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Synchronization of Lü System Based on Passivity Technique in Satellite CDMA System

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Abstract: This letter addresses the problem of controlled synchronization of Lü system in CDMA satellite system. A general model is studied using feedback passivity controls. The concept of semipassivity is defined to find simple conditions which ensure boundedness of the solutions of coupled Lü systems. Numerical results are presented to show the effectiveness of the proposed chaos synchronization methods.

Key words: CDMA, satellite system, sychronization methods

INTRODUCTION

Communications satellites have redefined our world. Satellites and other modern telecommunications networks, together with TV, have now altered the patterns and even many of the goals of modern society. Satellites, for better or worse, have made our world global, interconnected and interdependent. Worldwide access to rapid telecommunications networks via satellites and cables now creates widespread Internet links, enables instantaneous news coverage, facilitates global culture and conflict and stimulates the formation of true planetary markets. Satellites change our world and affect our lives[1].

The realization of the handover between different satellites and the multiple accesses is very important for a LEO satellite communication system. Both issues can be managed by using DS-CDMA (Direct Spread Code Division Multiple Access) technology. Unfortunately, the performance of a DS-CDMA system degrades with the delay of synchronization. For example, the Global star satellite system is taken as a basic model. Global star is a LEO satellite communication system with CDMA technology, which is already online[2,3]. Recently immediate synchronization has become very pertinent, due to the fact that it improves the system.

Chaos control and synchronization have drawn much attention in the last decade, for which fundamental research has recently been advanced[4]. To date, the main approach of controlling chaos is still dominated by linear feedback methodologies. The mathematical model of a chaotic system is often linearized around the desired equilibrium or target trajectory, to enable the application of the linear control theory. However, there are many limitations in applying linearization techniques: they are strictly local and usually not powerful enough for handling complex dynamics, not to mention the fact that many chaotic systems cannot be linearized. Moreover, most nonlinear systems with dimension higher than two cannot be exactly linearized via diffeomorphism and smooth feedback in terms of the Whitney topology. On the contrary, nonlinear feedback controls have many advantages, such as their global nature of effect, improvement of system transient behaviors, effectiveness in extending the regions of attraction, etc. Besides, designing a nonlinear feedback control may not be as difficult as one might imagine, if the mature nonlinear control theories are employed appropriately[5].

Many people had paid attention to passive network theory[6,7]. The passive system is a network theory concept and has dissipative network characteristics. A system's dynamical characteristic, such as stabilization, etc., can be analyzed by using passive network theory. This letter addresses the problem of controlled synchronization of Lü system in CDMA satellite system. A general model is studied using feedback passivity controls. The concept of semipassivity is defined to find simple conditions which ensure boundedness of the solutions of coupled Lü systems. Numerical result is presented to show the effectiveness of the proposed chaos synchronization methods.

Properties of passive system: Some preliminaries of passivity theory used in this paper will be shortly reviewed for the consistency of the presentation. Passivity is applied to non-linear systems which are modelled by ordinary differential equations with input vecteur u(t) and output vector y(t)[7]:

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$$\begin{aligned}\dot{x}(t) &= f(x(t),u(t)) \\ y(t) &= h(x(t))\end{aligned} \qquad (1)$$

The system (1) is dissipative with the supply rate W(u(t), y(t)) if it is not able to generate power by itself, that is the energy stored in the system is less than or equal to the supplied power:

$$\begin{aligned}&V(x(t)) \geq 0 \quad \text{and} \\ &V(x(T)) - V(x(0)) \leq \int_0^T W(u(t),y(t))dt\end{aligned} \qquad (2)$$

Furthermore, the storage function V(x(t)) must satisfy the requirements for a lyapunov function. If there exist a positive semidefinite layponov function, such that:

$$\begin{aligned}&\int u^T(\tau)y(\tau)d\tau \geq \int [\frac{\partial V(x(\tau))}{\partial x(\tau)} f(x(\tau),u(\tau)) + eu^T \\ &+\delta y^T(\tau)y(\tau) + \rho\phi(x(\tau))]d\tau\end{aligned} \qquad (3)$$

then the system (1) is passive. A passive system implies that any increase in storage energy is due solely to an external power supply.
Then the equilibrium point of the system:

$$\dot{x}(t) = f(t,x(t),0) \qquad (4)$$

is asymptotically stable in either of the two cases:

- $\rho>0$
- $e+\delta>0$ and the system is zeros-state observation.

