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Necessary and Sufficient Conditions Where One Γ-Semigroup is a Γ-Group

Sabri Sadiku
Faculty of Mining and Metallurgy in Mitrovica, University of Prishtina, R. Kosova

Abstract: In this study, researchers have studied the Γ-algebraic structures and some characteristics of them. According to Sen and Saha, we defined algebraic structures: Γ-semigroup, Γ-regular semigroup, Γ-idempotent semigroup, Γ-invers semigroup and Γ-group. Theorem 2, 3 and 4 proves the existence of Γ-group and gives necessary and sufficient conditions where one Γ-semigroup is a Γ-group. Finally, theorem 5 shows necessary and sufficient conditions where one Γ-regular semigroup is a Γ-group. In addition, for every Γ- algebraic structure that we mentioned before we give an original example.

Key words: Γ-algebraic structures, Γ-semigroups, (α, β)-inverse element, α-idempotent element, Γ-semigroup idempotent, Γ-group

INTRODUCTION

Idea for Γ-algebraic structures follows from Γ-ring which notion has been introduced by Nobusawa (1964). Theory of Γ-semigroup is expanded in a natural way while as foundation we take in the theory of semigroups.

Definition 1: Let:

$$S = \{a, b, c, ...\}$$

and

$$\Gamma = \{\alpha, \beta, \gamma, ...\}$$

two nonempty sets. A mapping f: S×Γ×S→S or f: (a, α, b)→c; a, b, c ∈ S; α ∈ Γ called ternary operation in *S* and Γ. This operation we denote by $(.)_\Gamma$ or by $(+)_\Gamma$. The element (a, α, b) we denote simple by aαb. Operation f is commutative if ∀a, b ∈ S, α ∈ Γ, satisfies condition:

$$a\alpha b = b\alpha a$$

Operation f is associative if it satisfies condition:

$$(a\alpha b)\beta c = a\alpha (b\beta c);\ \forall a, b, c \in S,\ \alpha, \beta \in \Gamma$$

Definition 2: Let:

$$S = \{a, b, c, ...\}$$

and

$$\Gamma = \{\alpha, \beta, \gamma, ...\}$$

two non-empty sets. Order pair $(S, (.)_\Gamma)$ is called Γ-algebraic structure.

Definition 3: A Γ-algebraic structure $(S, (.)_\Gamma)$ is called Γ-groupoid if it satisfies condition:

(i) $\forall a, b \in S,\ \alpha, \beta \in \Gamma \Rightarrow \alpha\alpha b \in S$

Example 1: Let:

$$S = \{a = 4z + 3,\ z \in Z\} = \{...-13, -9, -5, -1, 3, 7, 11, 15, ...\}$$

$$\Gamma = \{\alpha = 4z + 1,\ z \in Z\} = \{...-11, -7, -3, 1, 5, 9, 13, ...\}$$

two sets. If a = 4z + 3, b = 4z + 3 and $\alpha = 4z_3 + 1$ where a, b ∈ S and α ∈ Γ. Then:

$$a\alpha b = 4z_1 + 3 + 4z_3 + 1 + 4z_2 + 3$$
$$= 4(z_1 + z_2 + z_3 + 1) + 3 = 4z + 3 \in S$$

Therefore $(S, (+)_\Gamma$ is Γ-groupoid where operation $(+)_r$ or aαb is addition of integers.

MATERIALS AND METHODS

In definition 4, we defined Γ-semigroup by Γ-algebraic structure and examined few examples by using some of their characteristics. Next, briefly present definition of Γ-subsemigroup (Saha, 1987), also definition of ideal in Γ-semigroup (Saha, 1988).

Definition 4: A Γ-algebraic structure $(S\ (.)_\Gamma$ is called Γ-semigroup if it satisfies condition:

(i) $\forall a, b \in S,\ \alpha \in \Gamma \Rightarrow a\alpha b \in S$

(ii) $\forall a, b, c \in S; \alpha, \beta \in \Gamma\ (a\alpha b)\beta c = a\alpha(b\beta c)$

Γ-semigroup we can definite also in this way: A Γ-groupoid $(S, (.)_\Gamma$ satisfying the associative law is a Γ-semigroup:

$$(a\alpha b)\beta c = a\alpha(b\beta c); (\forall a, b, c \in S; \alpha, \beta \in \Gamma$$

Example 2: Let S be the set of all integers of the form 6z + 1:

$$S = \{6z + 1: z \in \mathbb{Z}\} = \{..., -11, -5, 1, 7, 13, ...\}$$

and Γ be the set of all integers of the form 6z+5:

$$\Gamma = \{6z + 5 : z \in \mathbb{Z}\} = \{\cdots, -13, -7, -1, 5, 11, 17, 23, \cdots\}$$

Then order pair $(S, (+)_\Gamma)$ is a Γ-semigroup where $a\alpha b$ denotes the addition of integers.

