Flight Control System Based On PID Controller for Pitch of Aircraft B747

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Abstract: Today's aircraft designs rely heavily on automatic control system to monitor and control many of aircraft's subsystem. The development of this work is to model a pitch controller based an autopilot that controls the pitch of an aircraft. The PID controller and model predictive controller are developed for controlling the pitch angle of an aircraft system. Simulation results for the response of pitch controller are presented in short period aircraft. Finally, the performances of pitch control systems are investigated and analyzed based on common criteria of step's response. PID controller give the best performance, it was optimal.

INTRODUCTION

Future aircraft are expected to routinely operate in nonlinear aerodynamic flight regimes to enhance performance and maneuverability. Novel advanced control design methodologies are required to address the complex nonlinear dynamic characteristics of such vehicles. The uncertainty associated with modeling and the complexity of the nonlinear phenomena associated with high alpha flight, present the main challenges in designing flight control systems for these regimes. The conventional flight control design methods make use of linearized models and gain scheduling.

The purpose of this study presents an algorithm in which the relationship between the control unit and aerodynamics equation are improved to be useful in control system. In this study, the application of aerodynamics on aircraft control system is improved through some more analysis and modifications. The equations of motion are derived from basic force and momentum with some remarkable versions. The next work is to reduce the complex equations and calculate the coefficients which are the main keys in the control system. Since, the performance of aerodynamics designed is poor, the effective control algorithm, especially PID

control system is improved to solve these problems. The responses of these transfer functions show the state and stability condition using MATLAB programming.

Aircraft control and movement: There are three primary ways for an aircraft to change its orientation relative to the passing air. Pitch (movement of the nose up or down), Roll (rotation around the longitudinal axis, that is, the axis which runs along the length of the aircraft) and Yaw (movement of the nose to left or right). Turning the aircraft (change of heading) requires the aircraft firstly to roll to achieve an angle of bank; when the desired change of heading has been accomplished the aircraft must again be rolled in the opposite direction to reduce the angle of bank to zero.

Flight dynamics: Flight dynamics is the science of air vehicle orientation and control in three dimensions. The three critical flight dynamics parameters are the angles of rotation in three dimensions about the vehicle's center of mass, known as pitch, roll and yaw (quite different from their use as Tait-Bryan angles).

Aerospace engineers develop control systems for a vehicle's orientation (attitude) about its center of mass. The control systems include actuators which exert forces

in various directions and generate rotational forces or moments about the aerodynamic center of the aircraft and thus rotate the aircraft in pitch, roll or yaw. For example, a pitching moment is a vertical force applied at a distance[1].

Roll, pitch and yaw refer to rotations about the respective axes starting from a defined equilibrium state. The equilibrium roll angle is known as wings level or zero bank angle, equivalent to a level heeling angle on a ship. Yaw is known as "heading". The equilibrium pitch angle in submarine and airship parlance is known as "trim" but in aircraft, this usually refers to angle of attack rather than orientation. However, common usage ignores this distinction between equilibrium and dynamic cases.

The most common aeronautical convention defines the roll as acting about the longitudinal axis, positive with the starboard (right) wing down. The yaw is about the vertical body axis, positive with the nose to starboard. Pitch is about an axis perpendicular to the longitudinal plane of symmetry, positive nose up.

A fixed-wing aircraft increases or decreases the lift generated by the wings when it pitches nose up or down by increasing or decreasing the Angle of Attack (AOA). The roll angle is also known as bank angle on a fixed wing aircraft which usually "banks" to change the horizontal direction of flight. An aircraft is usually streamlined from nose to tail to reduce drag making it typically advantageous to keep the sideslip angle near zero, though there are instances when an aircraft may be deliberately "sideslipped" for example a slip in a fixed wing aircraft[2].

Theory on the qualities of aircraft flight
Longitudinal modes: Oscillating motions can be described by two parameters, the period of time required for one complete oscillation and the time required to damp to half-amplitude or the time to double the amplitude for a dynamically unstable motion. The longitudinal motion consists of two distinct oscillations, a long-period oscillation called a phugoid mode and a short-period oscillation referred to as the short-period mode.

