Planning Student Number Growth using Progressive Time Series: A Case of a Newly Established University

Lebogang Eunice Sesale, Tshepo Ramarumo and Solly Matshonisa Seeletse
Sefako Makgatho Health Sciences University, Ga-Rankuwa Township, Molotlegi Street, Pretoria, Gauteng Province, South Africa

Abstract: Sefako Makgatho Health Sciences University (SMU) in South Africa aspires to expand access to education and training opportunities for youth by increasing the number of students in education, employment and training. Plans and anticipations were made to intensify ways to make it materialise. This study aim was to predict the growth rates of student numbers such that in 10 year's time, the campus should be able to reach the student count of 10000, starting from the current (year 2017) student count of 3600. It used algebraic and geometric sequences to generate time series student count data for the next 10 years. The methods were tested using Chi-square statistical tests to illustrate the growths that could be used for future planning.

INTRODUCTION

A new university in South Africa was established in 2015, the Sefako Makgatho Health Sciences University (SMU). This university was established in the Medunsa Campus of the University of Limpopo (UL) and thus, inherited students and staff who were originally under UL. It started with about 3600 students. SMU was instructed by the Minister of Higher Education and the Minister of Health to start growing to reach over 10.000 students within 10 years. This instruction was in line with Plessis and Smit[1]'s intimation that university expansion and growth are vital for the nation's economic growth.

The aim of this study was to reveal feasible growth patterns of student numbers in SMU. The study objective was to generate time series data of student numbers rate of growth for the anticipated future SMU.

Study context: This is a mathematical based study using time series methodology due to the chorological nature of the data sets[2] to be used in the investigation. The study benefitted from records of the student numbers collected from the UL student register according to the various faculties. These were tailed by the corresponding numbers during the SMU era. Then, based on the current student numbers and the years anticipated for the growth, various SMU student number growth patterns were developed.

Based on realities of SMU, the university planners can benefit for student recruitment methods to reach the required student count in the 10 years. SMU can also plan the required staffing and infrastructures as well as budget for the required funding.

Time series forecasting
Forecasting context for this study: Forecasting the future SMU student numbers is vital for the university to

plan ahead. There are several ways in which forecasting of the number of students can be done. According to the minister of education, in a message he sent through the then principal of the UL, the number of students was expected to reach a record of about 10.000. The campus had approximately, 2500 students. This study intended to forecast the number of students expected to move from 1 year of study to the next, using the existing student numbers. The context of the study was taken into consideration. This was used to assess possibilities of various forecasting patterns within the context.

Expectations and reality contest the stage. For example, if there are 100 students in year 1 of a 3 years programme it would be expected that all 100 move to year 2 in the next academic year and the same students should reach the third year in the 3rd year of enrolment. However, reality shows that some students repeat or restart year 1, others drop out before the end of their 1st year and others fail. Among those who fail, some come back to repeat while others do not return. This means that fewer than the original 100 students had successfully progressed to year 2. Therefore, if it is 90 of the original 100 who proceeded to year 2, the progression rate from year 1-2 would be 90%.

The progression can also be calculated for the same group when they get to year 3. Since, some of those who progress from year 1-2 do not progress to year 3, the number progressing from year 1-3 in the 3 years involved should be lower than the number from year 2-3. This means that for student numbers to increase, the intake at year 1 should increase dramatically.

In order to forecast the student population for the next academic year, the study considers each year of study separately. The student intake targets added to the expected number of students repeating or restarting year 1, form the forecast for year 1. The year 2 populations is forecast by applying the relevant progression rate to the current year 1 population and this is repeated for years 3-6. The population forecast is then used as the basis for calculating the forecast fee income for the university.

Time series as a forecasting tool: Time series is a sequence of data points measured typically at successive time instants spaced at uniform time intervals[3]. It is a set of observations X_t, each one being recorded at a specific time t. In this study, the time series of consideration are the number of students presented annually, from the year 2015 to the next 9 years, making the time periods to be 10 years. The main use of time series is in forecasting, both as a modelling tool as well as a tool to develop actual forecasts.

In order to forecast, time series uses a model to predict future values based on previously observed values.

Some advantages of time series[4, 6] include that time series analysis helps to identify trend and seasonal variations. This can benefit planning at different time periods according to the way the time series are presented. The forecasts developed can be reasonably accurate in the short-term if the study takes place in a stable environment.

