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On Properties of the Geometry $D_{n, n-2}$

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Abstract: We give a general case of the geometries $D_{5,3}$ and $D_{6,4}$, it is a point-line geometry of type $D_{n,n-2}$ (F), $n \geq 5$. We present a diagram and a complete definition of all varieties of such geometry to be isomorphic to the classical polar space of type Ω^+ (2n, F). This study includes a characterization of the geometry and the most important properties will be investigated; we prove that the geometry is a strong parapolar with diameter equal n-1.

Key words: Parapolar space, classical polar space, strong geometry, $D_{n,n-2}$

INTRODUCTION

Recently, in Abdelsalam (2007a, b) the point-line geometries of types $D_{5,3}$ and $D_{6,4}$ are characterized, respectively. Zayda presented a building and a complete definition of such geometries and the most important properties of them were investigated. The class of the geometries $D_{n,2}$ ($n \leq 5$), $D_{n,3}$ ($n \geq 6$) and $D_{n,4}$ ($n \geq 7$) had been studied and characterized as a point-line geometries (Abdelsalam, 2002.). The Half-spin geometry $D_{5,5}$ (F) was characterized as a group isomorphic to Ω^+ (10) by Cohen and Cooperstein (2003). Mohammed and zayda abdelsalam also were able to give the general case for the class of the geometries $D_{n,2}$ (n = 5), $D_{n,3}$ (n = 6) and $D_{n,4}$ ($n \geq 7$) by presenting a theorem which characterized, by axioms on points and lines, the geometry $D_{n,k}$ where, $k \geq 2$ and $n \geq k + 3$ and all properties of such geometry investigated (Abdelsalam, 2002). In this study, we give a general case of the 2 geometries $D_{5,3}$ and $D_{6,4}$.

First we present some definition of terminology's that will be used. For most of the following definitions (Cohen, 1984; Buekenhout and Shult, 1974).

Given a set I, a geometry Γ over I is an ordered triple $\Gamma = (X, *, D)$, where X is a set, D is a partition $\{X_i\}$ of X indexed by I, X_i are called components and $*$ is a symmetric and reflexive relation on X called incidence relation such that:

$x * y$ implies that either x and y belong to distinct components of the partition of X or x = y. Elements of X are called objects of the geometry and the objects within one component X_i of the partition are called the objects of type i. The subscripts that index the components are called types. The obvious mapping τ: $X \rightarrow I$, which takes each object to the index of the component of the partition containing it is called the type map τ.

A point-line geometry (P, L) is simply a geometry for which $|I| = 2$, one of the 2 types is called points; in this notation, the points are the members of P and the other type is called lines. Lines are the members of L. If $p \in P$ and $l \in L$, then $p * l$ if and only if $p \in l$. In point-line geometry (P, L), we say that 2 points of P are collinear if and only if they are incident with a common line (We use the symbol ~ for collinear).

The singular rank of a space Γ is the maximal number n (possibly ∞) for which there exist a chain of distinct subspaces $\varphi \neq X_0 \subset X_1 \subset ... \subset X_n$ such that X_i is singular for each i, $X_i \neq X_j$, $i \neq j$. For example rank (φ) = -1, rank ({p}) = 0 where p is a point and rank (l) = 1 where l a line.

$x^\perp$ means the set of all points in P collinear with x, including x itself.

A subspace of a point-line geometry $\Gamma = (P, L)$ is a subset $X \subseteq P$ such that any line which has at least 2 of its incident points in X has all of its incident points in X. $\langle X \rangle$ means the intersection over all subspaces containing X, where $X \subseteq P$. Lines incident with more than 2 points are called thick lines, those incident with exactly 2 points are called thin lines. In a point-line geometry Γ = (P, L), a path of length n is a sequence of n+1 ($x_0, x_1, .., x_n$) where, (x_i, x_{i+1}) are collinear, x_0 is called the initial point and x_n is called the end point. A geodesic from a point x to a point y is a path of minimal possible length with initial point x and end point y. We denote this length by d_Γ (x, y), the length of the geodesic from x to y is called the distance between x and y. The diameter of the geometry is the maximal distance of points.

A geometry Γ is called connected if and only if for any 2 of its points there is a path them. A subset X of P is said to be convex if X contains all points of all geodesics connecting 2 points of X.

