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Topical Function in Topological Space

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Abstract: In this study, we put new concept linking the topology and filter on a given space, we call it the topical function. Also, gave us the characteristics and advantages of this concept.

INTRODUCTION

One of the important topics in Mathematics is the position of the filter where it took a large part of this science and was linked in different branches of mathematics. Filters were introduced in 1937 by cartan[1]. In 1940 Borbaki[1] demonstrated several results in the topological space using filters and defined the filter as a non-fictional family of x that achieves two properties (1) $\hbar \in F$ and $\hbar \subseteq \varepsilon$ implies $\varepsilon \in F$;[1] $\hbar \in F$ and $\varepsilon \in F$ implies $\hbar \cap \varepsilon \in F$. In the same year, the scientist Tukey[1] studied groups, filters and various modifications of the two concepts. Then came the scientist Kuratowski[2] where he developed the topology depending on the filters. There are traces of the concept of filters, since, 1914 in the root article. More recently, filters play a key role in the development of fuzzy spaces that have applications in computer science and engineering. Filters are also an important tool used by researchers to describe non-topological convergence in functional analysis. For example, by Cascales and Oncina[3] studied the characteristics that associated with multi-stranded filters. The convergence of the topology of filters was provided by Stadler and Stadler[4] where he laid the theoretical basis for the approach to the filters and provided detailed evidence. There's many researchers used the fuzzy set theory as tool to define the filter in this spaces and study many properties including by Ramakrishnan and Nayagam[5] defined filter in the theory of the fuzzy sets in Algebra. In 2013, Vaidyanathaswamy[6] introduced and studied fuzzy filters and regular filters in Algebra. He derived all the equivalent conditions for fuzzy filters to become ordinary fuzzy filters and took Max for fuzzy filters in Algebra . Researchers Al-Swidi and Awaad[7] also invested idea soft set and soft ideal to defined a new points call it the soft turning and bench points[8, 9]. Ideal was studied in topological spaces by Kuratowski[2] where he showed that the characteristics of ideal which are a non-fictional subsets of X that satisfies the following conditions (1) $\hbar \in I$ and $\varepsilon \subseteq \hbar$ implies $\varepsilon \in I$; (2) if $\hbar \in I$ and ($\varepsilon \in I$ implies $\hbar \cup \varepsilon \in F$. Stadler and Stadler[4] showed ideal properties in the local function. And also (Maitara) 6 worked on the topological space of the ideal and its properties in general. In 2014, researchers Al-Swidi and Al-Rubaye[10, 11, 6] studying the compactness and the compactness and the local function on the special type of ideal. In this research, we present the concept of the local function in the field of the topological filter where we show the characteristics of the function by applying the conditions of the filter and showing some of the hypotheses, we also mention the concept of the closure in the filter and many concept, depending on the filter and study their properties.

Preliminaries: Throughout the present paper (X, ℑ) denoted a topological space (on simply denoted X) for a subset ħ of a topological space (X, ℑ), Cl (ħ) and Int (ħ) will denoted the closure and interior of ħ in (X, ℑ), respectively. An ideal I on a topological space (X, ℑ) is a non-empty collection subsets of X which satisfies (1) ħ∈I and ε⊆ħ implies j ε∈I (2) ħ∈I and ε∈I imlies j ħ∪ε∈I. A set operator (.)* :P(X)→P(X), called a local function[2, 12] of ħ with respect to ℑ and I is defined as follows: for ħ⊆X, ħ* (I , ℑ) = {x∈X: u∩ħ∉ I for every u∈ℑ(x)} where ℑ (x) = {u∈ℑ: x∈u}. A Kuratowski closure operator IcI* (.) for a topology ℑ* (I, ℑ), called the *-topology, ℑ finer than ℑ* which is defined by cl* (ħ) = ħ∪ħ* (I, ℑ)[6]. For every ideal space (X, ℑ, I), the collection β = {V/ħ; V∈ℑ and ħ∈I} is a basis for ℑ* subset ħ of X is called ℑ*_denser iff cl* ħ = X[13]. A topological spacer X is said to be hyperconnected[11] if every pair of non-empty open sets of X is dense. Also, we call the filter F is proper filer if it not containing the empty set.