Decomposition of time series: Time series data can be separated into the four components, namely trend, cycle, seasonal and irregular terms which is known as decomposition[7]. Decomposition is defined as a statistical method that deconstructs a time series into these four notional components. An example to decompose time series is to base it on the rates of change. This is when the analyst looks at the trend component T_t that reflects the long term progression of the series. It can also be based on predictability. The theory of time series analysis involves the idea of decomposing a time series into components defined as.

Trend: The trend component is a gradual increase or decrease in the average over time.

Seasonal influence: Seasonal influences are predictable short-term cycling behaviour due to time of day, week, month, season, year and so on.

Cyclical component: A cyclical movement is the unpredictable long-term cycling behaviour due to business cycle or product/service life cycle.

Error term: A random error (also known as irregular term) is the remaining variation that cannot be explained by the other four components.

Forecasting student numbers: Students are expected to start by enrolling at the beginning of the academic year until the end of that year, ideally without disruption. Their count is based on the number recorded on the last day on which cancellations of courses or registrations take place on campus. Those that decided to leave after this day appear as enrolled. Thus, they count as registered students for that academic year. As a result, by definition, students do not leave until the academic year ends. Physical absence is not considered as non-enrolment. This means that in any year the numbers of registered students are considered to be constant. Also, this means that in an academic setting there is no expectation of seasonality in the count of student numbers.

Study exclusions from time series: It was explained that for the whole academic year the students are neither allowed to deregister nor to decline in number. As a result

the numbers of students remain constant for the entire year once they appear in the register on the last day permissible to deregister. Thus, numbers of students in different months are the same. Due to this, seasonality is excluded in this project.

Cycles apply in long term contexts. This study does not extend to long term and thus, it excluded. Growing universities increase their student numbers annually. Thus, the focus of the study is only on the growth trend.

MATERIALS AND METHODS

The study sourced student numbers from the office of the university registrar and confirmed by the officials in the examination department.

Data analysis: Data consisted of student numbers of the various university schools presented as strata. Analysis consisted of the methods of time series for the next 10 years generated using algebraic and geometric sequences and Chi-square tests of hypothesis applied on the observed data. According to Bless and Kathuria[8], Chi-square tests can help in detecting if suitable fits to models have been achieved.

Chi-square comparisons: The student numbers in the various schools were compared using numbers, percentages and then tested to ensure that the methods were consistent in comparisons. The Chi-square tested if the proportions of student numbers were equal.

Generating future time series: The algebraic sequence and geometric sequence were used to generate the future data. These were the time series data for the 10 years to come and were compared using the algebraic and geometric sequences according to the anticipated growth of the SMU.

Mathematical exploration: The main focus is to generate prospect time series using algebraic and geometric series and sequences. Algebraic sequences allow an increase or decrease by a common value while a geometric sequence allows the change in terms of a common ratio[9, 10].

The total student numbers supplied by the SMU registration office for the year 2015 the SMU was for the schools of Medicine (Med.), Health Sciences (H.Sc.), Dentistry (Dent.) and Science and Technology (Sci and Tech). The numbers were given in Table 1.

The dominant faculty in terms of student numbers is medicine and followed closely by the health sciences. Science and technology has a reasonable representation following the health sciences while dentistry is much lower than the rest. However, these discrepancies are

Table 1: Breakdown of the student numbers

School	Student numbers	Percentages
Med.	1438	39.9
H. Sc.	1116	30.9
Dent.	317	8.8
Sci and Tech	736	20.4
Total	3607	100.0

Table 2: Contingency table of the student numbers

School	Observed student numbers	Expected student numbers
Med.	1438	901.75
H. Sc.	1116	901.75
Dent.	317	901.75
Sci and Tech	736	901.75

Table 3: Current and required student numbers

Faculty	Current student numbers	Required student numbers
Med.	1438	2500
H. Sc.	1116	2500
Dental	317	2500
Science and Tech.	736	2500

evaluated using the Chi-square tests. For testing the hypothesis, the null hypothesis states that each $p = 0.25$, it means that the four schools share the student number equally. The contingency table for this becomes (Table 2). The Chi-square statistic is calculated as follows:

$$\chi^2 = \sum_{i=1}^{4} \frac{(o-e)^2}{e}$$

$$= \frac{(1438-901.75)^2}{901.75} + \frac{(1116-901.75)^2}{901.75} +$$

$$\frac{(317-901.75)^2}{901.75} + \frac{(736-901.75)^2}{901.75}$$

$$= 779.4541$$

The Chi-square involved had k-1 = 3 degrees of freedom. At the 5% level of significance, the critical value is 7.815. Since, the test statistic is larger than this value, the hypothesis of equal proportions is rejected. Hence, the conclusion is that there is enough reason to believe that the student numbers in the four schools are different.