A polar space is a point-line geometry Γ = (P, L) satisfying the Buekenhout-Shult axiom:

For each point-line pair (p, l) with p not incident with l, p is collinear with one or all points of l, that is $|p^{\perp}\cap l| = 1$ or else $p^{\perp}\supset l$. Clearly this axiom is equivalent to saying that $p^{\perp}$ is a geometric hyperplane of Γ for every point $p\in P$.

A point-line geometry Γ = (P, L) is called a projective plane if and only if it satisfies the following conditions (Abdelsalam, 2007b):

- Γ is a linear space; every 2 distinct points x, y in P lie exactly on one line.
- Every 2 lines intersect in one point.
- There are 4 points no 3 of them are on a line.

A point-line geometry Γ = (P, L) is called a projective space if the following conditions are satisfied:

- Every 2 points lie exactly on one line.
- If l_1, l_2 are 2 lines $l_1\cap l_2 \neq \varnothing$, then $\langle l_1, l_2\rangle$ is a projective plane. ($\langle l_1, l_2\rangle$ means the smallest subspace of Γ containing l_1 and l_2).

A point-line geometry Γ = (P, L) is called a parapolar space if and only if it satisfies the following properties:

- Γ is a connected gamma space.
- For every line l; $l^{\perp}$ is not a singular subspace.
- For every pair of non-collinear points x, y; $x^{\perp}\cap y^{\perp}$ is either empty, a single point, or a non-degenerate polar space of rank at least 2.

If x, y are distinct points in P and if $|x^{\perp}\cap y^{\perp}| = 1$, then (x, y) is called a special pair and if $x^{\perp}\cap y^{\perp}$ is a polar space, then (x, y) is called a polar pair (or a symplectic pair). A parapolar space is called a strong parapolar space if it has no special pairs.

In Fig. 1, Consider the classical polar space $\Delta = \Omega^{+}$ (2n, F) that comes from a vector space of dimension 2n over a finite field F = GF (k) with a symmetric hyperbolic bilinear form. The 2 classes M_1, M_2 consist of maximal totally isotropic n-dimensional subspaces. Two-n-subspaces fall in the same class if their intersection is of odd dimension.

The geometry of type $D_{n,\ n-2}$ (F) is the point-line geometry (P, L), whose set of points P is corresponding to the class S_{n-2} that is: the collection of all totally isotropic (n-2)-dimensional subspaces of the vector space V and whose lines are corresponding to the collection of all n-dimensional subspaces of the vector space V that are fall in the class M_1. A point C is incident to a line B if and only if $C\subset B$ as a subspaces of V.

To define the collinearity, let C_1 and C_2 be two point (the points are the T.I (n-2)-spaces), then C_1 is collinear to C_2 if and only if the intersection of C_1 and C_2 is a T.I (n-4)-dimensional space. This intersection in addition to the complement of C_1 and C_2 must form a T.I n-dimensional space. The elements of the class M_2 are geometries of type $A_{n-1,\ n-2}$ (F).

The symplecta of $D_{n,\ n-2}$ (F) are the Grassmannians of type $A_{3,\ 2}$ (F) that are corresponding to the collection of TI (n-3)-dimensional spaces.

Notation: Let the map Ψ: $P\rightarrow V$ defined above, i.e., Ψ (p) is the T.I. (n-2)-dimensional subspace corresponding to the point p. We will use Ψ for the rest of the geometry; for example Ψ ($D_{4,\ 2}$) is the T.I. (n-4)-dimensional subspace corresponding to a geometry of type $D_{4,\ 2}$ and Ψ ($D_{5,\ 3}$) is the T.I. (n-5)-dimensional subspace corresponding to a geometry of type $D_{5,\ 3}$. The inverse map Ψ^{-1} will be used for the inverse; for example Ψ^{-1} (C) is the point corresponding to the T.I. (n-2)-dimensional subspace C.

OLD RESULTS

Recently, 2 point-line geometries of types $D_{5,\ 3}$ and $D_{6,\ 4}$ has been characterized (Abdelsalam, 2007a, b) The building of the geometries has diagrams (Fig. 2):

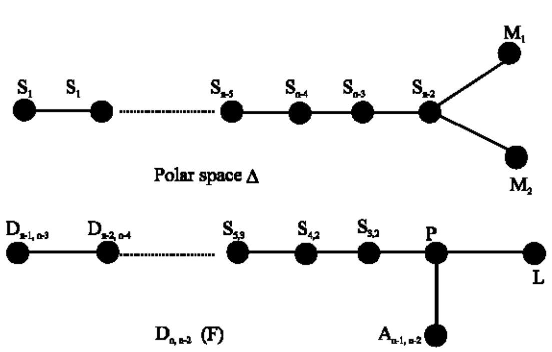

Fig. 1: Construction of $D_{n,\ n-2}$ (F)

