Journal of Engineering and Applied Sciences 15 (7): 1780-1789, 2020
ISSN: 1816-949X

Vibration Analysis of Elastically Supported Plates using Differential Quadrature Techniques

Ola Ragb and M.S. Matbuly
Department of Engineering Mathematics and Physics, Faculty of Engineering,
Zagazig University, P.O. 44519, Zagazig, Egypt

Abstract: Different schemes are examined for vibration analysis of elastically supported composite plate problems. Formulation of the problem is based on a first order transverse shear theory. Investigations are made over Winkler-Pasternak foundation model. Examined schemes are based on polynomial sinc discrete singular convolution differential quadrature methods. Numerical analysis is implemented to explore influence of different computational characteristics on convergence and accuracy of the obtained results. Further, a parametric study is introduced to investigate the influence of elastic and geometric characteristics of the vibrated plate on results.

Key words: Composite, vibration, elastic foundation, sinc, discrete singular convolution, differential quadrature

INTRODUCTION

Elastically supported plates have found significant applications in several engineering fields such as building infrastructures, tanks or silos foundations and aerospace engineering. Vibration analysis for like plates is very important for design, maintenance and structural health monitoring purposes. Due to its wide range of applications, there exists a lot of researches concerning with the research topic. The first studies were based on classical plate theory while the modern was based on transverse shear theories. These studies ranged from analytical to numerical treatments. Due to the difficulty of the problem, only few cases can be solved analytically (Akhavan *et al*., 2009; Wen, 2008; Kai *et al*., 2014; Li *et al*., 2009). So, approximate techniques such as Ritz method, finite difference, finite element, point collocation, boundary element and spectral element methods have been widely applied for such problems (Karasin *et al*., 2016; Bahmyari and Khedmati, 2017; Chakraverty and Pradhan, 2014; Moradi-dastjerdi *et al*., 2017; Tan and Zhang, 2013; Gupta *et al*., 2016; Karasin, 2016). The main disadvantage of these methods is their need for large number of grid points as well as a large computer capacity to attain a considerable accuracy (Karasin *et al*., 2016; Bahmyari and Khedmati, 2017; Chakraverty and Pradhan, 2014; Moradi-dastjerdi *et al*., 2017; Tan and Zhang, 2013; Gupta *et al*., 2016; Karasin, 2016). Further, computational ill-conditioning will be expected for such eigen-value problems.

Differential Quadrature Method (DQM) is an alternative technique for the numerical solution of differential and integral equations. Like some other approximate methods, DQM discretizes the spatial derivatives and therefore, reduces the governing equations into a standard eigenvalue problem. According to the selection of basis functions and influence domain for each point, there are more than versions of DQM. Polynomial based Differential Quadrature Method (PDQM) (Dehghan and Baradaran, 2011; Hsu, 2006; Wang and Wu, 2013), Sinc Differential Quadrature Method (SDQM) (Korkmaz and Dag, 2011; Secer, 2013; Trif, 2002) and Discrete Singular Convolution Differential Quadrature Method (DSCDQM) (Ng *et al*., 2004; Civalek and Kiracioglu, 2007; Civalek and Gurses, 2009; Civalek and Oeztuerk, 2008) are the most reliable versions.

The present work examines different schemes (PDQM, SDQM and DSCDQM) to solve vibration problems of composite plates. The plates are rested on linear elastic foundation of Winkler-Pasternak Model. The governing equations are formulated according to a first order transverse shear theory. The unknown field quantities and their derivatives are approximated using DQ approximations. The reduced eigen-value problem is solved using MATLAB. The angular frequencies and mode shapes are obtained and compared with the existing previous results. Numerical analysis is implemented to investigate convergence and efficiency of each scheme. Further a parametric study is introduced to investigate the influence of elastic and geometric characteristics of the vibrated plate on results.

Formulation of the problem: Consider a composite consisting of n plates interfacialy bonded and resting on linear elastic foundation of Winkler-Pasternak type as

Corresponding Author: Ola Ragb, Department of Engineering Mathematics and Physics, Faculty of Engineering, Zagazig University, P.O. 44519, Zagazig, Egypt

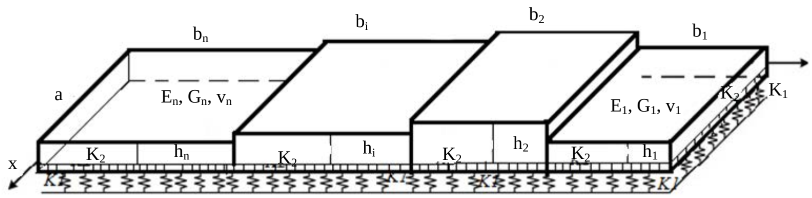

Fig. 1: Composite plate resting on Winkler-Pasternak foundation

shown in Fig. 1. Each plate occupies ($0\leq x\leq a$, $b_{i-1}\leq y\leq b_i$, $0\leq z\leq h_i$, i = 1, n) where h_i is the thickness of ith plate. b and a are width and length of the composite. Based on a first-order shear deformation theory, the equations of motion for each plate can be written as (Panc, 1975):

$$\frac{\partial M_{xx}}{\partial x}+\frac{\partial M_{xy}}{\partial y}-Q_x=I_1\frac{\partial^2\Phi_x}{\partial t^2} \quad (1)$$

$$\frac{\partial M_{xy}}{\partial x}+\frac{\partial M_{yy}}{\partial y}-Q_y=I_1\frac{\partial^2\Phi_y}{\partial t^2} \quad (2)$$

$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}+K_1 w-K_2\nabla^2 w=I_0\frac{\partial^2 w}{\partial t^2} \quad (3)$$

Where:

M_{xx}, M_{yy} and M_{xy} : The bending and twisting moment resultants
Q_x and Q_y : The shearing force resultants
I_O and I_1 : Mass moment of inertias (Reddy, 1997):

$$I_0, I_1=\int_{-h/2}^{h/2}(1, z^2)\,\rho\,dz \quad (4)$$

Where:

ρ : The plate mass density
K_1 and K_2 : Normal and shear modulus of foundation reaction
t : Time