In the following definition, we give a new concept that illustrates the connection between the topology and filter on the given set and we call it the topical function.

Definition (2-1): Let (X, ℑ) be topological space and is a filter F on X then the topical function of ħ is of the form $\hbar^{\#}$ = {x ∈ X; ux∩ ħ∈F foe each ux∈ ℑ (x)}.

Example (2-2): Let X = {1, 2, 3}, ℑ = {Ø, X, {2}, {3}, {2, 3}}, F = {X, {3}, {2}, {2,1}, {2,3}, {3,1}} ħ = {2}, ε = {2, 3}, then we have that $\hbar^{\#}$ = {1, 2} and $\varepsilon^{\#}$ = {1, 2, 3}.

Note(2-3): Let (X, ℑ ,I) be ideal topological space for any subset of ħ X $\hbar^{\#}$ (F) ⊆ h* (I) with the filter F = (ħ⊆X; $\hbar^{c}$ ∈ I) but the converse may be not true.

Proof: Let x∈$\hbar^{\#}$, then ∀ux∈ℑ (x) ∋ ux∩ħ∈ F which imply that $(ux\cap\hbar)^{c}$ ∈ I, then ux∩ħ∉ I, hance, x ∈$\hbar^{*}$.

Example (2-4): Let x = {1, 2, 3}, ℑ = {Ø, X, {1}, {2}, {1, 2}}, F = {x, {1, 2}} and I = {Ø, {3}} for ħ ={1, 2}, we have $\hbar^{*}$ = {1, 2, 3} and $\hbar^{\#}$ = {3}, so that, $\hbar^{*} \not\subseteq \hbar^{\#}$.

The topical function have many important properties are state in the following proposition.

Proposition (2-5): Let (X, ℑ) bel topological space, F, F1 and F2 are proper filters on X. The topical function has the following properties for any subset ħ and ε of X:

(1) ħ⊆ε→$\hbar^{\#} \subseteq \varepsilon^{\#}$
(2) $(\hbar\cap\varepsilon)^{\#} = \hbar^{\#}\cap\varepsilon^{\#}$
(3) $(\hbar^{\#})^{\#} \subseteq \hbar^{\#}$
(4) $\hbar^{\#}$ = cl ($\hbar^{\#}$) ⊆ cl ($\hbar^{\#}$)
(5) $\hbar^{\#}$-$\varepsilon^{\#}$ = $(\hbar\text{-}\varepsilon)^{\#}$-$\varepsilon^{\#}$ ⊆ $(\hbar\text{-}\varepsilon)^{\#}$
(6) If F1 ⊆ F2 then $\hbar^{\#}$ (F1) ⊆ $\hbar^{\#}$ (F2)
(7) $\hbar^{\#}$ = {emty if ħ ∉F/not empty if ħ∈F
(8) ∀F ∉F, we have $\hbar^{\#}$ = (ħ-F)
(9) $\hbar^{\#}$U$\varepsilon^{\#}$ ⊆ (ħUε)
(10) (ħ-$\hbar^{\#}$)∩(ħ- $\hbar^{\#}$)$^{\#}$ = Ø
(11) ¬∀G∈ℑ then G∩$\hbar^{\#}$ ⊆ $(G\cap\hbar)^{\#}$

Proof: (1) let x∈ħ then U∩ħ ∈F for each Ux ∈ℑ (x) but U∩ħ⊆U∩ε which imply that U∩ε∈F, hence, x ∈$\varepsilon^{\#}$. (2), since, ħ∩ε, so, ⊆ ħ ⊆ $(\hbar\cap\varepsilon)^{\#}$ and ħ∩ε ⊆ ε that $\varepsilon^{\#}$ ⊆ $(\hbar\cap\varepsilon)^{\#}$, thus, $\hbar^{\#}\cap\varepsilon^{\#}$. Now let x ∈ $(\hbar\cap\varepsilon)^{\#}$ then (ħ∩ε) ∩ u ∈F, ∀ux∈ℑ (x),..., 1

If possible x∉ ($\hbar^{\#}\cap\varepsilon^{\#}$), so, x ∉ $\hbar^{\#}$ or x ∉ $\varepsilon^{\#}$ then ∃ w, v ∈ ℑ (x) such that w ∩ħ∉ F or but w ∈ ∩ ℑ (x) and by (1) (ħ ∩ε)∩ w F but (ħ ∩ε)∩ w ⊂ħ ∩ w which imply that ħ ∩w∈F which contradiction.

