

## Design and Control of a Gantry Crane System with Limited Payload Angle using Robust and State Feedback Controllers

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**Key words:** Gantry crane, Trolley, Payload, H2 optimal, observer based controller

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**Abstract:** In this study, the performance improvement of a payload angle deflection of a gantry crane has been studied and simulated using MATLAB/Simulink toolbox successfully. H2 optimal and observer based controllers has been used to minimize the payload angular deflection by controlling the trolley position. The gantry crane has been compared with the proposed controllers to track the reference trolley position using step and sine wave signals and a promising results have been achieved.

## INTRODUCTION

A gantry crane is one of the many types of crane which is built at the top of a gantry which is a structure used to support an object to be lifted. They are huge machines that capable of lifting heavy loads like lifting automobile engines out of vehicles. The terms gantry crane and overhead crane (or bridge crane) are often used interchangeably as both types of crane straddle their workload. The distinction mass often drawn between the two is that with gantry cranes, the entire composition (including gantry) is usually wheeled (often on rails). By contrast, the promoting makeups of an overhead crane is fixed in location, often in the example of the walls or ceiling of a building, to which is attached a movable hoist running overhead along a bannister or ray (which may itself move). Further confusing the issue is that gantry cranes may also incorporate a movable beam-mounted hoist in addition to the entire structure entity wheeled and some overhead cranes are suspended from a freestanding gantry. Full gantry cranes (where the load remnants beneath the gantry structure, supported from a beam) are

well suited to lifting massive thing such as ship's engines as the entire disposition can resist the torque created by the load and counterweights are generally not required. These are often found in shipyards where they are used to move large boat part together for construction<sup>[1]</sup>.

## MATERIALS AND METHODS

**Mathematical modeling of gantry crane:** The gantry crane system design is shown in Fig. 1. To solve the mathematical model of the system, Lagrange's equations are used which is given as<sup>[2]</sup>:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} \right) = Q \quad (1)$$

$$L = T - V \quad (2)$$

Where:

$\theta$  = System variable

$L$  = Lagrange equation

$Q$  = Sum of forces or moments

$T$  = Kinetic energies

$V$  = Potential energies

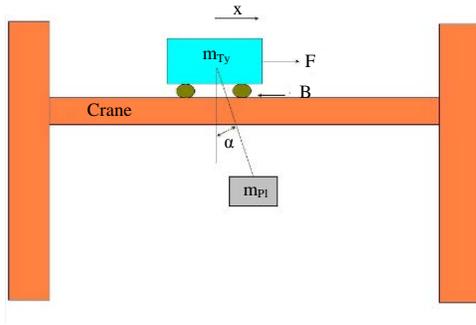


Fig. 1: Gantry crane system

The kinetic energy of the trolley given as:

$$T_1 = \frac{1}{2} m_{Ty} \dot{x}^2 \quad (3)$$

The kinetic energy of the payload is given as:

$$T_2 = \frac{1}{2} m_{pl} v^2 \quad (4)$$

And the velocity is found by taking the first derivative of the position:

$$v^2 = \dot{x}^2 + 2l\dot{\alpha}\dot{x} \sin \alpha + l^2\dot{\alpha}^2 \quad (5)$$

$$T_2 = \frac{1}{2} m_{pl} (\dot{x}^2 + 2l\dot{\alpha}\dot{x} \sin \alpha + l^2\dot{\alpha}^2)$$

The Lagrangian for this system can be written as:

$$L = \frac{1}{2} m_{pl} (\dot{x}^2 + 2l\dot{\alpha}\dot{x} \sin \alpha + l^2\dot{\alpha}^2) + \frac{1}{2} m_{Ty} \dot{x}^2 + m_{pl}gl \cos \alpha \quad (6)$$

And the equation of motion follows from:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} \right) = F - B\dot{x}$$

Where B is the damping friction coefficient:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\alpha}} - \frac{\partial L}{\partial \alpha} \right) = 0$$

Resulting in:

$$(m_{Ty} + m_{pl})\ddot{x} + B\dot{x} + m_{pl}l\ddot{\alpha} \cos \alpha - m_{pl}l\dot{\alpha}^2 \sin \alpha = F \quad (7)$$

$$m_{pl}l^2\ddot{\alpha} + m_{pl}l\dot{x} \cos \alpha + m_{pl}lg \sin \alpha = 0 \quad (8)$$

For small angle approximation:  $\alpha < 1^\circ$  the sine and cosine can be linearized as<sup>[3]</sup>:

Table 1: System parameters

Parameters	Symbols	Values
Trolley mass	$m_{Ty}$	80 kg
Payload mass	$m_{pl}$	32 kg
Surface friction	B	25 N-m/s
Cable length	l	3.5 m
Acceleration due to gravity	g	10 m sec <sup>-2</sup>

$$\alpha \approx 0$$

$$\sin \alpha \approx \alpha$$

$$\cos \alpha \approx 1$$

$$\dot{\alpha}^2 \approx 0$$

Hence, the derived Eq. 7 and 8 of the non-linear model can be approximately linearized as:

$$(m_{Ty} + m_{pl})\ddot{x} + B\dot{x} + m_{pl}l\ddot{\alpha} = F \quad (9)$$

$$m_{pl}l^2\ddot{\alpha} + m_{pl}l\dot{x} + m_{pl}lg\alpha = 0 \quad (10)$$

Rearranging Eq. 9 and 10:

$$\ddot{x} = -\frac{B}{m_{pl}}\dot{x} + \frac{m_{Ty}g}{m_{pl}}\alpha + \frac{F}{m_{pl}} \quad (11)$$

$$\ddot{\alpha} = \frac{B}{m_{pl}l}\dot{x} - \frac{(m_{pl} + m_{Ty})}{m_{pl}l}g\alpha + \frac{F}{m_{pl}l} \quad (12)$$

Let:

$$x_1 = x, x_2 = \dot{x}, x_3 = \alpha \text{ and } x_4 = \dot{\alpha}$$

The state space equation becomes:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{B}{m_{pl}} & \frac{m_{Ty}g}{m_{pl}} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{B}{m_{pl}l} & -\frac{(m_{pl} + m_{Ty})}{m_{pl}l}g & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_{pl}} \\ 0 \\ -\frac{1}{m_{pl}l} \end{bmatrix} F$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix}$$

The system parameters are shown in Table 1. Then the state space equation numerically becomes:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.22 & 25 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.22 & -10 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.03 \\ 0 \\ -0.009 \end{bmatrix} F$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix}$$

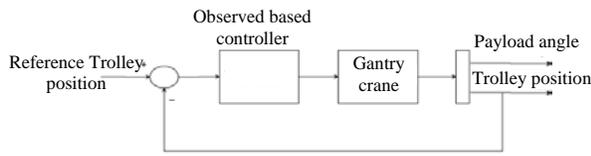


Fig. 2: Block diagram of the gantry crane system with the observer-based controller

**The proposed controllers design**

**Observer-based controller design:** The deal with the general argument where only a subset of the states or linear combinations of them are obtained from measurements and are available to our controller. Such a handbooks lines is referred to as the output feedback problem<sup>[4]</sup>. The output is of the form:

$$y = Cx + Du \tag{13}$$

The block diagram of the gantry crane system with the observer-based controller is shown in Fig. 2. The observer based controller  $G_c(s)$  can be further derived in the following form:

$$G_c(s) = I - K(sI - A + BK + HC)^{-1}B \tag{14}$$

With its state space realization:

$$G_c(s) = \begin{bmatrix} A - BK - HC & B \\ -K & I \end{bmatrix} \tag{15}$$

The controller  $G_c(s)$  in Eq. 15 is called the observer-based controller, since, the structural reference of the observer is reflected within the controller<sup>[5]</sup>.

Where the state space model of the plant,  $G$ , the state feedback gain vector  $K$  and the observer gain vector  $H$  are then returned, respectively. We select the weighting matrix  $Q$  and  $R$  as:

$$Q = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix} \text{ and } R = 5$$

And we select the observer gain vector as:

$$H = \begin{pmatrix} 1.5 & -0.3 \\ 0.4 & 1.2 \\ 0.7 & 0 \\ 0.9 & -0.5 \end{pmatrix}$$

And we obtain the state feedback gain vector  $K$  as:

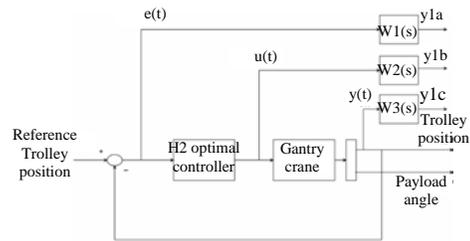


Fig. 3: Weighted control structure of the gantry crane system with the H2 optimal controller

$$K = [0.7746 \quad 86.0062 \quad 70.8594 \quad 212.6783]$$

The observer-based controller state space representation become:

$$\dot{x}_o = \begin{pmatrix} -1.5 & 1 & 0 & 0 \\ -0.42 & -2.8 & 22.87 & -6.38 \\ -0.7 & 0 & 0 & 1 \\ -0.9 & 1 & -9.362 & 1.9 \end{pmatrix} x_o + \begin{pmatrix} 1.5 \\ 0.4 \\ 0.7 \\ 0.9 \end{pmatrix} u_o$$

$$y_o = (0.77 \quad 86 \quad 71 \quad 212.7) x_o$$

**Augmentations of the model with weighting functions:**

In this study, we will center on the weighted control system shown in Fig. 3 where  $W1(s)$ ,  $W2(s)$  and  $W3(s)$  are weighting functions or weighting filters. We assume that  $G(s)$  (gantry crane system),  $W1(s)$  and  $W3(s)$   $G(s)$  are all proper; i.e., they are bounded when  $s \rightarrow \infty$ . It can be seen that the weighting function  $W3(s)$  is not required to be proper. In the two-port system, the output vector  $y1 = [y1a, y1b, y1c]^T$  is not used directly to construct the vector  $u2$ . Understand that  $y1$  is actually for the control outline attainment measurement. So, it is not strange to include the filtered “input signal”  $u(t)$  in the “output signal”  $y1$  because one may indispensability to quantities the control energy to assess whether the designed controller is perfect or not. Clearly, Fig. 3 represents a more general diagram of optimal and robust control systems. We can design an optimal H2 controller by using the thought of the augmented system model<sup>[6]</sup>.

The weighting function  $W1(s)$ ,  $W2(s)$  and  $W3(s)$  are chosen as:

$$W_1(s) = \frac{s+1}{s^2+6s+12} \quad W_2(s) = \frac{s+4}{s^2+16s+42}$$

$$W_3(s) = \frac{s+8}{s^2+21s+55}$$

The H2 optimal controller state space representation become:

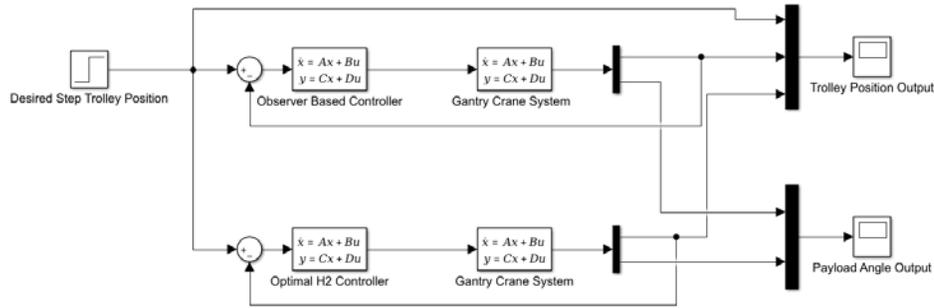


Fig. 4: Simulink model of the gantry crane system using augmentation based H2 optimal and observer based controllers using step input desired trolley position signal