The transverse deflection w(x, y, t) and the normal strain rotations Φ_x(x, y, t), Φ_y(x, y, t) are related to the moment and shear resultants through the following constitutive relations (Reddy, 1999):

$$\begin{bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{bmatrix}=\begin{bmatrix} -D\frac{\partial}{\partial x} & -\nu D\frac{\partial}{\partial y} \\ -\nu D\frac{\partial}{\partial x} & -D\frac{\partial}{\partial y} \\ \frac{1-\nu}{2}\frac{\partial}{\partial y} & \frac{1-\nu}{2}\frac{\partial}{\partial x} \end{bmatrix}\begin{bmatrix} \Phi_x \\ \Phi_y \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} Q_x \\ Q_y \end{bmatrix}=kGh_i\begin{bmatrix} \frac{\partial w}{\partial x} \\ \frac{\partial w}{\partial y} \end{bmatrix}-kGh_i\begin{bmatrix} \Phi_x \\ \Phi_y \end{bmatrix} \quad (6)$$

Where $D = E\,h_i^3/[12(1-v^2)]$ is the flexural rigidity of the plate. G, E and v are shear modulus, Young's modulus and Poisson's ratio of the plate. k is the shear correction factor (Liew *et al*., 2002, 2003) which is to be taken 5/6. Assuming harmonic behavior of the problem, the field quantities can be written as:

$$\Phi_x(x, y, t)=\varphi_x e^{j\omega t},\ \Phi_y(x, y, t)=\varphi_y e^{j\omega t},\ w(x, y, t)=We^{j\omega t} \quad (7)$$

where, ω is the natural frequency of the plate and $j=\sqrt{-1}$ φ_x, φ_y, W are the amplitudes for Φ_x, Φ_y and w, respectively. Substituting from Eq. 5-7 into (Eq. 1-4), one can reduce the problem to:

$$D\left(\frac{\partial^2\varphi_x}{\partial x^2}+\frac{(1-\nu)}{2}\frac{\partial^2\varphi_x}{\partial y^2}+\frac{(1+\nu)}{2}\frac{\partial^2\varphi_y}{\partial x\partial y}\right)+kGh\left(\frac{\partial W}{\partial x}-\varphi_x\right)=\omega^2 I_1\varphi_x \quad (8)$$

$$D\left(\frac{\partial^2\varphi_y}{\partial y^2}+\frac{(1-\nu)}{2}\frac{\partial^2\varphi_y}{\partial x^2}+\frac{(1+\nu)}{2}\frac{\partial^2\varphi_x}{\partial x\partial y}\right)+kGh\left(\frac{\partial W}{\partial y}-\varphi_y\right)=\omega^2 I_1\varphi_y \quad (9)$$

$$kGh\left(\begin{array}{l}\frac{\partial^2 W}{\partial x^2}+\frac{\partial^2 W}{\partial y^2}+\frac{K_1}{kGh}W-\frac{K_2}{kGh}\frac{\partial^2 W}{\partial x^2}- \\ \frac{K_2}{kGh}\frac{\partial^2 W}{\partial y^2}-\frac{\partial\varphi_x}{\partial x}-\frac{\partial\varphi_y}{\partial y}\end{array}\right)=\omega^2 I_0 W \quad (10)$$

According to the supporting type, the boundary conditions can be expressed as follows: simply supporting of the first kind: SS1:

$$W=0,\quad \overline{M}_{nn}=0,\quad \overline{M}_{ns}=0 \quad (11)$$

Simply Supporting of the second kind: SS2:

$$W=0,\quad \varphi_s=0,\quad \overline{M}_{nn}=0 \quad (12)$$

Clamped edge:

$$W=0,\quad \varphi_s=0,\quad \varphi_n=0 \quad (13)$$

Free edge:

$$\overline{Q}_n=0,\quad \overline{M}_{nn}=0,\quad \overline{M}_{ns}=0 \quad (14)$$

Where:

$$\overline{M}_{nn} = n_x^2\overline{M}_{xx} + 2n_x n_y \overline{M}_{xy} + n_y^2\overline{M}_{yy}$$

$$\overline{M}_{ns} = (n_x^2 - n_y^2)\overline{M}_{xy} + n_x n_y(\overline{M}_{yy} - \overline{M}_{xx})$$

$$\overline{Q}_n = n_x\overline{Q}_x + n_y\overline{Q}_y, \varphi_n = n_x\varphi_x + n_y\varphi_y, \varphi_s = n_x\varphi_y - n_y\varphi_x$$

n_x and n_y are the directional cosines at a point on the boundary edge. $\overline{M}_{xx}, \overline{M}_{yy}, \overline{M}_{xy}, \overline{Q}_x$ and $\overline{Q}_y$ denote the amplitudes of normal bending moments, twisting moment and shearing forces on the plate edge. Along the interface between ith plate and (i+1)th one, the continuity boundary conditions can be described as:

$$W(x, b_i^-) = W(x, b_i^+),\ \overline{M}_{xx}(x, b_i^-) = \overline{M}_{xx}(x, b_i^+)$$
$$\overline{M}_{xy}(x, b_i^-) = \overline{M}_{xy}(x, b_i^+)(i = 1, n)$$

Solution of the problem: Three different differential quadrature techniques are applied to reduce the governing equations into an eigenvalue problem as follows (Dehghan and Baradaran, 2011; Hsu, 2006; Wang and Wu, 2013; Korkmaz and Dag, 2011; Secer, 2013; Trif, 2002; Ng *et al*., 2004; Civalek and Kiracioglu, 2007; Civalek and Gurses, 2009; Civalek and Oeztuerk, 2008):

Polynomial based Differential Quadrature Method (PDQM): In this technique, Lagrange interpolation polynomial is employed as a shape function such that the unknown u and its derivatives can be approximated as a weighted linear sum of nodal values, u_i, (i = 1, N) as follows (Dehghan and Baradaran, 2011; Hsu, 2006; Wang and Wu, 2013):

$$u(x_i) = \sum_{j=1}^{N} \frac{\prod_{k=1}^{N}(x_i - x_k)}{(x_i - x_j)\prod_{j=1, j\neq k}^{N}(x_j - x_k)} u(x_j), (i = 1, N) \tag{16}$$

$$\left.\frac{\partial u}{\partial x}\right|_{x = x_i} = \sum_{j=1}^{N} C_{ij}^{x} u(x_j), \left.\frac{\partial^2 u}{\partial x^2}\right|_{x = x_i} = \sum_{j=1}^{N} C_{ij}^{xx} u(x_j),\ (i = 1, N) \tag{17}$$