Similarly, if V∩ε ∉ F. Therefor, x ∈ ($\hbar^{\#}\cap\varepsilon^{\#}$) (3). Let x ∈ $(\hbar^{\#})^{\#}$∀ ux∈ℑ (x) such that ux∩$\hbar^{\#}$ ∈ F, so, we have Ux∩$\hbar^{\#}$ ≠ Ø this empty ∃ z ∈ ux and z ∈ $\hbar^{\#}$, thus ∀ v ∈ℑ (z) ∋ V∩ħ∈ F but z ∈ ux, thus ux∩ħ∈F ∀ ux∈ ℑ (z), hance, x ∈ $\hbar^{\#}$. Therefor $(\hbar^{\#})^{\#} \subseteq \hbar^{\#}$. (4), since, ħ ⊆ cl ($\hbar^{\#}$) ∀ $\hbar^{\#}$ ⊆X, now to show that cl ($\hbar^{\#}$) ⊆$\hbar^{\#}$ Let x ∈ cl ($\hbar^{\#}$) and if possible that x ∉$\hbar^{\#}$, so, ∃ u∈ℑ (x) ∋ u∩ħ∉ F, ..., (1). Since, x∈cl ($\hbar^{\#}$), then ∀ v ∈ℑ (x) ∋v∩$\hbar^{\#}$ ≠ Ø, there exist z ∈X ∋z∈V and z∈$\hbar^{\#}$, now ∀ w ∈ ℑ (x) we have w∩ħ∈ F,.... (2).

Which contribution with (1), then x ∈$\hbar^{\#}$ that is $\hbar^{\#}$ = cl ($\hbar^{\#}$), now to prove cl ($\hbar^{\#}$) ⊆ cl (ħ), Let x∈ cl ($\hbar^{\#}$), so, $\hbar^{\#}$ ∩u ≠ Ø, ∀ u∈ ℑ (x) that is ∃ z ∈ ħ and z ∈ U, so, V ∩ħ ∈ F, ∀ v ∈ ℑ (x), this implies that u∩ħ ∈ F, then u ∩ħ ≠ Ø, ∀ u ∈ ℑ (x) that is x ∈ cl (ħ). Thus, cl ($\hbar^{\#}$) ⊆ cl (ħ), therefor ħ = cl ($\hbar^{\#}$) ⊆ cl (ħ). (5), since $(\hbar\text{-}\varepsilon)^{\#}$-$\varepsilon^{\#}$ ⊆ $(\hbar\text{-}\varepsilon)^{\#}$, now to show that $(\hbar\text{-}\varepsilon)^{\#}$-$\varepsilon^{\#}$ = $\hbar^{\#}$-$\varepsilon^{\#}$, since ħ-ε ⊆ ħ. We get that $(\hbar\text{-}\varepsilon)^{\#}$ ⊆ $\hbar^{\#}$, these are locating to $(\hbar\text{-}\varepsilon)^{\#}$-$\varepsilon^{\#}$ ⊆$\hbar^{\#}$-$\varepsilon^{\#}$, Let us x ∈ $\hbar^{\#}$-$\varepsilon^{\#}$ and if possible x ∉ $(\hbar\text{-}\varepsilon)^{\#}$-$\varepsilon^{\#}$, so, we have follow x ∈$\hbar^{\#}$, x ∉ $\varepsilon^{\#}$ and x ∉ $(\hbar\text{-}\varepsilon)^{\#}$, these are locating to:

- ∀ u ∈ℑ (x) ∋ u∩ħ∈ F,...., 1
- ,∃ v ∈ ℑ (x) ∋ v∩ε ∉ F,...., 2
- And ∃ w ∈ ℑ (x) ∋ w∩(ħ-ε) ∉ F,...., (3)
- But (w∩ħ)∩(w∩ε^{c}) = w∩(ħ∩ε^{c})