Solution

Closure property: If

$$a = 6z_1 + 1, b = 6z_2 + 1, z_1, z_2 \in \mathbb{Z}$$

and

$$\alpha = 6z' + 5, z' \in \mathbb{Z}$$

then:

$$\begin{aligned} a\alpha b &= 6z_1 + 1 + 6z' + 5 + 6z_2 + 1 \\ &= 6(z_1 + z_2 + z' + 1) + 1 = 6z + 1 \in S \end{aligned}$$

therefore, pair $(S, (+)_\Gamma$ is a grupoid.

Associative property: If

$$a = 6z_1 + 1, b = 6z_2 + 1, z_1, z_2 \in \mathbb{Z}$$

and

$$\alpha = 6z' + 5, \beta = 6z'' + 5, z', z'' \in \mathbb{Z}$$

then:

$$\begin{aligned} (a\alpha b)\beta c &= \{6(z_1 + z_2 + z' + 1) + 1\} + 6z'' + 5 + 6z_3 + 1 \\ &= 6(z_1 + z_2 + z_3 + z' + z'' + 2) + 1 = 6z + 1 \end{aligned}$$

From the other side:

$$\begin{aligned} a\alpha(b\beta c) &= 6z_1 + 1 + 6z' + 5 + (6z_2 + 1 + 6z'' + 5 + 6z_3 + 1) \\ & 6(z_1 + z_2 + z_3 + z' + z'' + 2) + 1 = 6z + 1 \end{aligned}$$

Consequently,

$$(a\alpha b)\beta c = a\alpha(b\beta c)$$

We now prove that the pair $(S, (+)_\Gamma$ is a Γ-semigroup.

Example 3: Let *S* be the set of all matrices of type:

$$S = \left\{\begin{pmatrix} a & 0 \\ b & 1 \end{pmatrix} : a, b \in R\right\}$$

and Γ be the set of all matrices:

$$\Gamma = \left\{\begin{pmatrix} x & 0 \\ 0 & 1 \end{pmatrix} : x \in R\right\}$$

We can prove that the system $(S, (.)_\Gamma$ is Γ-semigroup, where operation $(.)_\Gamma$ is the product of the matrices.

Solution

Closure property: If

$$A_1 = \begin{pmatrix} a_1 & 0 \\ b_1 & 1 \end{pmatrix}, A_2 = \begin{pmatrix} a_2 & 0 \\ b_2 & 1 \end{pmatrix}, A_1, A_2 \in S$$

and

$$\alpha = \begin{pmatrix} x & 0 \\ 0 & 1 \end{pmatrix}, \alpha \in \Gamma$$

then:

$$\begin{aligned} A_1\alpha A_2 &= \begin{pmatrix} a_1 & 0 \\ b_1 & 1 \end{pmatrix}\begin{pmatrix} x & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a_2 & 0 \\ b_2 & 1 \end{pmatrix} \\ &= \begin{pmatrix} a_1x & 0 \\ b_1x & 1 \end{pmatrix}\begin{pmatrix} a_2 & 0 \\ b_2 & 1 \end{pmatrix} \\ &= \begin{pmatrix} a_1xa_2 & 0 \\ b_1xa_2 + b_2 & 1 \end{pmatrix} = \begin{pmatrix} a & 0 \\ b & 1 \end{pmatrix} \end{aligned}$$

where,

$$a_1xa_2 = a, b_1xa_2 + b_2 = b$$

and

$$A_1\alpha A_2 \in S$$

Associative property is true because the product of the matrices is associative and relation is evident:

$$(A_1\alpha A_2)\beta A_3 = A_1\alpha(A_2\beta A_3)$$

since the algebraic system $(S, (.)_\Gamma$ is Γ-semigroup.

Semigoup S can be considered always like Γ-semigroup if operation in S expands in S∪{1} where 11 = 1 and 1a = a1 = a, (1≠a), ∀a ∈ S. Hence, S∪{1} is semigroup with identity element. If we define ab = a1b and Γ = {1} in this case S is Γ-semigroup. Therefore, principal results of the theory of semigroups can be expanded in the theory of a Γ-semigroup.