Where:
u : Terms to φ_x, φ_y and W
N : The number of grid points

The weighting coefficients C_{ij}^{x}, C_{ij}^{xx} be determined by differentiating (Eq. 16) as (Dehghan and Baradaran, 2011; Hsu, 2006; Wang and Wu, 2013):

$$C_{ij}^{x} = \begin{cases} \dfrac{1}{(x_i - x_j)} \prod_{k=1, k\neq i,j}^{N} \dfrac{(x_i - x_k)}{(x_j - x_k)} & i \neq j \\ -\sum_{j=1, j\neq i}^{N} C_{ij}^{x} & i = j \end{cases} \tag{18}$$

$$C_{ij}^{xx} = \begin{cases} 2(C_{ij}^{x}.C_{ii}^{x} - \dfrac{C_{ij}^{x}}{(x_i - x_j)}) & i \neq j \\ -\sum_{j=1, j\neq i}^{N} C_{ij}^{xx} & i = j \end{cases} \tag{19}$$

Similarly, one can approximate higher order derivatives.

Sinc Differential Quadrature Method (SDQM): In this technique, sine cardinal function is employed as a shape function such that the unknown *u* and its derivatives can be approximated as a weighted linear sum of nodal values, u_i , (i = -N, N), as follows (Korkmaz and Dag, 2011; Secer, 2013; Trif, 2002):

$$u(x_i) = \sum_{j=-N}^{N} \frac{\sin[\pi(x_i - x_j)/h_x]}{\pi(x_i - x_j)/h_x} u(x_j),\quad (i = -N, N) \tag{20}$$

where, h_x is the step size. Derivatives of u can be approximated as a weighted linear sum of u_i (I = -N, N) such as (Korkmaz and Dag, 2011; Secer, 2013; Trif, 2002):

$$\left.\frac{\partial u}{\partial x}\right|_{x = x_i} = \sum_{j=-N}^{N} C_{ij}^{x} u(x_j), \left.\frac{\partial^2 u}{\partial x^2}\right|_{x = x_i} = \sum_{j=-N}^{N} C_{ij}^{xx} u(x_j), (i = -N, N) \tag{21}$$

Where:

$$C_{ij}^{x} = \begin{cases} \dfrac{(-1)^{i-j}}{h_x(i-j)}, & i \neq j \\ 0 & i = j \end{cases},\quad C_{ij}^{xx} = \begin{cases} \dfrac{2(-1)^{i-j+1}}{h_x^2(i-j)^2}, & i \neq j \\ -\dfrac{\pi^2}{3h_x^2} & i = j \end{cases} \tag{22}$$

Discrete Singular Convolution Differential Quadrature Method (DSCDQM): In this technique, Regularized Shannon Kernel (RSK) may be used as a shape function such that the unknown u(x) and its derivatives can be approximated over a narrow bandwidth ($x - x_M$, $x + x_M$) as (Ng *et al*., 2004; Civalek and Kiracioglu, 2007; Civalek and Gurses, 2009; Civalek and Oeztuerk, 2008):

$$u(x_i)=\sum_{j=-M}^{M}\left\langle \frac{\sin[\pi(x_i\text{-}x_j)/h_x]}{\pi(x_i\text{-}x_j)/h_x}\, e^{-(\frac{(x_i-x_j)^2}{2\sigma^2})}\right\rangle u(x_j) \quad (i=\text{-}N,N) \tag{23}$$

Where:

- h_x : The step size
- 2M+1 : The effective computational band width
- σ : Regularization parameter, $\sigma = r\,h_x$
- r : A computational parameter

Derivatives of u can be approximated as a weighted linear sum of $u_i(i = \text{-}N, N)$ as (Ng *et al*., 2004; Civalek and Kiracioglu, 2007; Civalek and Gurses, 2009; Civalek and Oeztuerk, 2008):

$$\left.\frac{\partial u}{\partial x}\right|_{x=x_i}=\sum_{j=-M}^{M}C_{ij}^{x}u(x_j),\ \left.\frac{\partial^2 u}{\partial x^2}\right|_{x=x_i}=\sum_{j=-M}^{M}C_{ij}^{xx}u(x_j)\ (i=\text{-}N,N) \tag{24}$$

Where:

$$C_{ij}^{x}=\begin{cases}\dfrac{(-1)^{i-j}}{h_x(i\text{-}j)}e^{-h_x(\frac{(i-j)^2}{2\sigma^2})}, & i\neq j\\ 0 & i=j\end{cases} \tag{25}$$

$$C_{ij}^{xx}=\begin{cases}\left(\dfrac{2(-1)^{i-j+1}}{h_x^{2}(i-j)^2}+\dfrac{1}{\sigma^2}\right)e^{-h_x^2(\frac{(i-j)^2}{2\sigma^2})}, & i\neq j\\ -\dfrac{1}{\sigma^2}-\dfrac{\pi^2}{3h_x^{2}} & i=j\end{cases} \tag{26}$$

As well as Delta Lagrange Kernel (DLK) can be applied as a shape function such that the unknown u(x) and its derivatives can be approximated as (Ng *et al*., 2004; Civalek and Kiracioglu, 2007; Civalek and Gurses, 2009; Civalek and Oeztuerk, 2008):

$$u(x_i)=\sum_{j=-M}^{M}\frac{\prod_{k=-M}^{M}(x_i\text{-}x_k)}{(x_i\text{-}x_j)\prod_{j=-M,j\neq k}^{M}(x_j\text{-}x_k)}u(x_j)(i=\text{-}N,N) \tag{27}$$

Derivatives of u can be approximated as a weighted linear sum of $u_i(i = \text{-}N, N)$ as (Ng *et al*., 2004; Civalek and Kiracioglu, 2007; Civalek and Gurses, 2009; Civalek and Oeztuerk, 2008):

$$\left.\frac{\partial u}{\partial x}\right|_{x=x_i}=\sum_{j=-M}^{M}C_{ij}^{x}u(x_j),\ \left.\frac{\partial^2 u}{\partial x^2}\right|_{x=x_i}=\sum_{j=-M}^{M}C_{ij}^{xx}u(x_j)(i=-N,N) \tag{28}$$