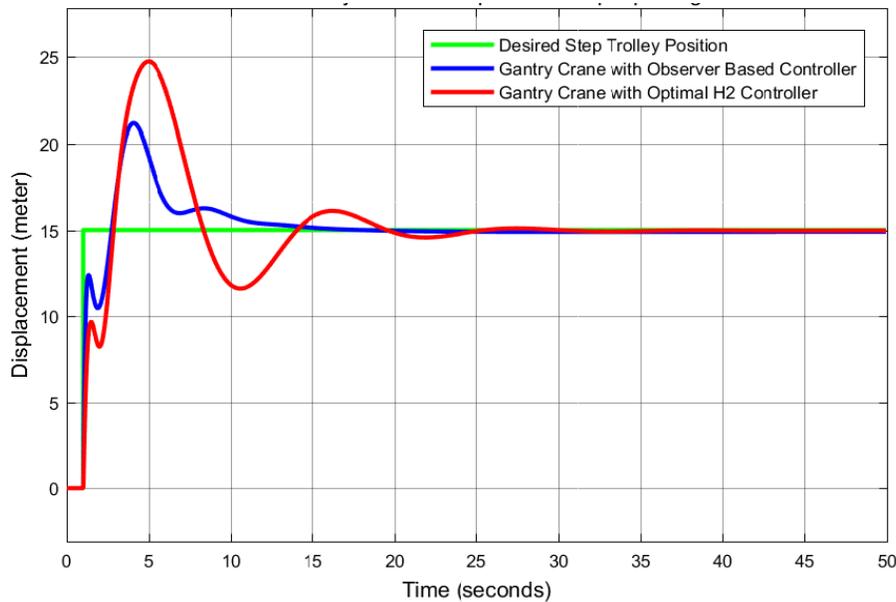


Fig. 5: Step response of the trolley position comparison

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} 0.43 & 1.27 & 2.05 & 23 \\ 20 & 32 & 45 & 10 \\ 0.9 & -0.35 & 12 & 1 \\ 34 & 22 & -0.45 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} 20 \\ 13 \\ 0.54 \\ -5 \end{bmatrix} u$$

$$w = [27.98 \ 0 \ 0.936 \ 0] \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix}$$

**RESULTS AND DISCUSSION**

**Comparison of the gantry crane system using augmentation based H2 optimal and observer based controllers for a step input trolley position signal:** The Simulink model of the gantry crane system using augmentation based H2 optimal and observer based

Table 2: Trolley step response data

Performance data	Observer based controller	H2 optimal controller
Rise time	1 sec	1.1 sec
Per. overshoot	26.6%	46.6%
Settling time	15 sec	30 sec

controllers using step input desired trolley position signal is shown in Fig. 4. The simulation result of the trolley and payload comparison for the proposed controllers and the input force to the gantry crane system are shown in Fig. 5-8, respectively<sup>[7]</sup>.

The input force of the gantry crane system with the observer based controller shows improvement in reducing the force amplitude. The data of the rise time, percentage overshoot and settling time of the trolley is shown in Table 2.

As Table 2 shows that the gantry crane system with the observer based controller improves the performance