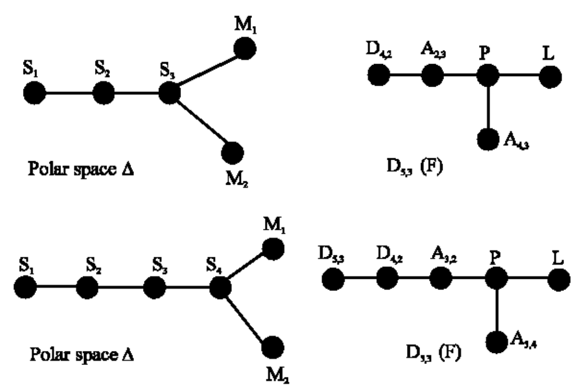

Fig. 2: The building of the geometries

and it has been proved that the geometries are strong parapolar with diameters equal 4 and 5, respectively by the following theorems:

Theorem: Let $\Gamma = (P, L)$ be a point-line geometry of type $D_{5,3}$, then the following are satisfied:

- Γ is a strong parapolar space of diameter 4.
- If (p, S) is a pair of non-incident point-symplecton, then rank $(p^{\perp} \cap S) = -1, 0, 2$.
- If S_1 and S_2 are 2 different symplecta of $D_{5,3}$ then rank $(S_1 \cap S_2) = -1, 0$.

Proof: (Abdelsalam, 2007a)

Theorem: Let $\Gamma = (P, L)$ be a point-line geometry of type $D_{6,4}$, then the following are satisfied:

- Γ is a strong parapolar space of diameter 5.
- If (p, S) is a pair of non-incident point-symplecton, then rank $(p^{\perp} \cap S) = -1, 0, 2$.
- If S_1 and S_2 are 2 different symplecta of $D_{5,3}$, then rank $(S_1 \cap S_2) = -1, 0$.

Proof: (Abdelsalam, 2007b).

THE MAIN RESULT

The following Theorems represent the first part of the main result in which we prove that the diameter of the geometry $D_{n,n-2}$ is equal n-1 and the geometry is a strong parapolar. The second part of the result, will be proved later, is to show that the relation between a point and a symplecton is either empty a point or a plane and the relation between 2 different symplecta is either empty or a point.

Theorem: let $\Gamma = (P, L)$ be the point-line geometry of type $D_{n,n-2}$ (F), then the following conditions are satisfied:

- The diameter of Γ equals n-1.
- Γ is strong geometry.

Proof: We prove that for any 2 points p and q, max {d (p, q): p, q are points} = n-1. Let $\Psi(p) = \langle x_1, x_2,\dots,x_{n-2}\rangle$, $\Psi(q) = \langle y_1, y_2,\dots,y_{n-2}\rangle$ be the corresponding of p and q, respectively. Then $\Psi(p) \cap \Psi(q)$ has the following cases:

- $\Psi(p) \cap \Psi(q)$ = TI (n-4) dimensional space, then $\Psi(p) \cap \Psi(q) = \langle u_1, u_2,\dots,u_{n-4}\rangle$ where $u_1 = x_3 = y_3$, $u_2 = x_4 = y_4,\dots,u_{n-4} = x_{n-2} = y_{n-2}$ and

$$x_1^{\perp} \cap \Psi(q) = \Psi(q)$$
$$x_2^{\perp} \cap \Psi(q) = \Psi(q)$$

Then the subspace $\langle x_1, x_2, u_1, u_2,\dots,u_{n-4}, y_1, y_2\rangle$ form the TI n-space which corresponds to the line incident to the points p and q. Then p is collinear to q and d (p, q) = 1.