Where:

$$C_{ij}^{x}=\begin{cases}\dfrac{1}{(x_i\text{-}x_j)}\prod_{k=-M,k\neq i,j}^{M}\dfrac{(x_i\text{-}x_k)}{(x_j\text{-}x_k)} & i\neq j\\ -\sum_{j=-M,j\neq i}^{-M}C_{ij}^{x} & i=j\end{cases} \tag{29}$$

$$C_{ij}^{xx}=\begin{cases}2\left(C_{ij}^{x}.C_{ii}^{x}-\dfrac{C_{ij}^{x}}{(x_i\text{-}x_j)}\right) & i\neq j\\ -\sum_{j=-M,j\neq i}^{M}C_{ij}^{xx} & i=j\end{cases} \tag{30}$$

Similarly, one can approximate u_y, u_{yy} and calculated C_{ij}^{y}, C_{ij}^{yy}. On suitable substitution from Eq. 16-30 into (Eq. 8-10), the problem can be reduced to the following eigenvalue problem:

$$\sum_{j=1}^{N}\left[kGhc_{ij}^{x}W^{j}+D\left(c_{ij}^{xx}+\frac{1\text{-}\nu}{2}c_{ij}^{yy}\text{-}kGh\right)\varphi_x^{j}+D\left(\frac{1+\nu}{2}c_{ik}^{x}c_{kj}^{y}\right)\varphi_y^{j}\right]=\omega^2 I_1\varphi_x^{j},(i,k=1,N) \tag{31}$$

$$\sum_{j=1}^{N}\left[kGhc_{ij}^{y}W^{j}+D\left(\frac{1+\nu}{2}c_{ik}^{x}c_{kj}^{y}\right)\varphi_x^{j}+D\left(c_{ij}^{yy}+\frac{1\text{-}\nu}{2}c_{ij}^{xx}\text{-}kGh\right)\varphi_y^{j}\right]=\omega^2 I_1\varphi_y^{j},(i,k=1,N) \tag{32}$$

$$\sum_{j=1}^{N}kGh\left[\left((1\text{-}\frac{K_2}{kGh})c_{ij}^{xx}+(1\text{-}\frac{K_2}{kGh})c_{ij}^{yy}\right)W^{j}+\frac{K_1}{kGh}W^{j}\text{-}c_{ij}^{x}\varphi_x^{j}\text{-}c_{ij}^{y}\varphi_y^{j}\right]=\omega^2 I_0 W^{j},(i=1,N) \tag{33}$$

The boundary conditions (Eq. 11-14) can also be approximated using DQMs as: simply supporting of the first kind: SS1:

$$\begin{aligned}&W^{i}=0,\\&\sum_{j=1}^{N}\left[\left((n_x^2+\nu n_y^2)\,c_{ij}^{x}+(1\text{-}\nu)n_x n_y c_{ij}^{y}\right)\varphi_x^{j}+\begin{pmatrix}(\nu n_x^2+n_y^2)c_{ij}^{y}+\\(1\text{-}\nu)n_x n_y c_{ij}^{x}\end{pmatrix}\varphi_y^{j}\right]=0\\&\text{-}\frac{1\text{-}\nu}{2}D\sum_{j=1}^{N}\left[\left((n_x^2\text{-}n_y^2)\,c_{ij}^{y}\text{-}2n_x n_y c_{ij}^{x}\right)\varphi_x^{j}+\begin{pmatrix}(n_x^2\text{-}n_y^2)\,c_{ij}^{x}+\\2n_x n_y c_{ij}^{y}\end{pmatrix}\varphi_y^{j}\right]=0\\&(i=1,N)\end{aligned} \tag{34}$$

Simply supporting of the second kind: SS2:

$$W^i = 0,\ n_x\varphi_y^i - n_y\varphi_x^i = 0,$$

$$\sum_{j=1}^{N}\left[\left((n_x^2+\nu n_y^2)c_{ij}^x+(1-\nu)n_x n_y c_{ij}^y\right)\varphi_x^j + \begin{pmatrix}(\nu n_x^2+n_y^2)c_{ij}^y+\\(1-\nu)n_x n_y c_{ij}^x\end{pmatrix}\varphi_y^j\right]=0 \quad (i=1,N) \qquad (35)$$

Clamped edge:

$$W^i = 0,\ n_x\varphi_y^i - n_y\varphi_x^i = 0,\ n_x\varphi_x^i + n_y\varphi_y^i = 0,\ \ (i=1,N) \qquad (36)$$

Free edge:

$$kG\ h\left(\sum_{j=1}^{N}(n_x c_{ij}^x + n_y c_{ij}^y)\ W^j\right) - kG\ h(n_x\varphi_x^i + n_y\varphi_y^i) = 0$$

$$\sum_{j=1}^{N}\left[\left((n_x^2+\nu n_y^2)c_{ij}^x+(1-\nu)n_x n_y c_{ij}^y\right)\varphi_x^j + \begin{pmatrix}(\nu n_x^2+n_y^2)c_{ij}^y+\\(1-\nu)n_x n_y c_{ij}^x\end{pmatrix}\varphi_y^j\right]=0 \qquad (37)$$

$$-\frac{1-\nu}{2}D\sum_{j=1}^{N}\left[\left((n_x^2-n_y^2)c_{ij}^y-2n_x n_y c_{ij}^x\right)\varphi_x^j + \begin{pmatrix}(n_x^2-n_y^2)c_{ij}^x+\\2n_x n_y c_{ij}^y\end{pmatrix}\varphi_y^j\right]=0 \quad (i=1,N)$$

RESULTS AND DISCUSSION

Numerical results: This section presents numerical results that demonstrate convergence and efficiency of each one of the proposed schemes for vibration analysis of elastically supported composite plate. For all results, the boundary conditions (Eq. 34-37) are augmented in the governing (Eq. 31-33). The computational characteristics of each scheme are adapted to reach accurate results with error of order $\leq 10^{-8}$. The obtained frequencies are normalized such as: $\Omega = (\omega\sqrt{I})/\Omega_0$ where Ω_0 is the fundamental frequency of isotropic squared plate. For PDQM the problem is solved over a non-uniform grids with Gauss-Chebyshev-Lobatto discretizations such as (Dehghan and Baradaran, 2011; Hsu, 2006; Wang and Wu, 2013):

$$x_i = \frac{1}{2}\left[1-\cos\left(\frac{i-1}{N-1}\pi\right)\right],\ (i=1,N) \qquad (38)$$

Where the dimensions of the grid (N*N) ranges from 7*7-25*25. The obtained results agreed with previous analytical ones (Lam *et al*., 2000; Yang and Shen, 2001) over 18*18 grid size as shown in Table 1.