The system (1) can be represented as the normal form:

$$\begin{aligned}\dot{x} &= f(z) + g(z,y)y \\ \dot{y} &= l(z,y) + k(z,y)u\end{aligned} \qquad (5)$$

The non linear system (5) may be rendred bay a tate feedback of the form[7]:

$$u = \alpha(x) + \beta(x)\upsilon \qquad (6)$$

Next we define a semipassive system. This notion was introduced in Pogromsky[6] and Polushin[8], an equivalent notion was called quasipassivity. Roughly speaking, a semipassive system behaves like a passive system for sufficiently large $|x|$. More precisely, assume that there exists a nonnegative function V: $\Re^n \to \Re_+$ such that for all admissible inputs, for all initial conditions and for all t for which the corresponding solution of (1) exists, we have

$$\dot{V} \leq y^T u - H(x) \qquad (7)$$

where the function H: $\Re^n \to \Re$ is nonnegative outside some ball:

$$\exists \rho > 0, \forall |x| \geq \rho \Rightarrow H(x) \geq \eta(|x|) \qquad (8)$$

For some continuous nonnegative function η defined for $|x| \geq \rho$. If the function H is positive outside some ball, i.e., (8) holds for some continuous positive function η, then the system (3) is said to be strictly semipassive.

The concept of semipassivity allows one to find simple conditions which ensure boundedness of the solutions of synchronization systems.

Consider a coupled chaotic Lü system of the form (1) as

$$\begin{cases}\dot{x}_t(t) = f(x_t) \\ y_t(t) = Cx_t\end{cases} \text{and} \quad \begin{cases}\dot{x}_r(t) = f(x_r) + Bu \\ y_r(t) = Cx_r\end{cases} \qquad (9)$$

Where: f(0) = 0 and B,C constant matrices of appropriate dimension. Define the symmetric 2*2 matrix Γ as:

$$\Gamma = \begin{vmatrix}\gamma_{11} & -\gamma_{12} \\ -\gamma_{21} & \gamma_{11}\end{vmatrix} \qquad (10)$$

$\gamma_{i,j} = \gamma_{j,i} \geq 0$ all row sums are zero. The matrix Γ is symmetric and therefore all its eigenvalues are real.

Moreover, applying Gerschgorin's theorem about localization of eigenvalues, one can see that all eigenvalues of Γ are nonnegative, that is, the matrix is positive semidefinite[6].

We say that the systems (9) are diffusively coupled if the matrix CB is similar to a positive definite matrix and the systems (9) are interconnected by the following feedback:

$$u = -\gamma_{21}(y_r - y_t) \qquad (11)$$

where $\gamma_{i,j} = \gamma_{j,i} \geq 0$ are constants.

In this case, we rewrite the systems (9) in a form which can be obtained from (9) via a linear change of coordinates due to the nonsingularity of:

$$\begin{cases}\dot{z} = q(z,y) \\ \dot{y} = a(z,y) + CBu\end{cases} \qquad (12)$$

Theorem 1 Pogromsky[6]: Consider the smooth diffusively coupled systems (9) and (12), which, because of the nonsingularity of CB are rewritten as (9) and (12). Assume the following.