Lemma 1: If S is a semigroup, Γ = {1} and ab = a1b then S is Γ-semigroup.

Proof: From definition 4, it is evident.

Definition 5: Let S be a Γ-semigroup. Subset M of S is Γ-subsemigroup of Γ-semigroup S if MΓM⊆M, where:

$$M\Gamma M = \{m\alpha n : m, n \in M; \alpha \in \Gamma\}$$

Other alternative way of defining Γ-subsemigroup M of semigroup S can be found by Saha (1987).

Example 4: Let S = [0. 1] and

$$\Gamma = \left\{\frac{1}{n} : n \in Z^+\right\}$$

Then S is Γ-semigroup. But subset:

$$M = \left[0, \frac{1}{2}\right]$$

is Γ- subsemigroup of Γ-semigroup S. We can see that:

$$M \subset S \text{ and } m\alpha n \in M, \forall m, n \in M; \forall \alpha \in \Gamma$$

Determination of ideal in Γ-semigroup S, although not according to the definition provided by Saha, the idea and some characteristics were taken from Saha (1988).

Definition 6: A left (right) ideal of a Γ-semigroup S is non-empty subset I of S (I⊂S, I≠ϕ) such that SΓI⊂I (IΓS⊂I). If I is both a left and right ideal, then we say that I is an ideal of S. Let Q non-empty set of S.Q is quasi-ideal of Γ-semigroup S if QΓS∩SΓQ⊆Q.

Definition 7: A Γ-semigroup S is left (right) simple if it has no proper left (right) ideal. A Γ-semigroup S is said to be simple if it has no proper ideal.

Definition 8: An element a∈S is said to be a regular in the Γ-semigroup S if a∈aΓSΓa, where:

$$a\Gamma S\Gamma a = \{(a\alpha b)\beta a : b \in S; \alpha, \beta \in \Gamma\}$$

A Γ-semigroup S is said to be regular if every element of S is regular.

Example 5: Let A = {1, 2, 3} and B = {4, 5} be two non-empty sets. Let S = {f, g, h} Γ = {α, β, γ, δ, θ, ϕ}, where f, g, h are maping from the set A to the set B and α, β, γ, δ, θ, ϕ are maping from the set B to the set A. They are defined by:

$$f = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 4 & 4 \end{pmatrix}, g = \begin{pmatrix} 1 & 2 & 3 \\ 5 & 5 & 5 \end{pmatrix}, h = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 4 & 5 \end{pmatrix}$$

and:

$$\alpha = \begin{pmatrix} 4 & 5 \\ 1 & 1 \end{pmatrix}, \beta = \begin{pmatrix} 4 & 5 \\ 2 & 2 \end{pmatrix}, \gamma = \begin{pmatrix} 4 & 5 \\ 3 & 3 \end{pmatrix},$$
$$\delta = \begin{pmatrix} 4 & 5 \\ 2 & 1 \end{pmatrix}, \theta = \begin{pmatrix} 4 & 5 \\ 2 & 3 \end{pmatrix}, \phi = \begin{pmatrix} 4 & 5 \\ 1 & 3 \end{pmatrix}$$

Where, 1f = 4, 2f = 4, 3f = 4 and similarly others. We can see that:

$$f\alpha f\delta f = f, g\alpha g\gamma g = g, h\vartheta h\phi h = h$$

For example: if 1∈A∧f∈S, we have 1f = 4. From the other side:

$$1(f\alpha f\delta f) = 1f(\alpha f\delta f) = 4\alpha(f\delta f)$$
$$= 1f(\delta f) = 4\delta(f) = 2f = 4$$

We proof fαfδf = f. Therefore, S is regular Γ-semigroup. We can give another definition about regular Γ-semigroups.

Definition 9: A Γ-semigroup S is called regular Γ-semigroup if for any a∈S exists b∈S, α, β∈Γ such that a = (aαb) βa.

Example 6: Let S be the set of all 2×3 matrices over the field and Γ be the set of all 3×2 matrices over the same field:

$$S = (a_{ij})_{2\times 3}; \Gamma = (x_{ij})_{3\times 2}$$

then S is regular Γ-semigroup where AαB (A, B∈S, α∈Γ) denote the product of the matrices. Indeed for A∈S, we can chose α∈Γ such that:

$$(A\alpha A)\alpha A = A\alpha A = A$$

Definition 10: An element e∈S is said to be an idempotent of Γ-semigroup S, if eαe = e for some α∈Γ. In this case, we call e an α-idempotent. S is a idempotent Γ-semigroup if and only if every element of S is idempotent.