For SincDQ scheme, the problem is solved over a regular grids ranging from 5*5-25*25. Table 2 shows convergence of the obtained results. They agreed with exact ones (Lam *et al*., 2000; Yang and Shen, 2001) over grid size $\geq$18*18. Also, this table shows that execution time of SincDQ scheme is less than that of PDQM. Therefor, it is more efficient than PDQM for vibration analysis of elastically supported plates.

For DSCDQ scheme based on delta Lagrange kernel, the problem is also solved over a uniform grids ranging from 5*5-25*25. The bandwidth 2M+1 ranges from 3-17. Table 3 shows convergence of the obtained fundamental frequency which agreed with exact ones (Lam *et al*., 2000; Yang and Shen, 2001) over grid size 17*17 and bandwidth. Table 4 shows that the obtained results are more accurate than that were obtained using finite element method (Omurtag *et al*., 1997). The table also shows that execution time of DSCDQM-DLK is less than that of PDQM but it is greater than that of SincDQM.

Table 1: Comparison between the obtained normalized frequencies, due to PDQM and the previous exact and numerical ones, for various grid sizes: simply supported plate, $K_1 = K_2 = 0$

Normalized frequencies/Grid size	Ω_1	Ω_2	Ω_3	Ω_4
11×11	19.1921	49.0983	49.0983	78.6932
13×13	19.5467	49.17948	49.17948	78.7856
15×15	19.7349	49.3387	49.3387	78.9546
18×18	19.7361	49.3480	49.3480	78.9568
21×21	19.7361	49.3480	49.3480	78.9568
Exact results (Lam *et al*., 2000; Yang and Shen, 2001)	19.7361	49.3480	49.3480	78.9568
Finite element results (Omurtag *et al*., 1997)	19·911	50.112	50.112	80.090
Execution time (sec)	3.704655 -- over 18*18 non-uniform grid			

Table 2: Comparison between the obtained normalized frequencies, due to SincDQM and the previous exact and numerical ones, for various grid sizes: simply supported plate, $K_1 = K_2 = 0$

Normalized frequencies/Grid size	Ω_1	Ω_2	Ω_3	Ω_4
11×11	19.2825	49.12568	49.12568	78.77975
13×13	19.6479	49.27635	49.27635	78.8539
15×15	19.7357	49.3478	49.3478	78.9553
18×18	19.7361	49.3480	49.3480	78.9568
21×21	19.7361	49.3480	49.3480	78.9568
Exact results (Lam *et al*., 2000; Yang and Shen, 2001)	19.7361	49.3480	49.3480	78.9568
Finite element results (Omurtag *et al*., 1997)	19·911	50.112	50.112	80.090
Execution time (sec)	2.425466 -- over 18*18 uniform grid			

Further, it records the least excutsion time among the examined DQ schemes. Therefore, DSCDQM-RSK scheme is the best choise for vibration analysis of elastically supported plates.

For DSCDQ scheme based on Regularized Shannon Kernel (RSK), the problem is also solved over a uniform grids ranging from 5*5-25*25. The bandwidth 2M+1 ranges from 3-17 and the regularization parameter $\sigma = r\, h_x$ ranges from 1.8 h_x to 3 h_x where h_x = 1/N-1. Figure 2 shows convergence of the obtained fundamental frequency to the exact ones (Lam *et al*., 2000; Yang and Shen, 2001) over grid size 15*15, bandwidth and regulization parameter σ = 2.86 h_x. Table 4 and 5 also ensures that the obtained results from DQ schemes are more accurate than that of finite element methods. Further, execution time of this scheme is the least. Therefore, DSCDQM-RSK scheme is the best choice among the examined quadrature schemes for vibration analysis of elastically supported plates. Also, for different boundary conditions and sub-grade reactions, Table 6

Table 3: Variation of the fundamental frequency with bandwidth and grid size for a simply supported plate by using DSCDQM based on delta Lagrange kernel

Bandwidth/Grid size	M = 1	M = 2	M = 4	M = 5	M = 6	M = 8
5×5	7.32010	8.89900	8.89900			
7×7	7.85320	9.43850	13.1225	13.1225	13.1225	
9×9	9.01750	11.9113	16.5983	16.5983	16.5983	16.5983
11×11	10.5489	13.3475	17.7605	18.6565	18.6565	18.6565
13×13	11.9631	13.7948	17.9836	18.9969	19.17948	19.17948
15×15	13.9512	14.3387	18.1041	19.2346	19.7349	19.7352
17×17	14.5120	15.7238	18.2283	19.2723	19.7361	19.7361
19×19	14.9846	16.3479	18.4663	19.3365	19.7361	19.7361
21×21	15.4190	16.9482	18.7568	19.4931	19.7361	19.7361
23×23	15.7889	17.3184	18.9210	19.5604	19.7361	19.7361
25×25	16.4974	17.7605	19.3276	19.5822	19.7361	19.7361

Table 4: Comparison between the obtained normalized frequencies, due to DSCDQM-DLK and the previous exact and numerical ones, for various grid sizes: bandwidth=13; simply supported plate, $K_1 = K_2 = 0$

Normalized frequencies/Grid size	Ω_1	Ω_2	Ω_3	Ω_4
15×15	19.7349	49.3412	49.3412	78.9501
17×17	19.7361	49.3480	49.3480	78.9568
23×23	19.7361	49.3480	49.3480	78.9568
Exact results (Lam *et al*., 2000; Yang and Shen, 2001)	19.7361	49.3480	49.3480	78.9568
Finite element results (Omurtag *et al*., 1997)	19.911	50.112	50.112	80.090
Execution time (sec)	3.221545 --over 17*17 uniform grid and M = 6			