- $\Psi(p) \cap \Psi(q)$ = (n-3) space, then $\Psi(p) \cap \Psi(q) = \langle u_1, u_2,\dots,u_{n-3}\rangle$ which means that p is not collinear to q. If $x_1^{\perp} \cap \Psi(q) = \Psi(q)$, then $\langle y_1, u_1, u_2,\dots,u_{n-3}, x_1\rangle$ forms a TI (n-1) space and contained in a maximal TI n-space, say $\langle y_1, u_1, u_2,\dots,u_{n-3}, x_1, u\rangle$. Then we can find many points collinear to both p and q, for this purpose select a point r such that $\Psi(r) = \langle u, x_1, y_1, u_1, u_2,\dots,u_{n-5}\rangle$, then $\Psi(r) \cap \Psi(p)$ = (n-4)–space and $\Psi(r) \cap \Psi(q)$ = (n-4)-space. Then r is collinear to both p and q, so d (p, q) = 2.
- At the following cases: $\Psi(p) \cap \Psi(q)$ = (n-5) space, $\Psi(p) \cap \Psi(q)$ = (n-6) space,..., $\Psi(p) \cap \Psi(q)$ = 1-space we get d (p, q)∩n-2.
- If $\Psi(p) \cap \Psi(q)$ = 0-space, $x_i^{\perp} \cap \Psi(q) = \Psi(q)$ and $x_j^{\perp} \cap \Psi(q) = \Psi(q)$ ($i \neq j$ and i, j = 1, 2,..., n-2), then we have d (p, q)∩n-2. If $\Psi(p) \cap \Psi(q)$ = 0-space, then we can find a geodesic of n points beginning with the point p and ending with the point q. To obtain such a geodesic let $\Psi(q)$ be contained in a maximal TI n-space $\langle y_1, y_2,\dots,y_{n-2}, u, v\rangle$ and let r_1 be the first point that is collinear to p corresponds to $\Psi(r_1) = \langle u, v, x_3,\dots,x_{n-2}\rangle$, the second point that is collinear to r_1 corresponds to $\Psi(r_2) = \langle u, v, y_1, y_2, x_5,\dots,x_{n-2}\rangle$ and we repeat the same process to get the following point at the geodesic by replacing the 2 vectors x_5 and x_6 by y_3 and y_4 to get the following point at the geodesic which is $\Psi(r_3) = \langle u, v, y_1, y_2, y_3, y_4, x_7,\dots,x_{n-2}\rangle$ finally we reach to the last point before q at the geodesic that is $\Psi(r_{n-2}) = \langle u, v, y_1, y_2, y_3, y_4,\dots,y_{n-4}\rangle$ and its collinear to the end point q. Then we get a sequence of n point of a geodesic that are p, $r_1, r_2, r_3,\dots,r_{n-2}$, q which means that d (p, q) = n-1, so, we have max{d (p, q): p, q are 2 points} = n-1.

To prove that the geometry has no special points, let p and q be 2 any point in the geometry and $\Psi(q) = \langle y_1, y_2,\dots,y_{n-2}\rangle$, $\Psi(p) = \langle x_1, x_2,\dots,x_{n-2}\rangle$ be correspondence of q and p, respectively. In part 1 of this theorem, we discussed all cases of $\Psi(p) \cap \Psi(q)$ and then at all cases of d (p, q) except d (p, q) = 2 we find that (p, q) is not a special pair.

If d (p, q) = 2, we prove that (p, q) is not also a special pair by showing that $|p^{\perp} \cap q^{\perp}| > 1$. If $\Psi(p) \cap \Psi(q)$ = (n-3)-space we can find many points such as r_1 and r_2 where $\Psi(r_1) = \langle u, x_1, y_1, u_3, u_4,\dots,u_{n-3}\rangle$ and $\Psi(r_2) = \langle u, x_1, y_1, u_1, u_2, u_5, u_6,\dots,u_{n-3}\rangle$. Then $\Psi(r_1) \cap \Psi(p)$ = (n-4)-space, $\Psi(r_1) \cap \Psi(q)$ = (n-4)-space, $\Psi(r_2) \cap \Psi(p)$ = (n-4)-space and $\Psi(r_2) \cap \Psi(q)$ = (n-4)-space. Then $|p^{\perp} \cap q^{\perp}| > 1$, so (p, q) is not a special pair which mean that $D_{n,n-2}$ is a strong geometry.

Theorem: $D_{n, n-2}$ (F) is a parapolar geometry.