Example 7: Let

$$S = \left\{E_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, E_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, E_3 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\right\} \text{and } \Gamma = \left\{\alpha = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right\}$$

Then S is a idempotent Γ-semigroup.

Definition 11: Let S be a Γ-semigroup and a∈S. If for b∈S exists α, β∈Γ such that a = (aαb)βa and b = (bβa) αb then b is called an (α, β) invers of a. In this case researcher write:

$$b \in V_\alpha^\beta(a)$$

Proposition 1: If S is a regular Γ-semigroup and a∈S, then $V_\alpha^\beta(a) \neq \emptyset$ for some α, β∈Γ.

Proof: If S is a regular Γ-semigroup, then for some a∈S exists b∈S, α, β∈Γ such that a = (aαb)βa. For element x = bβaαb∈S, we can prove that $x \in V_\alpha^\beta(a)$. Indeed:

$$(a\alpha x)\beta a = (a\alpha b\beta a\alpha b)\beta a = (a\alpha b\beta a)\alpha b\beta a = a\alpha b\beta a = a$$

and

$$x\beta a\alpha x = b\beta(a\alpha b\beta a)\alpha b\beta a\alpha b = b\beta(a\alpha b\beta a)\alpha b = b\beta a\alpha b = x$$

Then:
$$x \in V_\alpha^\beta(a)$$

Definition 12: A Γ-semigroup S is called a inverse Γ-semigroup if every element a of S has a unique (α, β)-inverse, whenever (α, β)-inverse of a exists.

Proposition 2: A regular Γ-semigroup S is a inverse Γ-semigroup if $|V_\alpha^\beta(a)| = 1$ for all a∈S and for all α, β∈Γ.

Proof: It is evident from definition 12.

Theorem 1: Let S be a inverse Γ-semigroup and a∈S, α, β∈Γ. If $\alpha^{-1} \in V_\alpha^\beta(\alpha)$, then $a\alpha a^{-1}$ is β-idempotent and $a^{-1}\beta a$ is α-idempotent of S.

Proof: If S is a inverse Γ-semigroup, then for a∈S and α, β∈Γ exists unique inverse element $\alpha^{-1} \in V_\alpha^\beta(\alpha)$ such that a = $a\alpha a^{-1}\beta a$ and $a^{-1} = a^{-1}\beta a\alpha a^{-1}$. Element $(a\alpha a^{-1})$∈S is a β-idempotent:

$$(a\alpha a^{-1})\beta(a\alpha a^{-1}) = a\alpha(a^{-1}\beta a\alpha a^{-1}) = a\alpha a^{-1}$$

and element $a^{-1}\beta a$ is a α-idempotent i S:

$$(a^{-1}\beta a)\alpha(a^{-1}\beta a) = a^{-1}\beta(a\alpha a^{-1}\beta a) = a^{-1}\beta a$$

RESULTS AND DISCUSSION

Initially we create semigroup S_α (Chinram and Siammai, 2009). In Theorem 3 and 4, researchers proof necessary and sufficient condition where semigroup G_α is a Γ-group. Furthermore, we have determined necessary and sufficient condition where regular and inverse semigroup G_α is a Γ-group. Now we will create the semigroup S_α.

Let S be a Γ-semigroup and *α* be a fixed element of Γ. If a, b∈S, define operation ∘ in S by, a∘b = aαb, ∀a, b∈S. Then, S is a semigroup.

Proof:
(i) ∀a, b∈S; α∈Γ ⇒ a∘b = aαb∈S, α-fixed element of Γ
(ii) ∀a, b, c∈S, α∈Γ we have a∘(b∘c) = a∘(bαc) = aα(bαc)
On the other hand:

$$(a \circ b) \circ c = (a\alpha b) \circ c = (a\alpha b)\alpha c = a\alpha(b\alpha c)$$

Since:
$$(a \circ b) \circ c = a \circ (b \circ c) \Rightarrow (S, \circ) \text{ is semigroup}$$

We denote this semigroup by S_α.

Example 8: If

$$S = \left\{ \begin{pmatrix} a & 0 \\ b & 1 \end{pmatrix} : a, b \in R \right\} \text{and } \alpha = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

is fixed element of Γ. Then S_α is semigroup, where operation is the product of matrices. We took definition of Γ-group from Sen and Saha (1986) which in a free form appears as:

Definition 13: A Γ-semigroup G is called a Γ-group if G_α is group for some α∈Γ (Sen and Saha, 1986).