Table 5: Comparison between the obtained normalized frequencies, due to DSCDQM-RSK and the previous exact and numerical ones, for various grid sizes: bandwidth = 13; σ = 2.86 h_x, simply supported plate, $K_1 = K_2 = 0$

Normalized frequencies/Grid size	Ω_1	Ω_2	Ω_3	Ω_4
9×9	18.2759	48.9731	48.9731	77.2551
13×13	19.7357	49.3462	49.3462	78.9533
15×15	19.7361	49.3480	49.3480	78.9568
19×19	19.7361	49.3480	49.3480	78.9568
Exact results (Lam *et al*., 2000; Yang and Shen, 2001)	19.7361	49.3480	49.3480	78.9568
Finite element results (Omurtag *et al*., 1997)	19.911	50.112	50.112	80.090
Execution time (sec)	1.556069 --over 15*15uniform grid			

Table 6: Comparison between the obtained fundamental natural frequenciesdue to DSCDQM –RSKand the previousresults for different boundary conditions and modulus of subgrade reactions

Subgrade reaction/Boundary condition		CSCS			CSSS		
K_1	K_2	Element free Galerkin (Bahmyari *et al*., 2013)	Obtained results	Exact results (Lam *et al*., 2000; Yang and Shen, 2001)	Element free Galerkin (Bahmyari *et al*., 2013)	Obtained results	Exact results (Lam *et al*., 2000; Yang and Shen, 2001)
0	0	29.0033	28.95	28.95	23.6649	23.65	23.65
	100	54.7225	54.68	54.68	51.3359	51.32	51.32
	1000	-	146.73	146.73	-	144.24	144.24
100	0	-	60.63	60.63	-	25.67	25.67
	100	55.6285	55.59	55.59	52.3006	52.29	52.29

Table 6: Continue

Subgrade reaction/Boundary condition		CSCS			CSSS		
K_1	K_2	Element free Galerkin (Bahmyari *et al*., 2013)	Obtained results	Exact results (Lam *et al*., 2000; Yang and Shen, 2001)	Element free Galerkin (Bahmyari *et al*., 2013)	Obtained results	Exact results (Lam *et al*., 2000; Yang and Shen, 2001)
	1000	-	147.13	147.13	-	144.61	144.61
1000	0	42.9070	42.87	42.87	39.4949	39.49	39.49
	100	-	63.17	63.17	-	60.28	60.28
	1000	-	150.12	150.12	-	147.62	147.62
Subgrade reaction/Boundarycondition		SSSS			SFSF		
0	0	19.7421	19.7361	19.7361	9.6356	9.63	9.63
	100	48.6146	48.62	48.62	32.9047	32.90	32.90
	1000	-	141.87	141.87	-	99.83	99.83
100	0	22.1299	22.13	22.13	13.8866	13.88	13.88
	100	49.6323	49.63	49.63	34.3905	34.39	34.39
	1000	-	142.20	142.20	-	100.33	100.33
1000	0	37.2771	37.28	37.28	33.0570	31.62	31.62
	100	-	58.00	58.00	-	45.64	45.64
	1000	-	145.36	145.36	-	104.72	104.72

Table 7: Comparison between the obtained natural frequencies due to DSCDQM-RSK and the previous results for simply supported plate: h/a = 0.01, K_2 = 10

	K_1 = 100				K_1 = 500			
Subgrade reaction/Results	ω_1	ω_2	ω_3	ω_4	ω_1	ω_2	ω_3	ω_4
Obtained DSCDQM-RSK	26.2048	54.9915	54.9915	84.2914	32.9645	58.5139	58.5139	86.6305
Exact results (Lam *et al*., 2000)	26.2048	54.9915	54.9915	84.2914	32.9645	58.5139	58.5139	86.6305
Element free Galerkin (Bahmyari *et al*., 2013)	26.2127	55.0714	55.0714	84.4355	32.9704	58.5889	58.5889	86.7706
Ritz method (Zhou *et al*., 2004)	26.2048	54.9905	54.9905	84.2923	32.9625	58.5119	58.5119	86.6305
Radial basis (Ferreira *et al*., 2010)	26.2127	54.9915	54.9915	84.2706	32.9704	58.5119	58.5119	86.6097

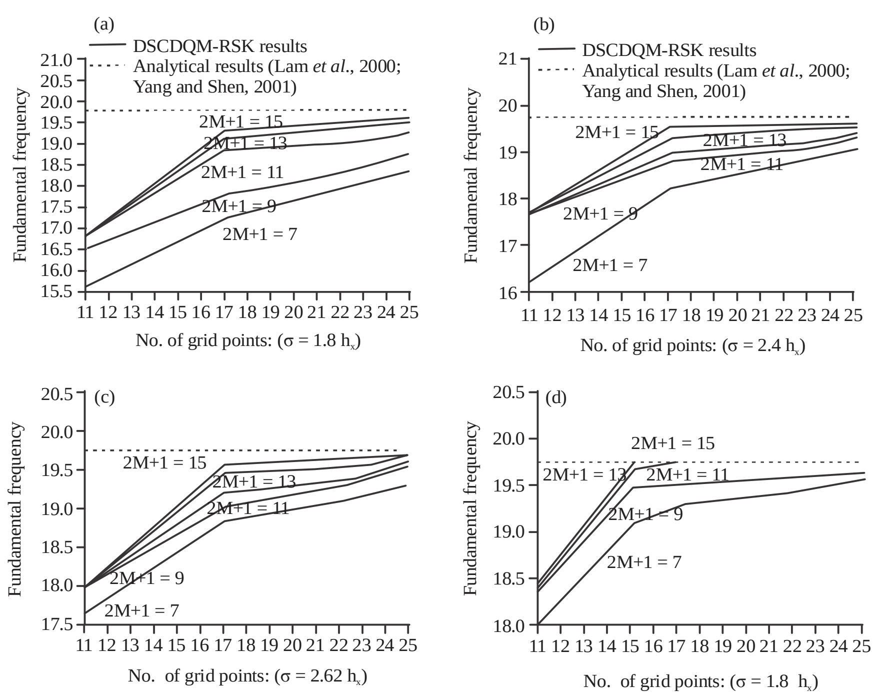

Fig. 2(a-d): Variation of the normalized fundamental frequency with the bandwidth, regularization parameter σ and grid size for a simply supported plate by using DSCDQM-RSK