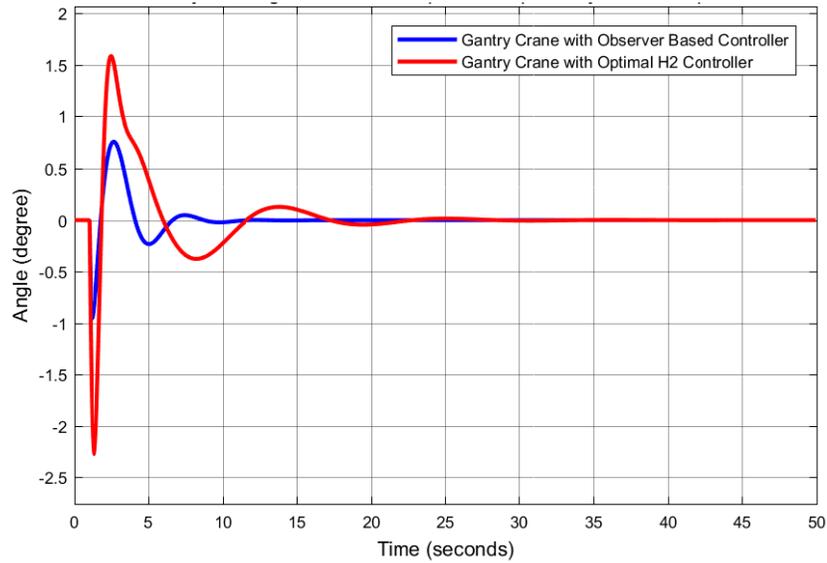


Fig. 6: Step response of the trolley position comparison

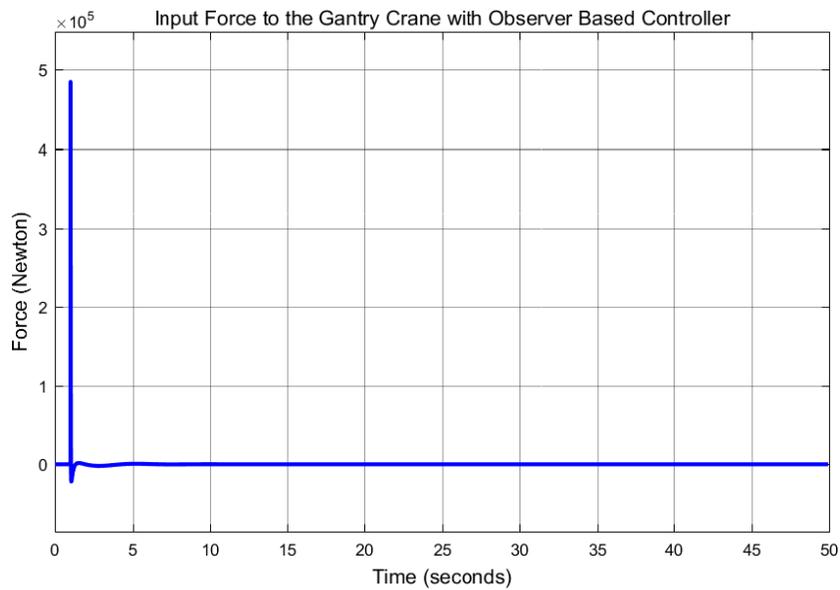


Fig. 7: Input force to the system with observer based controller

Table 3: Payload step response data

Performance data	Observer based controller	H2 optimal controller
Rise time	1 sec	1 sec
Per. overshoot	18.6%	35.6%
Settling time	10 sec	28 sec

of the Trolley position by minimizing the percentage overshoot and settling time. The data of the rise time, percentage overshoot and settling time of the payload is shown in Table 3.

As Table 3 shows that the gantry crane system with the observer based controller improves the performance of the payload angle by minimizing the percentage overshoot and settling time.

**Comparison of the gantry crane system using augmentation based H2 optimal and observer based controllers for a sine wave input trolley position signal:** The Simulink model of the gantry crane system using augmentation based H2 optimal and observer based

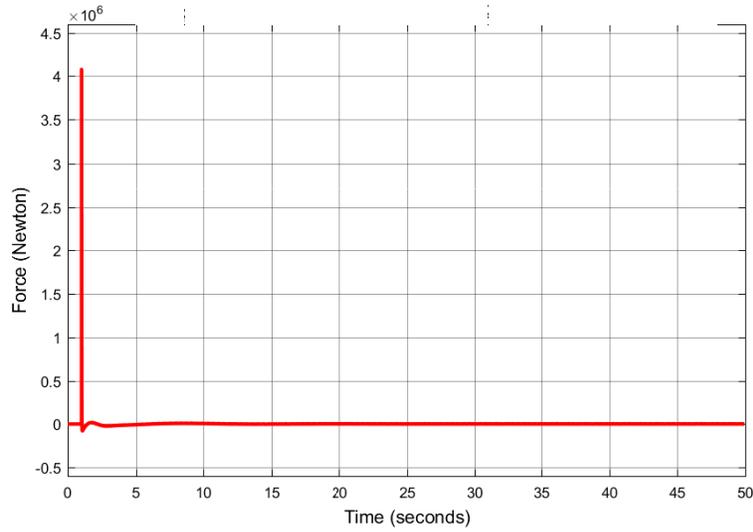


Fig. 8: Input force to the system with H2 optimal controller

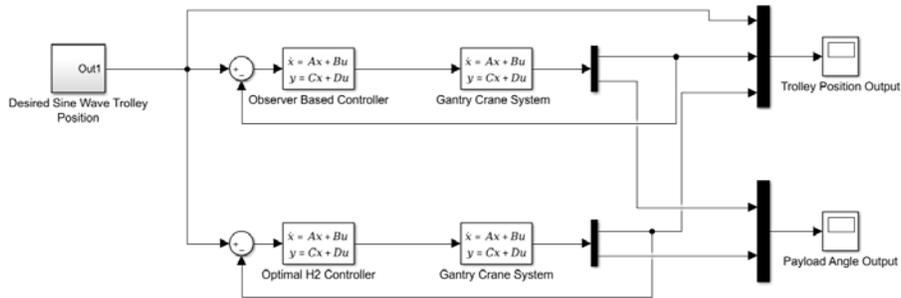


Fig. 9: Simulink model of the gantry crane system using augmentation based H2 optimal and observer based controllers using sine wave input desired trolley position signal

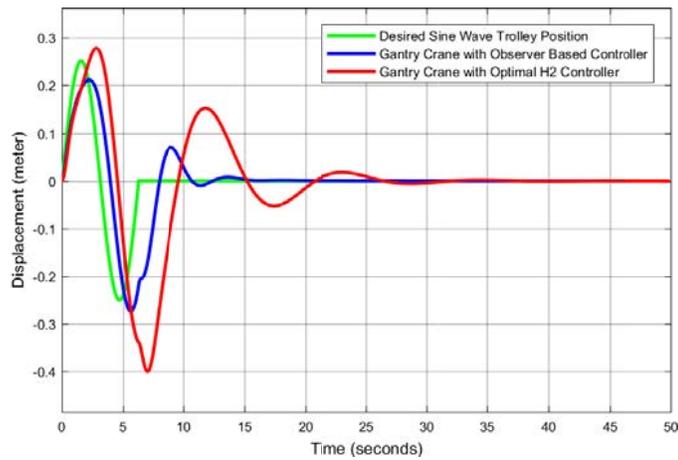


Fig. 10: Sine wave response of the trolley position comparison

controllers using sine wave input desired trolley position signal is shown in Fig. 9. The simulation result of the trolley and payload comparison for the proposed

controllers and the input force to the gantry crane system are shown in Fig. 10-13, respectively. The input force of the gantry crane system with the observer based controller