Phugoid (longer period) oscillations: The longer period mode, called the "phugoid mode" is the one in which there is a large-amplitude variation of air-speed, pitch angle and altitude but almost no angle-of-attack variation. The phugoid oscillation is really a slow interchange of kinetic energy (velocity) and potential energy (height) about some equilibrium energy level as the aircraft attempts to re-establish the equilibrium level-flight condition from which it had been disturbed. The motion is so slow that the effects of inertia forces and damping forces are very low. Although the damping is very weak,
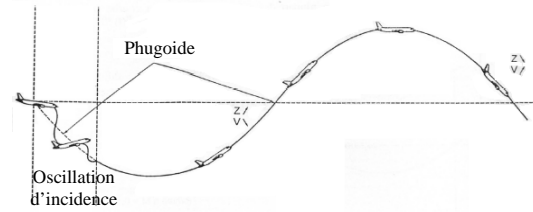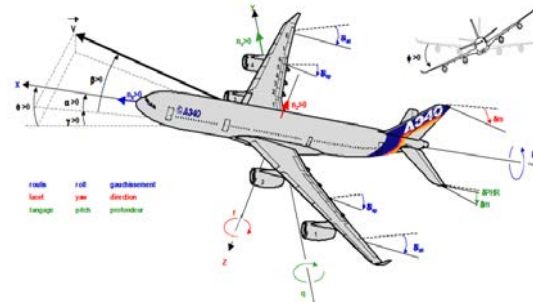

Fig. 1: Comportment of longitudinal mode

Fig. 2: Aerodynamic reference

the period is so long that the pilot usually corrects for this motion without being aware that the oscillation even exists. Typically the period is 20-60 sec[2].

Short period oscillations: With no special name, the shorter period mode is called simply the "short-period mode". The short-period mode is a usually heavily damped oscillation with a period of only a few seconds. The motion is a rapid pitching of the aircraft about the center of gravity. The period is so short that the speed does not have time to change, so, the oscillation is essentially an angle-of-attack variation. The time to damp the amplitude to one-half of its value is usually on the order of 1 sec. Ability to quickly self damp when the stick is briefly displaced is one of the many criteria for general aircraft certification[2] (Fig. 1).

Dynamics longitudinal
Equations of movements: The general equations of the movement are governed by the equations of mechanics (Fig. 2):

$$\begin{cases} m\dfrac{\overrightarrow{du}}{dt} = \sum \overrightarrow{Fe} \\[2mm] \dfrac{\overrightarrow{dc}}{dt} = \sum \overrightarrow{M_e} \end{cases} \tag{1}$$

Equation of longitudinal motion:

$$\beta = p = r = \Phi = 0 \tag{2}$$

Longitudinal equations can be rewritten as:

$$\begin{cases} \dot{u} = \dfrac{X_u}{m}u + \dfrac{X_w}{m}w - \dfrac{g\cos\Theta_0}{m}\theta + \Delta X^c \\[2mm] \dot{w} = \dfrac{Z_u}{m-Z_{\dot{w}}}u + \dfrac{Z_w}{m-Z_{\dot{w}}}w + \dfrac{Z_{q+mU_0}}{m-Z_{\dot{w}}}q - \dfrac{mg\sin\Theta_0}{m-Z_{\dot{w}}}\theta + nZ^c \\[2mm] \dot{q} = \dfrac{\left[M_u + Z_u\Gamma\right]}{I_{yy}}u + \dfrac{\left[M_u + Z_u\Gamma\right]}{I_{yy}}w + \dfrac{\left[M_q + \left(Z_q + mU_0\right)\Gamma\right]}{I_{yy}} \\[2mm] \qquad - \dfrac{mg\sin\Theta_0\Gamma}{I_{yy}}\theta + \Delta M^c \\[2mm] \dot{\theta} = q \end{cases} \qquad (3)$$