So, by condition (2) of filter and the fact if ħ∈F, then $\hbar^{c}$ ∉ F, hence, we get that w∩ħF which contradiction with (1). Thus x ∈ $(\hbar\text{-}\varepsilon)^{\#}$-$\varepsilon^{\#}$. (6) let x ∈ #$\hbar^{\#}$ (F1), so, ∀ u ∈ ℑ (x) ∋ u∩ħ ∈ F1 but F1 ⊆ F2 that is u∩ħ ∈ F2, then x ∈ $\hbar^{\#}$ (F2), then $\hbar^{\#}$ (F1) ⊆ $\hbar^{\#}$ F2. (7) ħ∉ F if possible that $\hbar^{\#}$ ≠ Ø. Then, there exist x ∈ $\hbar^{\#}$, so, ∀ Ux ∈ℑ (x), we have Ux∩ħ ∈ F but Ux ∩ħ⊆ħ. We get that ħ ∈ F which contradiction, thus, $\hbar^{\#}$ = Ø, ∀ ħ ∈F. Now, if ħ∈F that means ħ ≠Ø, then ħ contains at last one point say a , ħ = {a} but ∀ u ∈ ℑ (a) → u∩{a} = {a} ∈ F, thus a ∈ $F^{\#}$ and we get that $F^{\#} \neq Ø$ (8), since, ħ-F⊆ħ imply that $(ħ^{\#}\text{-}F) \subseteq ħ^{\#}$ but b $F^{\#} = Ø$ and $ħ^{\#}\text{-}F^{\#} \subseteq (ħ\text{-}F)^{\#}$, then, we get that $ħ^{\#} \subseteq (ħ\text{-}F)^{\#}$, thus, $ħ^{\#} = (ħ\text{-}F)^{\#}$ (9), since, ħ⊆ħ∪ε, then by part (1) we have $ħ^{\#} \subseteq (ħ\cup ε)^{\#}$, similarly that $ε^{\#} \subseteq (ħ\cup ε)^{\#}$, thus $ħ^{\#}$ U$ε^{\#} \subseteq (ħ\cup ε)^{\#}$. (10) Let x ∈ (ħ-$ħ^{\#}$) and x ∈ $(ħ\text{-}ħ^{\#})^{\#}$, then x ∈ħ, x ∉ $ħ^{\#}$, so, ∃ u ∈ 𝔍 (x) ∋ u ∩ ħ ∉ F...(1).

Also, ∀v ∈ 𝔍 (x) ∋ v ∩ (ħ- $ħ^{\#}$) ∈F,...., (2). But v ∩ (ħ-$ħ^{\#}$) ⊆ v∩ħ imply that v∩ħ ∈ F, ∀ v ∈ 𝔍 (x) which contradiction, thus $(ħ\text{-}ħ)^{\#} \cap (ħ\text{-}ħ)^{\#} = Ø$. (11) le x ∈ G ∩ $ħ^{\#}$, then x ∈ G and $ħ^{\#}$, so, ∀ Ux ∩ ħ ∈F. Now if possible that x ∉ $(G\cap ħ)^{\#}$, so ∃ Vx ∈ 𝔍 (x) ∋ Vx ∩G ∩ ħ ∉ F but Vx ∩ G ∈ 𝔍 (x) which contradiction, hense, x ∈ $(G\cap ħ)^{\#}$. From the a above proposition and the definition of topical function we have the following facts.

1- $(\text{Int } ħ^{\#})^{\#} \subseteq ħ\#$ 2-If F = P (x)/{Ø}, then $ħ^{\#}$ = cl (ħ) for any topological space (X, 𝔍) 3-for any non-indiscrete topological space (X, 𝔍) and any h subset ħ of X with filter F = {X}, then $ħ^{\#} = Ø$. 4-If (X, 𝔍) is indiscrete topological spacer and filter F = {X}, then:

$$ħ^{\#} = \begin{cases} Ø \text{ if } ħ \subset X \\ X \text{ if } ħ = X \end{cases}$$

Proposition (2-6): Let (X, 𝔍) be topological space for any x∈X and ħ ⊆ X, we have:

$$ħ^{\#}(F_x) = \begin{cases} Ø \text{ if } x \notin ħ \\ cl\{x\} \text{if } x \in ħ \end{cases}$$