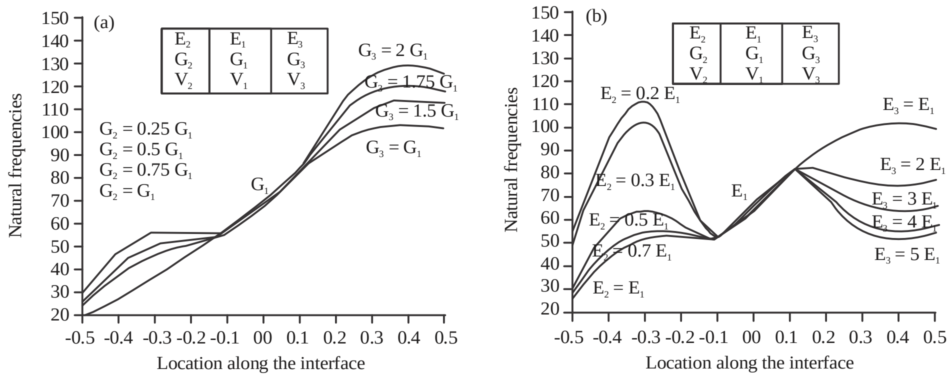

Fig. 3(a, b): Variation of the natural frequencies with Shear and Young's modulus gradation ratio of a squared simply supported composite (K_1 = 200, K_2 = 10, h/a = 0.1, $v_1 = v_2 = v_3$)

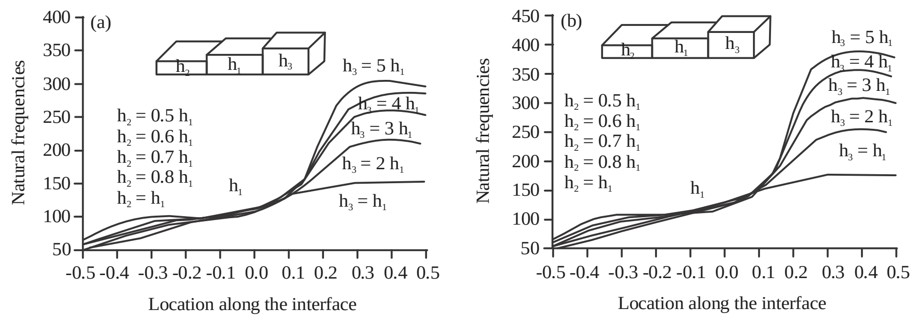

Fig. 4(a, b): Variation of the natural frequencies with thickness of a squared elastically supported composite (K_1 = 500, K_2 = 100, $E_1 = E_2 = E_3$, $G_1 = G_2 = G_3$, $v_1 = v_2 = v_3$) (a) Simply supported plates and (b) Clamped plates

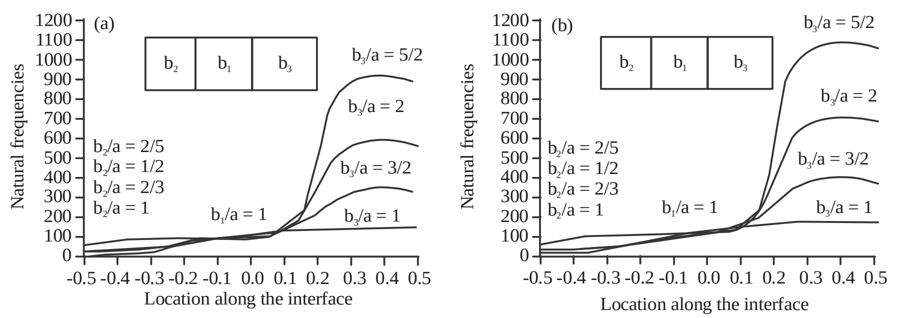

Fig. 5(a, b): Variation of the natural frequencies with aspect ratio (a/b) for elastically supported composite (K_1 = 500, K_2 = 100, $E_1 = E_2 = E_3$, $G_1 = G_2 = G_3$, $v_1 = v_2 = v_3$, $h_1 = h_2 = h_3$), (a) Simply supported plates and (b) Clamped plates

and Table 7 also insist that DSCDQM-RSK scheme is the best choice for vibration analysis of elastically supported plates. Furthermore, a parametric study is introduced to investigate the influence of elastic and geometric characteristics of the composite on the values of natural frequencies. Figure 3 shows that the natural frequencies decrease with increasing Young's modulus gradation ratio, (E_2/E_1) and (E_3/E_1). As well as, Fig. 3-5 show that the natural frequencies are increased with increasing shear modulus gradation ratio (G_2/G_1 and G_3/G_1) thickness ratio (h_2/h_1 and h_3/h_{1s}) and aspect ratio b/a. The case of ($E_1 = E_2 = E_3$, $G_1 = G_2 = G_3$ and $h_1 = h_2 = h_3$) is a limiting case of this study which was previously solved by Lam *et al*. (2000), Bahmyari *et al*. (2013), Zhou *et al*. (2004) and Ferreira *et al*. (2010).

CONCLUSION

Different quadrature schemes have been successfully applied for vibration analysis of elastically supported composite plates. A MATLAB program is designed for each scheme such that the maximum error (comparing with the previous exact results) is also execution time for each scheme is determined. It is concluded that discrete singular convolution differential quadrature method based on regularized Shannon kernel (DSCDQM-RSK) with grid size 15*15, bandwidth 2M+1 and regulization parameter $\sigma = 2.86\ h_x$ leads to best accurate efficient results for the concerned problem. Based on this scheme, a parametric study is introduced to investigate the influence of elastic and geometric characteristics of the vibrated plate on results. It is aimed that these results may be useful for design purposes of engineering fields.

REFERENCES

Akhavan, H., S.H. Hashemi, H.R.D. Taher, A. Alibeigloo and S. Vahabi, 2009. Exact solutions for rectangular Mindlin plates under in-plane loads resting on Pasternak elastic foundation. Part II: Frequency analysis. Comput. Mater. Sci., 44: 951-961.

Bahmyari, E. and M.R. Khedmati, 2017. Stochastic analysis of thin plates on elastic foundation by combining the generalized polynomial chaos and element free Galerkin method. J. Mech. Sci. Technol., 31: 1813-1824.