Example 9: If S = {6z+1: z∈ ℤ } and Γ = {5} from example 2. We can see that S_5 is semigroup:

(i) $\forall a = 6z_1 + 1, b = 6z_2 + 1 \in S, \alpha = 5 \in \Gamma$

(ii)
$$a \circ b = a\alpha b = (6z_1 + 1) + 5 + (6z_2 + 1) = 6(z_1 + z_2 + 1) + 1 = 6z + 1 \in S$$

$$\begin{aligned} a \circ (b \circ c) &= [(6z_1 + 1) \circ (6z_2 + 1 + 5 + 6z_3 + 1)] \\ &= 6(z_1 + z_2 + z_3 + 2) + 1 = 6z + 1 \; (a \circ b) \circ c \\ &= [(6z_1 + 1) \circ (6z_2 + 1)] \circ (6z_3 + 1) \\ &= (6z_1 + 1 + 5 + 6z_2 + 1) \circ (6z_3 + 1) \\ &= 6\,(z_1 + z_2 + z_3 + 2) + 1 = 6z + 1 \end{aligned}$$

Consequently, S_5 is Γ-semigroup and denoted by G_5. We can see that G_5 is Γ-group. Identity element in G_5 is -5∈G_5:

$$-5 \circ (6z + 1) = -5 + 5 + 6z + 1 = 6z + 1 \in G_5$$

$$\forall a = 6z + 1 \in G_5, \exists! b = -6(z + 2) + 1 \in G_5$$

$$a \circ b = 6z + 1 + 5 + [-6(z + 2) + 1] = 6z + 6 - 6z - 12 + 1 = -5$$

Consequently, G_5 is Γ-group. Similarly, we can see that G_α is Γ-group for all α∈Γ.

Theorem 2: G_α is a Γ-group if and only if G is simple Γ- semigroup .

Proof: Suppose G is simple, then G is right simple and left simple. Let a∈G, consider the set aαG. We can show that aαG is a right ideal in G. Since G is right simple we have aαG = G. Similarly, we can show that Gαa = G. Hence, a∘G = G and G∘a = G for any a∈G. Then from the known (Ljapin, 1960) result it follows that G_α is a Γ-group.

Conversely assume that G_α is a group and e_α be the identity element in G_α. Let I be a left ideal in the Γ-semigroup G and a∈I. Then there exists b∈G such that b∘a = e_α. Hence, e_α = b∘a = bαa = ∈I. Let c∈G. Then c = c∘e_α = cαe_α∈I. This shows that G = I. Similarly, we can show that G has no proper right ideal. Hence G both left simple and right simple Γ-semigroup. Then G is simple Γ-semigroup.

Corollary 1: G_α is a Γ-group if and only if G is a Γ-semigroup and if it has not proper ideal.

Proof : From Theorem 2 and definition 6 and 7 is evident.

Theorem 3: Let G a Γ-semigroup and α∈Γ. G_α is Γ-group if and only if G does not have proper quasi-ideal.

Proof: Suppose that G does not have proper quasi-ideal. Let a∈G, consider the set aαG. Then:

$$(a\alpha G)\Gamma G \cap G\Gamma(a\alpha G) \subseteq (a\alpha G)\Gamma G = a\alpha(G\Gamma G) \subseteq a\alpha G$$

Therefore, aαG is quasi-ideal. But G does not have proper quasi-ideal, then aαG = G. Similarly we can show that Gαa = G. Hence, for every a∈G we have a∘G = G∘a = G. It shows that $G_\alpha$is a Γ-group.

Conversely, suppose that G_α is a group and Q is quasi-ideal of G. Let a∈Q, then:

$$G = G\alpha a = a\alpha G = a\alpha G \cap G\alpha a \subseteq Q\Gamma G \cap G\Gamma Q \subset Q$$

Consequently, G = Q. This shows that G does not have proper quasi-ideal.

Theorem 4: Let G be a Γ-semigroup if G_α is Γ-group for some α∈Γ then G_α is Γ-group for all α∈Γ.

Proof: Let G_α be a group. Consider the sets aβG and Gβa; a∈G, β∈Γ. Now (aβG) αG = aβ (GαG) ⊂ aβG and Gα (Gβa) = (GαG) βa ⊂ Gβa. Hence, aβG is a right ideal and Gβa is a left ideal in G_α. Since $G_\alpha$is a group we have aβG = G and Gβa = G. Then a∘G_β = G and G_β∘a = G for all a∈G. Hence, from known result it follows that G_β is a group.