Where:
F_x = The collection of all supset
ħ = Containing x which is filter on X

Proof: Let x ∉ ħ, then Fx, so, by proposition (2-5) part (7), we have $ħ^{\#}(F_x) = Ø$, now let x ∈ ħ but ∀ p ∈ $ħ^{\#}(F_x)$ iff v ∩ ħ ∈ F ∋∀ v ∈ 𝔍 (p) if f x ∈ v∩ħ ≠Ø, thus, x ∈v → ∀ v ∈ 𝔍 (p)that is v ∩ {x} ≠Ø → ∀ v ∈ 𝔍 (p) iff p ∈ cl {x}, thus, $ħ^{\#}(F_x)$ = cl {x} if x ∈ħ.

Proposition (2-7): Let (X, 𝔍) be topology space, then the following statement tare equivalent:

- (1) $𝔍^c$ ∩ F/X = Ø, when $𝔍^c$ = {G ⊆ X; G^c ∈𝔍}
- (2) If J ∈ f/x, then ext (j) = s Ø

Proof: (1)→ 2) if possible ext (J) ≠Ø that mean there exist ∃x∈ ext (J), then ∃ Ux ∈𝔍 ∋ ux ⊆ J^c that is J⊆ux^c, so, we get that ux^c ∈ F/X which contradiction, thus, ext (J) = Ø. (2)→(1) if possible $𝔍^c$∩ F/X ≠ Ø, so, we get that u ∈ $𝔍^c$ and u ∈ F/X thus u^c ∈ 𝔍 imply ent (U^c) ≠ Ø but ent (U^c) = ext (U) ≠ Ø and ∈F/X which contradiction with (2), thus, $𝔍^c$ ∩ F/x = Ø.

Proposition (2-8): Let (X, 𝔍) be topology t for any subset ħ of X the following are equivalent:

- (1) If $ħ\cap ħ^{\#} = Ø$, then $ħ^{\#} = Ø$
- (2) $(ħ\text{-}ħ^{\#})^{\#} = Ø$

Proof: (1)→(2) let ε = ħ-$ħ^{\#}$, then propositin (2-5) part (10) we have $ε\cap ε^{\#} = Ø$. Thus, by (1), we get, $ε^{\#} = Ø$, so $(ħ\text{-}ħ^{\#})^{\#} = Ø$. (2) → (1) let $ħ\cap ħ^{\#} = Ø$ nut ħ = (ħ-$ħ^{\#}$)#∪($ħ\cap ħ^{\#}$), so, $ħ^{\#} = (ħ\text{-}ħ^{\#})^{\#} = Ø$.

Definition (2-9): Let (X, 𝔍) be topological space and the topical closure of a subset ħ of X is denoted by $cl^{\#}$ ħ and $cl^{\#}$ ħ = ħ∪$ħ^{\#}$.

From the definition (2-9) and proposition(2-5), we have the following proposition.

Proposition(2-10): Let (X, 𝔍) be topological space and for and subset ħ, ε of X .The following are hold:

- (1) $cl^{\#}(cl^{\#}ħ) = cl^{\#}ħ$
- (2) $cl^{\#}(ħ\cup ε) = cl^{\#}ħ \cup cl^{\#} ε$
- (3) $cl^{\#}(ħ\cup ε) = cl^{\#}ħ \cup cl^{\#} ε$
- (4) $cl^{\#}$ X = X
- (5) $cl^{\#}$ Ø = Ø

And so from definition (2-9) and note (2-3), we get that the topology generated by topical function is finer than the topology generated by local function, i.e 𝔍* (I, 𝔍) ⊆ $𝔍^{\#}$ (F, 𝔍) where $𝔍^{\#}$ (F, 𝔍) = {ħ ⊆ X; $cl^{\#}(ħ)^c = ħ^c$} is topology generated by filter F and the topology 𝔍 and is also that is finer than the topology 𝔍. So, that, the subset ħ of a space X is $𝔍^{\#}$- closed iff $cl^{\#}$ ħ = ħ.

Theorem (2-11): Let (X, 𝔍) be topological space and F filter on X. Then the family β = {V∩$ħ^c$; V ∈ 𝔍 and ħ ∉ F}is a basis for $𝔍^{\#}$ (F, 𝔍) (simply $𝔍^{\#}$).