With:

$$\Delta X^c = \frac{X_{\delta_e}}{m}\delta_e + \frac{X_p}{m}\delta_p$$

$$\Delta Z^c = \frac{Z_{\delta_e}}{m-Z_{\dot{\omega}}}\delta_e + \frac{Z_{\delta_P}}{m-Z_{\dot{\omega}}}\delta_p$$

$$\Delta M^c = \frac{M_{\delta_e} + Z_{\delta_e}\Gamma}{I_{yy}}\delta_e + \frac{M_{\delta_p} + Z_{\delta_P}\Gamma}{I_{yy}}\delta_p$$

Rewrite in state space form as:

$$A = \begin{bmatrix} \dfrac{X_u}{m} & \dfrac{X_\omega}{m} & 0 & -g\cos\Theta_0 \\[2mm] \dfrac{Z_u}{m-Z_{\dot{\omega}}} & \dfrac{Z_\omega}{m-Z_{\dot{\omega}}} & \dfrac{Z_g + mU_0}{m-Z_{\dot{\omega}}} & \dfrac{-mg\sin\Theta_0}{m-Z_{\dot{\omega}}} \\[2mm] I_{yy}^{-1}\left[M_u + Z_u\Gamma\right] & I_{yy}^{-1}\left[M_u + Z_u\Gamma\right] & I_{yy}^{-1}\left[M_g + \left(Z_g + mU_0\right)\Gamma\right] & -I_{yy}^{-1}mg\sin\Theta_0\Gamma \\[2mm] 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \dfrac{X_{\delta_e}}{m} & \dfrac{X_{\delta_p}}{m} \\[2mm] \dfrac{Z_{\delta_e}}{m-Z_{\dot{\omega}}} & \dfrac{Z_{\delta_p}}{m-Z_{\dot{\omega}}} \\[2mm] \mathbf{I}_{yy}^{-1}\left[M_{\delta_E} + Z_u\Gamma\right] & \mathbf{I}_{yy}^{-1}\left[M_{\delta_p} + Z_u\Gamma\right] \\[2mm] 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad (4)$$

Since, $u \approx 0$ in this mode, then $\dot{u} \approx 0$ and can eliminate the X force equation:

$$\begin{bmatrix} \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dfrac{Z_w}{m-Z_{\dot{w}}} & \dfrac{Z_q + mU_0}{m-Z_{\dot{w}}} & \dfrac{-mg\sin\Theta_0}{m-Z_{\dot{w}}} \\[2mm] \dfrac{\left[M_w + Z_w\Gamma\right]}{I_{yy}} & \dfrac{\left[M_q + \left(Z_q + mU_0\right)\Gamma\right]}{I_{yy}} & \dfrac{-mg\sin\Theta_0}{I_{yy}} \\[2mm] 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} \Delta Z^c \\ \Delta M^c \\ 0 \end{bmatrix} \qquad (5)$$

Typically find that $Z_{\dot{w}} \leq m$ et $Z_q \leq mU_0$. Check for 747:

$$-Z_{\dot{w}} = 1909 \leq m = 2.8866*10e5$$

$$-Z_q = 4.5*10e5 \leq mU_0 = 6.8*10e7$$

$$\Gamma = \frac{M_{\dot{w}}}{m-Z_{\dot{w}}} \Rightarrow \Gamma \approx \frac{M_{\dot{w}}}{m}$$

$$\begin{bmatrix} \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dfrac{Z_w}{m} & U_0 & -g\sin\Theta_0 \\ \dfrac{\left[M_w + Z_w \dfrac{M_{\dot{w}}}{m} \right]}{I_{yy}} & \dfrac{\left[M_q + (mU_0) \dfrac{M_{\dot{w}}}{m} \right]}{I_{yy}} & \dfrac{-mg\sin\Theta_0 \dfrac{M_{\dot{w}}}{m}}{I_{yy}} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} \Delta Z^c \\ \Delta M^c \\ 0 \end{bmatrix} \tag{6}$$