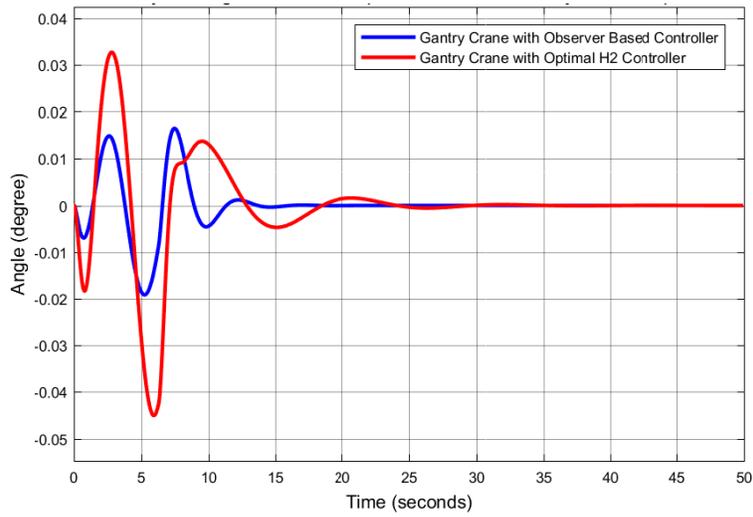


Fig. 11: Sine wave response of the trolley position comparison

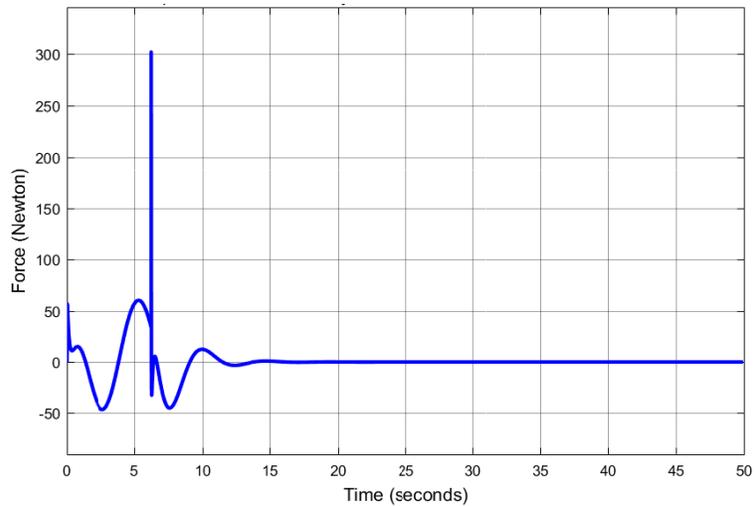


Fig. 12: Input force to the system with observer based controller

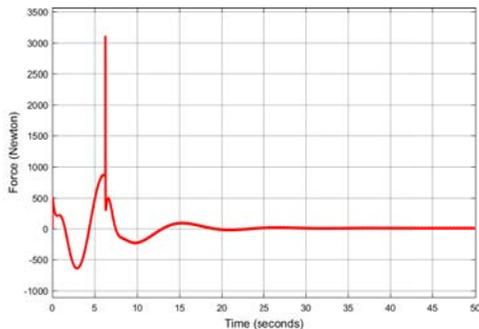


Fig. 13: Input force to the system with H2 optimal controller

shows improvement in reducing the force amplitude. Figure 10 and 11 shows that the gantry crane system with the observer based controller improves the performance of the trolley position and payload angle by minimizing the percentage overshoot and settling time.

### CONCLUSION

In this study, a minimum payload angular deflection has been achieved for a gantry crane using a feedback controller. This achievement has been done by controlling the trolley position instead. Comparison of the system

with the proposed controllers shows that the system with observer based controller improves the payload deflection and vibration better than the proposed H2 optimal controller with a minimum force input to the system.

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