronic components like television, DVD, Freezers etc. There are also shortages.

Since almost all items undergo either direct spoilage or physical decay in the course of time, deterioration factor has an important role to play in an inventory.

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