Theorem 5: A regular Γ-semigroup G will be a Γ-group if and only if eαf = fαe = f and eβf = fβe = e for any two idempotents e = eαe and f = fβf of G.

Proof: Suppose G is a Γ-group. Let e = eαe and f = fβf of G be two idempotents in G. Then e is the identity element of the group G_α and f is dhe identity element of the group G_β. Now f∈G_α. Hence e∘f = f∘e (e-identity of G_α) and e∘f = eαf, f∘e = fαe. This shows that eαf = fαe = f. Similarly eβf = fβe = e. Conversely, suppose that the given condition holds in a regular Γ-semigroup G. Let a∈G. Then there exist α, β∈Γ and b∈G such that a = aα (bβa) = (aαb)βa. Let e = bβa and f = aαb. Hence eαe = e and fβf = f are two idempotents in G . We shall show that G_α is a group. Let c = (cγd)δc be an element in G where γ, δ∈Γ and d∈G. Then:

$$(c\gamma d)\delta(c\gamma d) = [(c\gamma d)\delta c]\gamma d = c\gamma d \text{ and } (d\delta c)\gamma(d\delta c) = d\delta c$$

are idempotents. Now:

$$e\alpha c = e\alpha[(c\gamma d)\delta c] = [e\alpha(c\gamma d)]\delta c = (c\gamma d)\delta c = c$$

and

$$c\alpha e = [c\gamma(d\delta c)]\alpha e = c\gamma[(d\delta c)\alpha e] = c\gamma(d\delta c) = c$$

Hence, e∘c = e∈c = c∘e = cαe = c for all. Again fβe = e. Hence, (aαb)βe = e. Then aα(bβe) = e. Also

$$(b\beta e)\alpha a = b\beta[e\alpha(a\alpha b)\beta a] = b\beta[e\alpha(f\beta a)] =$$
$$b\beta[(e\alpha f)\beta a] = b\beta(f\beta a) = b\beta a = e$$

Hence, for a there exists bβe in G_α such that:

$$a \circ (b\beta e) = (b\beta e) \circ a = e$$

Hence, G_α is a group.

Example 10: If

$$S = \left\{\begin{pmatrix} a & 0 \\ b & c \end{pmatrix} : a,b,c \in \mathbb{Z}\right\}, \Gamma = \left\{\begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} x,y \in \mathbb{Z}\right\}$$

then S is Γ-semigroup. For fixed element:

$$\alpha = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \alpha\varepsilon\Gamma$$

We can see that G_α is Γ-group. Identity element is:

$$E = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} E \circ A = E + \alpha + A = A$$

For:

$$A = \begin{pmatrix} a & 0 \\ b & c \end{pmatrix}, \exists! A^{-1} = \begin{pmatrix} -4+a & 0 \\ -b & -4+c \end{pmatrix}; A, A^{-1} \in G_\alpha : A \circ A^{-1} = A$$

Theorem 6: An invers Γ-semigroup G will be a Γ-group if and only if eαf = fαe = f and eβf = fβe = e for any two idempotents e = eαe and f = fβf of G.

Proof: Inverse Γ-semigroup G always is a regular Γ-semigroup. Hence, the remaining part of proof of Theorem 6 follows from Theorem 5.

CONCLUSION

A Γ-semigroup G is a Γ-group, if G is simple or G does not have proper quasi-ideal or ideal. A regular or invers Γ- semigroup G will be a Γ-group if G satisfies condition in theorem 5 or 6.

REFERENCES

Chinram, R. and P. Siammai, 2009. Green's Lemma and Greens's Theorem for Ã-semigroups. Lobachevskii J. Mathemat., 30: 208-213.

Ljapin, E.S., 1960. Semigroups. Fizmatigiz, Moscow.

Nobusawa, N., 1964. On a generalization of dhe ring theory. Osaka J. Math., 1: 81-89.

Saha, N.K., 1987. On Ã-semigroup-II. Calcutta Mathemat. Soc., 79: 331-335.

Saha, N.K., 1988. On Ã-semigroup III. Bull. Calcutta Mathemat. Soc., 80: 1-12.

Sen, M.K. and N.K. Saha, 1986. On Ã-semigoup- I. Calcutta Mathemat. Soc., 78: 180-186.