Set $\Theta_0 = 0$ and remove θ from the model (it can be derived from q). With these approximations, the longitudinal Dynamics reduce to:

$$\dot{x}_{sp} = A_{sp}x_{sp} + B_{sp}\delta_e \tag{7}$$

where, δ_e is the elevator input and:

$$x_{sp} = \begin{bmatrix} w \\ q \end{bmatrix}, \quad A_{sp} = \begin{bmatrix} \dfrac{Z_w}{m} & U_0 \\ \dfrac{\left(M_w + \dfrac{M_{\dot{w}}Z_w}{m}\right)}{I_{yy}} & \dfrac{(M_q + M_{\dot{w}}U_0)}{I_{yy}} \end{bmatrix},$$

$$B_{sp} = \begin{bmatrix} \dfrac{Z_{\delta_e}}{m} \\ \dfrac{\left(M_{\delta_e} + \dfrac{M_{\dot{w}}Z_{\delta_e}}{m}\right)}{I_{yy}} \end{bmatrix} \tag{8}$$

$$s^2 + 2\xi_{sp}\omega_{sp}s + \omega_{sp}^2 = 0 \tag{9}$$

where the full approximation gives:

$$2\xi_{sp}\omega_{sp} = -\left(\frac{Z_\omega}{m} + \frac{M_q}{I_{yy}} + \frac{M_{\dot{\omega}}}{I_{yy}}U_0 \right)$$

$$\omega_{sp}^2 = \frac{Z_\omega M_q}{mI_{yy}} - \frac{U_0 M_q}{I_{yy}}$$

Given approximate magnitude of the derivatives for a typical aircraft can develop a coarse approximate:

$$\begin{cases} 2\xi_{sp}\omega_{sp} \approx -\dfrac{M_q}{I_{yy}} \\ \omega_{sp}^2 \approx -\dfrac{U_0 M_\omega}{I_{yy}} \end{cases} \Rightarrow \begin{cases} \xi_{sp} \approx -\dfrac{M_q}{2}\sqrt{\dfrac{-1}{U_0 M_\omega I_{yy}}} \\ \omega_{sp} \approx \sqrt{\dfrac{-U_0 M_\omega}{I_{yy}}} \end{cases} \tag{10}$$

Changes to ω and q are very small compared to u, so, we can:

- Set $\dot{\omega} \approx 0$ and $\dot{q} \approx 0$, Set $\Theta_0 = 0$
- $\left(Z_{\dot{w}} \leq \text{met } \Gamma \approx \dfrac{M_{\dot{\omega}}}{m} \right)$ and $\left(Z_q \approx mU_0 \right)$
- $|M_u Z_\omega| << |M_\omega Z_u|$
- $|M_\omega U_0 m| >> |M_q Z_\omega|$ et $|M_u X_\omega / M_\omega| << X_u$

With these approximations, the longitudinal dynamics reduce to the coarse approximation:

$$\dot{x}_{ph} = A_{ph}x_{ph} + B_{ph}\delta_e \tag{11}$$

And:

$$x_{ph} = \begin{bmatrix} u \\ \theta \end{bmatrix}, \quad A_{ph} = \begin{bmatrix} \dfrac{X_u}{m} & -g \\ -\dfrac{Z_u}{mU_0} & 0 \end{bmatrix} \tag{12}$$

$$B_{ph} = \begin{bmatrix} \dfrac{\left(X_{\delta_e} - \left[\dfrac{X_\omega}{M_\omega}\right] M_{\delta_e} \right)}{m} \\ \dfrac{\left(-Z_{\delta_e} + \left[\dfrac{Z_\omega}{M_\omega}\right] M_{\delta_e} \right)}{mU_0} \end{bmatrix} \tag{13}$$

Which gives:

$$\begin{cases} 2\xi_{ph}\omega_{ph} = -X_u/m \\ \omega_{ph}^2 = -\dfrac{gZ_u}{mU_0} \end{cases} \tag{14}$$

From examining Fig. 3, it can be seen that dynamical behavior of an aircraft is not acceptable considering overshoot, rise time, settling time and steady-state error values and must be modified using feedback control.