Bahmyari, E., M.M. Banatehrani, M. Ahmadi and M. Bahmyari, 2013. Vibration analysis of thin plates resting on Pasternak foundations by element free Galerkin method. Shock Vibr., 20: 309-326.

Chakraverty, S. and K.K. Pradhan, 2014. Free vibration of functionally graded thin rectangular plates resting on Winkler elastic foundation with general boundary conditions using rayleigh-ritz method. Int. J. Applied Mech., Vol. 6, No. 4. 10.1142/S1758825114500434

Civalek, O. and B. Oeztuerk, 2008. Discrete singular convolution method for free vibration analysis of tapered rectangular plates. Adv. Vibration Eng., 7: 261-274.

Civalek, O. and M. Gurses, 2009. Discrete singular convolution for free vibration analysis annular membranes. Math. Comput. Applic., 14: 131-138.

Civalek, O. and O. Kiracioglu, 2007. Discrete singular convolution for free vibration analysis of anisotropic rectangular plates. Math. Comput. Applic., 12: 151-160.

Dehghan, M. and G.H. Baradaran, 2011. Buckling and free vibration analysis of thick rectangular plates resting on elastic foundation using mixed finite element and differential quadrature method. Applied Math. Comput., 218: 2772-2784.

Ferreira, A.J.M., C.M.C. Roque, A.M.A. Neves, R.M.N. Jorge and C.M.M. Soares, 2010. Analysis of plates on Pasternak foundations by radial basis functions. Computat. Mech., 46: 791-803.

Gupta, A., M. Talha and V.K. Chaudhari, 2016. Natural frequency of functionally graded plates resting on elastic foundation using finite element method. Procedia Technol., 23: 163-170.

Hsu, M.H., 2006. Vibration characteristics of rectangular plates resting on elastic foundations and carrying any number of sprung masses. Int. J. Applied Sci. Eng., 4: 83-89.

Kai, X., W. Jiufa, L. Qiuhong, W. Weiyuan and W. Ping, 2014. An exact series solution for the vibration of Mindlin rectangular plates with elastically restrained edges. Shock Vibr., Vol. 2014, 10.1155/2014/286710

Karasin, A., 2016. Vibration of rectangular plates on elastic foundations by finite grid solution. Int. J. Math. Comput. Methods, 1: 140-145.

Karasin, A., R. Akdogan, P. Gulkan and M.A. Ugurlu, 2016. Usage of finite grid solution for vibration of rectangular plates on elastic foundations. Wseas Trans. Applied Theor. Mech., 11: 60-66.

Korkmaz, A. and I. Dag, 2011. Shock wave simulations using sinc differential quadrature method. Eng. Computations, 28: 654-674.

Lam, K.Y., C.M. Wang and X.Q. He, 2000. Canonical exact solutions for Levy-plates on two-parameter foundation using Green’s functions. Eng. Struct., 22: 364-378.

Li, Q.L., W.D. Luan and Z.Q. Zhu, 2017. Shooting method for free vibration of FGM reissner-mindlin circular plates resting on elastic foundation in thermal environments. J. Vibroengineering, 19: 4423-4439.

Li, W.L., X. Zhang, J. Du and Z. Liu, 2009. An exact series solution for the transverse vibration of rectangular plates with general elastic boundary supports. J. Sound Vibr., 321: 254-269.

Liew, K.M., Y.Q. Huang and J.N. Reddy, 2002. A hybrid moving least squares and differential quadrature (MLSDQ) meshfree method. Int. J. Comput. Eng. Sci., 3: 1-12.

Liew, K.M., Y.Q. Huang and J.N. Reddy, 2003. Vibration analysis of symmetrically laminated plates based on FSDT using the moving least squares differential quadrature method. Comput. Methods Applied Mech. Eng., 192: 2203-2222.

Moradi-dastjerdi, R., H. Momeni-Khabisi and R. Baghbani, 2017. Mesh-free dynamic analyses of FGM sandwich plates resting on a pasternak elastic foundation. Mech. Adv. Compos. Struct.?, 4: 153-168.

Ng, C.H.W., Y.B. Zhao and G.W. Wei, 2004. Comparison of discrete singular convolution and generalized differential quadrature for the vibration analysis of rectangular plates. Comput. Methods Applied Mech. Eng., 193: 2483-2506.

Omurtag, M.H., A. Ozutok, A.Y. Akoz and Y. Oezcelikoers, 1997. Free vibration analysis of Kirchhoff plates resting on elastic foundation by mixed finite element formulation based on gateaux differential. Int. J. Numer. Methods Eng., 40: 295-317.

Panc, V., 1975. Theories of Elastic Plates. Springer Science and Business Media, New York, USA., ISBN-13: 9789028601048, pp: 13-41.

Reddy, J.N., 1997. Mechanics of Laminated Composite Plates Theory and Analysis. CRC Press, Boca Raton.

Reddy, J.N., 1999. Theory and Analysis of Elastic Plates. Taylor and Francis, Philadelphia.

Secer, A., 2013. Numerical solution and simulation of second-order parabolic PDEs with Sinc-Galerkin method using maple. Abstr. Applied Anal., Vol. 2013, 10.1155/2013/686483

Tan, F. and Y.L. Zhang, 2013. A hybrid boundary node method for the bending of plates on elastic foundation. Eng. Mech., 30: 35-41.

Trif, D., 2002. On sinc methods for partial differential equation. Semin. Fixed Point Theor. Cluj-Napoca, 3: 173-182.

Wang, X.W. and Z. Wu, 2013. Differential quadrature analysis of free vibration of rhombic plates with free edges. Applied Math. Comput., 225: 171-183.

Wen, P.H., 2008. The fundamental solution of Mindlin plates resting on an elastic foundation in the laplace domain and its applications. Int. J. Solids Struct., 45: 1032-1050.

Yang, J. and H.S. Shen, 2001. Dynamic response of initially stressed functionally graded rectangular thin plates. Compos. Struct., 54: 497-508.

Zhou, D., Y.K. Cheung, S.H. Lo and F.T.K. Au, 2004. Three-dimensional vibration analysis of rectangular thick plates on Pasternak foundation. Int. J. Numer. Methods Eng., 59: 1313-1334.