Proof: Let u ∈$𝔍^{\#}$ and x ∈ u, so, X-u is $𝔍^{\#}$-closed, if f $(X\text{-}U)^{\#} \subseteq (X\text{-}u)$ iff U⊆ X-$(X\text{-}u)^{\#}$ that is x ∉ $(X\text{-}U)^{\#}$, hence, exists V ∈ 𝔍 (x) such that V∩(X-u) ∉ F, put ħ = V∩(X-u) and x ∉ ħ, so, x ∈ $ħ^c$ = V^c∪u and V∩ħ ⊆ u. There, for, β is basis for $𝔍^{\#}$.

Definition (2-12): Let (X, 𝔍) be topology space and F be filter on X then a ħ subset of X is called:

- (1) F-open j iff ħ ⊆ Int $ħ^{\#}$
- (2) F-dense iff $ħ^{\#}$ = X
- (3) Locally in F iff $ħ\cap ħ^{\#} = Ø$
- (4) $𝔍^{\#}$-dense in itself iff ħ⊆$ħ^{\#}$
- (5) $𝔍^{\#}$-dense iff $cl^{\#}$ ħ = X

Note (2-13): 1-that every F-dense is $𝔍^{\#}$-dense and the coverage may not true and each *-dense is F-dense. Also, that for any subset ħ of X such that:

- 1-Ifj ħ ∉F, then ħ is F+-open
- 2-If ħ ⊆ $ħ^{\#}$ then ħ is $𝔍^{\#}$-dense iff is 𝔍-dense

- 3-i) For any subset ħ of X and u∈ℑ we have u ∩cl# ħ⊆cl# (ħ∩U)
- ii) If ħ is ℑ#-dense and U∈ℑ then j U ⊆ cl# (ħ∩U)
- 4-If ħ is F-dense and U∈ℑ then j U ⊆ (ħ∩U)#

Note (2-14): Let (X, ℑ) be topology and F1, F2 are filters on X with F1 ⊆ F2. If a subset ħ of X is ℑ# (F1)-dense, then ħ is ℑ# (F2)-dense.

Proof: By using Proposition (2-5) part (6).

Note (2-15): Let ℑ ⊆ σ be two topologies on X and ħ ⊆X for any proper filter F on X if ħ is σ#-dense (σ-dense), then σ is ℑ#-dense (ℑ-dense).

Proof: Since, $cl^{\#}_{\sigma}$ ħ ⊆ cl# ℑ ħ but ħ is σ#-dense→ X = $cl^{\#}_{\sigma}$ ħ→X ⊆ cl# ℑ ħ→ħ is ℑ#-dense.

Defination (2-16): Let (X, ℑ) be a topological space. Then we call that 1-X is F-hypperconnected iff every non-empty ℑ-open set is F-densev. 2-X is ℑ#-hypperconnected iff every non-empty ℑ -open set is ℑ#-dense (3) ℑ#-connected if X cannot be writeen as the union of non-empty and disjoint an open set and a ℑ#-open set of X.

Note (2-17): Every F-hypperconnected→hypperconnected and every F-hypperconnected→ℑ#-hypperconnected →hypperconnected.

Remark (2-18): (1) Generally, it is known that every hypperconnected topological space is connected but not conversely (2). For an topological space (X, ℑ), ℑ⊂ℑ# and we have the following properties: (X, ℑ) is ℑ#-hyperconnected ⇒(X, ℑ) is hyperconnected (X, ℑ) is ℑ#-connected ⇒ (X, ℑ) is connected k. The implications in the diagram are not reversible as shown in the following examples.

Examples (2-19): Let X = {a, b, c}, ℑ = {X, Ø, {a}, {a, b}, {a, c}} and F {X, {a, b, c}}. Then the space (X, ℑ) is hyperconnected but (X, ℑ) is ℑ# connected.

Example (2-20): Let X = {a, b, c, d}, ℑ = {X, Ø, {a}, {c}, {a, b}, {a, c}, {a, b, c, {a, c, d}} and F = {X, {b}, {a, b}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {b, c, d}}. Then, the topological space (X, ℑ) is ℑ#-connected but it is not hyper-connected.

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