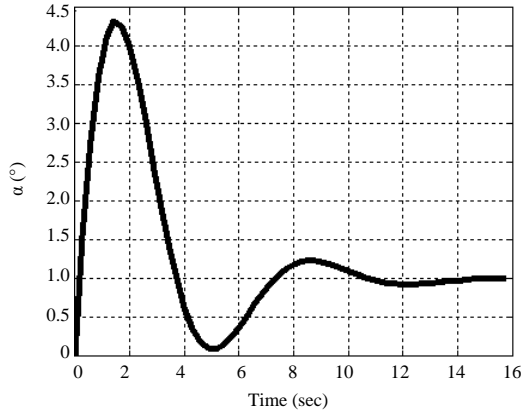$X = [u, \omega, q, \gamma]^T$ and $\gamma = \theta - \alpha$ represent flight path angle with:

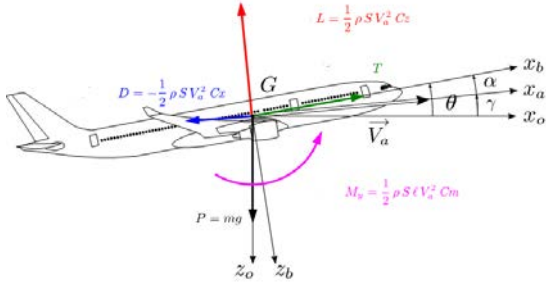Fig. 3: Open loop of pitch angle response

Fig. 4: Aerodynamics angles

$$\alpha = \omega, \ u = \begin{bmatrix} \delta_e \\ \delta_p \end{bmatrix}$$

The input (elevator deflection angle, δ_e) will be 0.2 rad (11 degrees) and the output is the pitch angle (theta) (Fig. 4).

PID controller: The components of a pitch attitude control system are shown in Fig. 5. For this design the reference pitch angle is compared with the actual angle measured by a gyro to produce an error signal to activate the control servo. In general, the error signal is amplified and sent to the control surface causes the aircraft to achieve a new pitch orientation which is fed back to close the loop. The elevator servo transfer function can be represented as a first-order system[3, 4]:

$$\frac{\Delta\delta(s)}{\Delta\delta_v(s)} = \frac{k_a}{s + \dfrac{1}{\tau}}$$

where, δe, v, K_a and τ tare the elevator deflection angle, input voltage, elevator serve gain and servomotor time constant. Time constant for typical servomotors falls in a
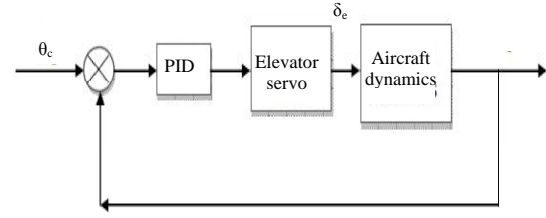

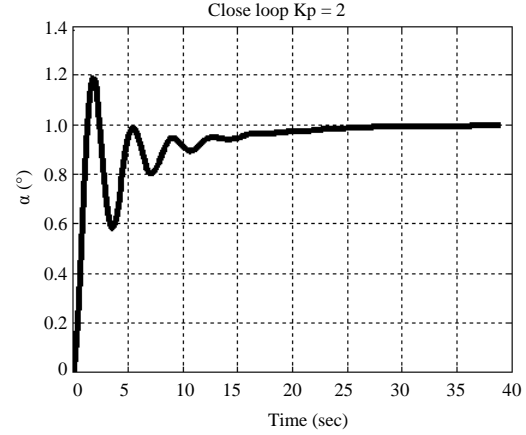Fig. 5: Pitch displacement control without rate feedback

Fig. 6: Time response for P-controller

range 0.05-0.25s. In this design, assume time constant is 0.1s and ka is -1 (Fig. 6). The transfer function of the servo actuator is:

$$G = \frac{10}{s + 8.8889}$$

The state space of the aircraft as:

$$x_{sp} = \begin{bmatrix} w \\ q \end{bmatrix}, \ A_{sp} = \begin{bmatrix} \dfrac{Z_w}{m} & U_0 \\ \dfrac{\left(M_w + \dfrac{M_{\dot w}Z_w}{m}\right)}{I_{yy}} & \dfrac{\left(M_q + M_{\dot w} U_0\right)}{I_{yy}} \end{bmatrix},$$

$$B_{sp} = \begin{bmatrix} \dfrac{Z_{\delta_e}}{m} \\ \dfrac{\left(M_{\delta_e} + \dfrac{M_{\dot w} Z_{\delta_e}}{m}\right)}{I_{yy}} \end{bmatrix} \quad (15)$$

For now, let the proportional gain (K_p) equal 2 and observe the system behavior. Enter the following commands and run it in the MATLAB command window. You should obtain the step response similar to the one shown below[5, 6].