Proof: The geometry $D_{n, n-2}$ is connected, 1 of Theorem 3 to show that $D_{n, n-2}$ is a gamma space, let (p, l) be a non-incidence pair of a point p and a line l such that Ψ (p) = $<x_1, x_2,..., x_{n-2}>$ and Ψ (l) = $<u_1, u_2,...,u_n>$. To be specified we must identify 2 points r and s that define the line l say, Ψ (r) = $<u_1, u_2, u_3,...,u_{n-2}>$ and Ψ (s) = $<u_3, u_4,...,u_{n-2}, u_{n-1}, u_n>$. Then the intersection Ψ (p)$\cap\Psi$ (l) has 3 cases:

- If Ψ (p)$\cap\Psi$ (l) = 0-space or 1-space,...or (n-5)-space, then there is no any (n-4)-space contained in Ψ (l) and intersect Ψ (p) in (n-4)-space which means that $p^{\perp}\cap$ l = φ.
- Ψ (p)$\cap\Psi$ (l) = (n-4)-space = $<u_3, u_4,...,u_{n-2}>$, where $x_3 = u_{3,...,}u_{n-2} = x_{n-2}$. Then $x_2^{\perp}$, $x_1^{\perp}\cap\Psi$ (l) = (n-1)-space = $<u_1, u_2,...,u_{n-2}, u_{n-1}>$. Since, Ψ (r)$\subseteq\Psi$ (l), Ψ (p)$\cap\Psi$ (r) = $<u_3, u_4,...,u_{n-2}>$ and $<x_1, x_2, u_1,...,u_{n-2}>$ is a TI n-space, p~r mean while $<x_1, x_2, u_3, u_4,...,u_{n-1}, u_n>$ is not TI n-space, then p is not collinear to s. Then $p^{\perp}\cap$ l = {r}.
- Ψ (p)$\cap\Psi$ (l) = (n-3)-space = $<u_2, u_3,...,u_{n-2}>$, $x_1^{\perp}\cap\Psi$ (l) = (n-1)-space = $<u_1, u_2,...,u_{n-1}>$. Then there is a unique point, say, t incident to the line l such that Ψ (t) = $<u_3, u_4,...,u_{n-2}, u_{n-1}, u_n>$. Since, Ψ (t)$\cap\Psi$ (p) = (n-4)-space and $<x_1, u_2,...,u_n>$ forms a TI n-space, t is collinear to p i.e., $p^{\perp}$nl = {t}. Then according to the a above cases $D_{n, n-2}$ is gamma space. The remaining part of the proof is to show that for any 2 non-collinear points p and q, $p^{\perp}\cap q^{\perp}$ is either empty, a single point, or a non-degenerate polar space of rank at least 2. By Theorems 3 and 2 we showed that for any pair of non-collinear points p and q, d (p, q) = 1, 3, or ...,or n-1 which means that $p^{\perp}$n $q^{\perp}$ is empty. For d (p, q) = 2, we proved that $p^{\perp}$n$q^{\perp}$ is a non degenerate polar space and then for any line l, $l^{\perp}$ is not singular subspace. Then $D_{n, n-2}$ is a parapolar geometry.

The following theorems presents the second part of the result as a general case of Theorems 2.1 and 2.2 in (Abdelsalam, 2007a, b).

Theorem: Let S_1 and S_2 be 2 distinct symplecta in the geometry $D_{n, n-2}$. Then rank ($S_1\cap S_2$) = -1 or 0.

Proof: Ψ (S_1) = $<x_1, x_2,...,x_{n-3}>$ and Ψ (S_2) = $<y_1, y_2,...,y_{n-3}>$ are corresponding (n-3)-spaces to the symplecta S_1 and S_2, respectively. Then we have the following cases for Ψ (S_1) $\cap$ Ψ (S_2):

- If Ψ (S_1) $\cap\Psi$ (S_2) = (n-4)-space, i.e., Ψ (S_1) $\cap\Psi$ (S_2) = $<u_1, u_2,...,u_{n-4}>$, where $u_1 = x_1 = y_1$, $u_2 = x_2 = y_2$,... and $u_{n-4} = x_{n-4} = y_{n-4}$, then if $x_{n-3}^{\perp}\cap\Psi$ (S_2) = Ψ (S_2), then the point r such that Ψ (r) = $<x_{n-3}, y_{n-3}, u_1, u_2,...,u_{n-4}>$ is contained in S_1 and S_2 which means that rank ($S_1\cap S_2$) = 0.
- If Ψ (S_1) $\cap$ Ψ (S_2) = 0-space, 1-space,..., or (n-5)-space, then there is no any TI (n-2)-space containing Ψ (S_1) and Ψ (S_2), i.e., $S_1\cap S_2 = \varphi$ and rank ($S_1\cap S_2$) = -1. Then rank ($S_1\cap S_2$) = -1 or 0.