Assumption 1: The system

$$\begin{cases} \dot{z} = q(z,y) \\ \dot{y} = a(z,y) + CBu \end{cases} \quad (13)$$

is strictly semipassive with respect to the input u and outputy with a radially unbounded storage function V

Assumption 2: There exists a -smooth c^2 positive definite function and a positive number α such that V_0 the following inequality is satisfied :

$$(\nabla V_0(z_1 - z_2))^T (q(z_1, y_1) - q(z_2 - y_2)) \le -\alpha |z_1 - z_2|^2 \quad (14)$$

Theorem 2[6]**:** Assume that there exists a positive definite matrix $P = P^T$ such that all eigenvalues of the symmetric matrix

$$\frac{1}{2}\left[P\left(\frac{\partial q}{\partial z}(z,\xi)\right) + P\left(\frac{\partial q}{\partial z}(z,\xi)\right)^T P \right] \quad (15)$$

are negative and separated from zero for all z ϵ $\Re$ and ξ ϵ D. Then the system z = q(z,0)) is noncritically convergent in the class ID.
We consider Lü system [9]:

$$\begin{cases} x_t' = a(y_t - x_t) \\ y_t' = -x_t z_t + c y_t \\ z_t' = x_t y_t - b x_t \end{cases} \quad (16)$$

We assume Eq. (16) to be the transmitter and we select its output as:

$$y = y_t \quad (17)$$

Then the receiver equations are

$$\begin{cases} x_r' = a(y_r - x_r) \\ y_r' = -x_r z_r + c y_r + u \\ z_r' = x_r y_r - b x_r \end{cases} \quad (18)$$

To force the two systems to synchronize, a connection is needed between the two systems. If the purpose of the control is to force $y_r \to y_t$ as $t \to \infty$, we can add control terms u into the reveiver system in the form of

$$u = -\gamma_{21}(y_r - y_t) \quad (19)$$

First, we check that the receiver system is strictly semipassive with respect to the input u and output y_r
To this end consider the smooth function:

$$V(x_r, y_r, z_r) = \frac{1}{2}(x_r^2 + y_r^2 + (z_r - a)^2) \quad (20)$$

Its time derivative with respect to the uncontrolled system satisfies:

$$\dot{V} = -a x_r^2 - y_r^2 - b\left(z_r - \frac{a}{2}\right)^2 + b\frac{(a)^2}{4} \quad (21)$$

It is seen that V =) determines an ellipsoid outside of which the derivative of V is negative. If K satisfies

$$K^2 = \frac{1}{4} + \frac{b}{4}\max\left\{\frac{1}{a}, 1\right\} \quad (22)$$

Then this ellipsoid lies inside the ball

$$\Xi = \left\{x, y, z : x_r^2 + y_r^2 + (z_r - a)^2 \le K^2 a^2\right\} \quad (23)$$

Which means that all solutions of the uncontrolled system approach within some finite time the set defined by (23). Calculating the time derivative of along solutions of the system (18) yields

$$\dot{V}(x_r, y_r, z_r, u) = \dot{V}(x_r, y_r, z_r, 0) + y_r u \quad (24)$$

Therefore, the function V is a storage function which proves strict semipassivity of the system (18) from the input to the output.

Secondly, we find the zero dynamics by imposing the external constraints $y_r = y_t$

$$\begin{cases} x_t' = a(y_t - x_t) \\ z_t' = x_t y_t - b x_t \end{cases} \text{and} \begin{cases} x_r' = a(y_t - x_t) \\ z_r' = x_r y_t - b x_r \end{cases} \quad (25)$$

Now we show that the receiver system is noncritically convergent for any bounded y_r (t). Indeed, the symmetrized Jacobi matrix for this system has two eigenvalues -a and -b and, therefore, according to Theorem 2, there exists a quadratic function which satisfies Assumption A2 of Theorem1.

Thus, all the conditions of Theorem 1 are satisfied and coupled Lü systems has an asymptotically stable compact subset of the set $\{x_t = x_r, y_t = y_r, z_t = z_r\}$.