As you see, both the overshoot and the settling time need some improvement (Fig. 7). The derivative controller will reduce both the overshoot and the settling time. Let's try a PD controller. The closed-loop transfer function of the system with a PD controller[7].
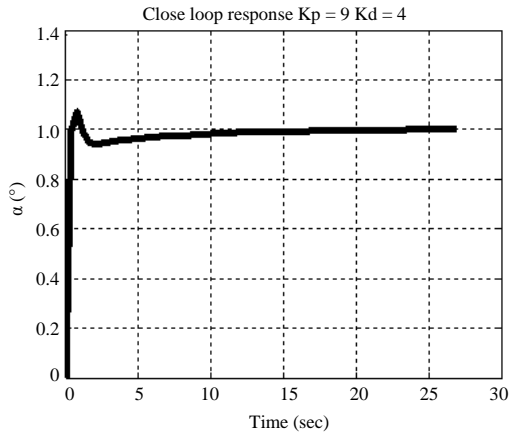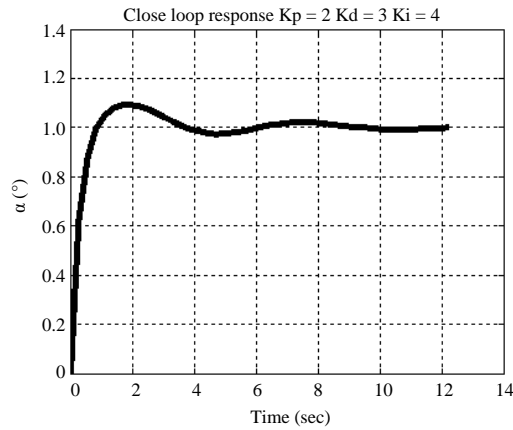
Fig. 7: Time response for PD-controller

Fig. 8: Time response for PID-controller

Table 1: Parameter of aircraft

Parameters	Values
Xu	-1.982e3
Xw	4.025e3
Zu	-2.595e4
Zw	-030e4
Zq	-4.524e5
δ_e	1.909e3
μ	1.593e4
g	9.81
M_ω	1.563e5
Mq	-1.521e7
Mwd	-1.702e4
S	511
Θ_0	0
U_0	235
$\bar{c}$	8.324

Using the commands shown below and with several trial-and-error runs, a proportional gain (K_p) of 9 and a derivative gain (K_d) of 4 provided the reasonable response. To confirm this, you should obtain the step response similar to the one shown[8].

This step response shows the rise time of <2 sec, the overshoot of <10%, the settling time of <10 sec and the steady-state error of <2%. All design requirements are satisfied (Fig. 8).

Even though all design requirements were satisfied with the PD controller, the integral controller (K_i) can be added to reduce the sharp peak and obtain a smoother response. After several trial-and-error runs, the proportional gain (K_i) of 2, the integral gain (K_i) of 4 and the derivative gain (K_d) of 3 provided a smoother step response that still satisfies all design requirements. To confirm this, enter the following commands to an m-file and run it in the command window. You should obtain the step response shown (Table 1)[9].

CONCLUSION

The overall meaning of this paper addresses the model of aerodynamics which is the input of control system driving the control surfaces. The design and calculation of this model is based on the general application of aircraft, so that, the required aerodynamics of different kinds of aircraft can be designed and improved according to these aircraft's different characteristics. It is needed to design, test and optimize a control system that can be able to control all actuators simultaneously. Similarly, the controllers for other control loops are also needed to, design and choose suitable one from P, PI and PID controllers using MATLAB. And then these control loops require to be implemented into the selected micro-controller in order to get a reliable completed On Board Computer (OBC) for the stability and purposes of aircraft system. It is also needed to pass many simulations and ground tests step by step until the system has been reached to a predefined level before trying flight test[10].

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