Theorem: Let (p, S) be a non-incidence pair of a point p and a symplecton S in $D_{n, n-2}$. Then rank ($p^{\perp}\cap$S) = -1, 0 or 2.

Proof: Let Ψ (p) = $<x_1, x_2,...,x_{n-2}>$, Ψ (S) = $<y_1, y_2,...,y_{n-3}>$ be the correspondence of the point p and the symplecton S, respectively. Then there is the following cases for Ψ (p)$\cap\Psi$ (S):

- Ψ (p)$\cap\Psi$ (S) = (n-4)-space, Ψ (p)$\cap\Psi$ (S) = $<u_1, u_2,..., u_{n-4}>$ where $u_1 = x_1 = y_1$, $u_2 = x_2 = y_2$,... and $u_{n-4} = x_{n-4} = y_{n-4}$, now if $y_{n-3}^{\perp}\cap\Psi$ (p) = Ψ (p), then the subspace $<y_{n-3}, x_{n-3}, x_{n-2}, u_1, u_2,...,u_{n-4}>$ is contained in a TI n-space $<u, y_{n-3}, x_{n-3}, x_{n-2}, u_1, u_2,...,u_{n-4}>$. Then we can find a point r such that Ψ (r) = $<u, y_{n-3}, u_1, u_2,...,u_{n-4}>$. Since, Ψ (S) $\subseteq$ Ψ (r), r is a point in the symplecton S and since Ψ (r) $\cap$ Ψ (p) = (n-4)-space, r is collinear to the point p. Then $p^{\perp}\cap$ S is a point, i.e., rank ($p^{\perp}\cap$ S) = 0.
- Ψ (p)$\cap\Psi$ (S) = (n-5)-space, Ψ (p)$\cap\Psi$ (S) = $<u_1, u_2,..., u_{n-5}>$ where $u_1 = x_1 = y_1$, $u_2 = x_2 = y_2$,... and $u_{n-5} = x_{n-5} = y_{n-5}$. If $y_{n-3}^{\perp}\cap\Psi$ (p) = Ψ (p) and $y_{n-4}^{\perp}\cap\Psi$ (p) = Ψ (p), the we find 3 points r_1, r_2 and r_3 such that Ψ (r_1) = $<y_{n-3}, y_{n-4}, x_{n-4}, u_1, u_2,...,u_{n-5}>$, Ψ (r_2) = $<y_{n-3}, y_{n-4}, x_{n-3}, u_1, u_2,...,u_{n-5}>$ and Ψ (r_3) = $<y_{n-3}, y_{n-4}, x_{n-2}, u_1, u_2,...,u_{n-5}>$. Since, following: Ψ (S)$\subseteq\Psi$ (r_1), Ψ (S)$\subseteq\Psi$ (r_2) and Ψ (S)$\subseteq\Psi$ (r_3), then r_1, r_2 and r_3 are points in the symplecton S and since:

- Ψ (r_1)$\cap\Psi$ (p) = (n-4)-space.
- Ψ (r_2)$\cap\Psi$ (p) = (n-4)-space.
- Ψ (r_3)$\cap\Psi$ (p) = (n-4)-space.

Then each of point of r_1, r_2 and r_3 is collinear to the point p. Then $p^{\perp}\cap$ S is a plane, i.e., rank ($p^{\perp}\cap$ S) = 2.

If Ψ (p)$\cap\Psi$ (S) = 0-space or 1-space or,...,or (n-5)-space, then any selected (n-2)-space containing Ψ (S) must intersect Ψ (p) in 0-space, 1-space or,...,or in (n-5)-space, respectively which means that no points in S collinear to p, i.e., $p^{\perp}\cap$ S = φ. Then for the above 3 cases we have rank ($p^{\perp}$n S) = -1, 0 or 2.

Finally, Theorem 3, 4, 5 and 6 form a characterization for the geometry as follow:

Theorem: Let Γ = (P, L) be a point-line geometry of type $D_{n, n-2}$ (F), then the following are satisfied:

- Γ is a strong parapolar space of diameter n-1.
- If (p, S) is a pair of non-incident point-symplecton, then rank ($p^{\perp}\cap$ S) = -1, 0, 2.

- If S_1 and S_2 are 2 different symplecta of $D_{5,3}$, then rank ($S_1 \cap S_2$) = -1, 0.

Proof: Theorem 3, 4, 5 and 6.

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