RESULTS AND DISCUSSION

Lü System has attractor for some typical parameter values: a = 36, b = 3, c = 20. The typical Lü chaotic attractors is showing in Fig. 1.

In Fig. 2 we show the proposed schema for synchronization and dispread sequence in CDMA system.

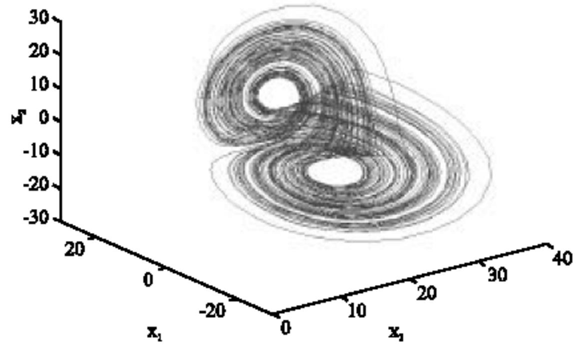

Fig. 1: The typical Lü chaotic attractors

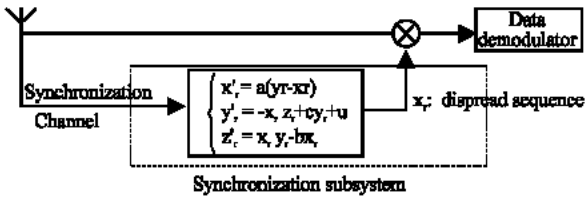

Fig. 2: Proposed schema for synchronization and dispread sequence

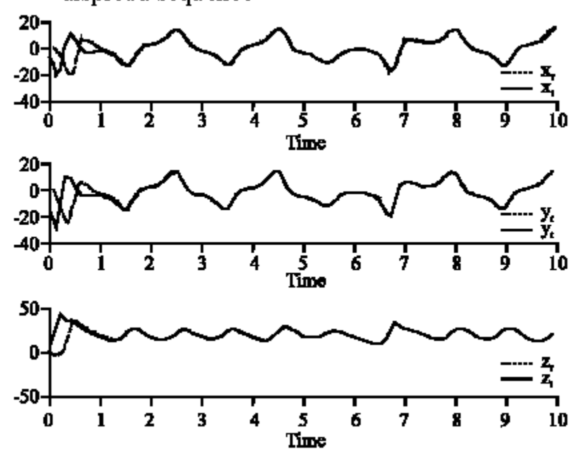

Fig. 3: Synchronization of two Lü system

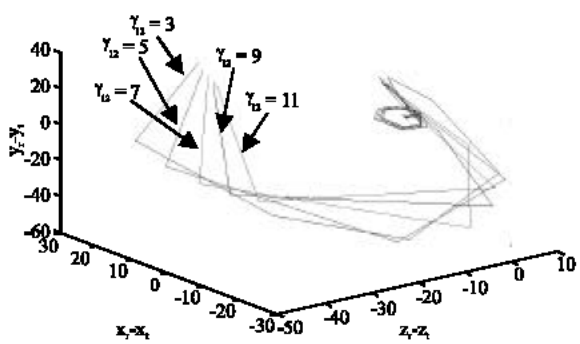

Fig. 4: Phase-space trajectory

The goal is to force the two systems to synchronize under control, while they have different initial conditions. The transmitter system stars from [6 -6 5] and the receiver system from [-3 -2 -4]. We select $\gamma_{12} = 4$ and use the controller as in (19): $u = -4(y_r-y_t)$.

In Fig. 3 and 4, we note that the system is quickly and perfectly synchronized, also the bigger γ_{12} give the best performance.

CONCLUSION

In this study, the problem of controlled synchronization of Lü system in CDMA satellite system was presented. passivity nonlinear control techniques are presented and semipassivity technique was defined to find simple conditions which ensure boundedness of the solutions of coupled systems. The Numerical result presented show that system is quickly and perfectly synchronized with the proposed method.

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