With the student numbers currently, the statement from the Ministers of Education and Health was that in 10 year's time the new university should have 10.000 students. Thus this can be explained in Table 3.

Generating forecast time series over 10 years: Algebraic and geometric series are used to generate time series of expected student numbers in 10 year's time, first for the entire student body and then for each individual school. The total of all students = 3607.

Algebraic difference: Algebraic sequence is: a, a+2d, ..., a+(n-1)d. Here, a is the student number count at the start

Table 4: Projected time series of the student numbers

Years (t)	f-values
1	3607
2	4318
3	5029
4	5740
5	6451
6	7162
7	7873
8	8584
9	9295
10	10006

t = years 1-10; f = student numbers for the years given

of the study, d is the common difference describing the annual increase of the student numbers. Given that in 2015:

$$n = 10 \text{ years } (n-1) = 9, a = 3607; \text{ then}$$
$$a+(n-1)d = 3607+9d = 10000$$
$$9d = 1000-3607 = 6393$$
$$d = 710.33$$

The total for the increases in student numbers for years 1-10 to reach the anticipated 10000 students, distributed according to an annual increase of about 711 students (Table 4). Thus, using this method, the projected time series of student numbers on campus for 10 years (year t = 1-10) is:

Students numbers in medicine:

$$a+(n-1)d = 1438+9d = 2500$$
$$9d = 2500-1438 = 1062$$
$$d = 118$$

Thus, using algebraic increase method, the awaited time series of student numbers for the next 10 years is (Table 5).

Students numbers in health sciences:

$$a+(n-1)d = 1116+9d = 2500$$
$$9d = 2500-1116 = 1384$$
$$d = 154$$

Subsequently, using algebraic sequencing method, the anticipated time series of the student numbers on campus for the next 10 years is Table 6.

Students numbers in dentistry:

$$a+(n-1)d = 317+9d$$
$$317+9d = 2500$$
$$9d = 2500-317 = 2183$$
$$d = 243$$

Table 5: Projected time series of the medicine student numbers

Years (t)	f-values
1	1438
2	1556
3	1674
4	1792
5	1910
6	2028
7	2146
8	2264
9	2382
10	2500

Table 6: Projected time series of the health sciences student numbers

Years (t)	f-values
1	1116
2	1270
3	1424
4	1578
5	1732
6	1886
7	2040
8	2194
9	2348
10	2502

Table 7: Projected time series of the dental student numbers

Years (t)	f-values
1	317
2	560
3	803
4	1046
5	1289
6	1532
7	1775
8	2018
9	2261
10	2504

Accordingly, using algebraic sequencing method, the anticipated time series of the student numbers on campus for the next 10 years is Table 7.

Students numbers in science and technology:

$$a+(n-1)d = 736+9d = 2500$$
$$9d = 2500-736 = 1764 = 196$$

Hence, using the algebraic sequence method, the estimated time series of student numbers for the next 10 years is Table 8.

Geometric ratio: The geometric sequence is given by: a, ar, ar^2, ..., ar^{n-1}. Here, a is the initial student number count at the beginning of the investigation, r is the common ratio which describes the annual increment of the student numbers:

$$ar^{n-1} = 3607r^9 = 10.000$$
$$r^9 = 2.772387025$$
$$r = \sqrt[9]{2.772387025} = 1.12$$

Therefore, using this method, the anticipated time series of the student numbers on campus for the next 10 years is Table 9.

Table 8: Projected time series of the science and technology student numbers

Years (t)	f-values
1	736
2	932
3	1128
4	1328
5	1520
6	1716
7	1912
8	2108
9	2304
10	2500

Table 9: Projected time series of the student numbers

Years (t)	f-values
1	3607
2	4040
3	4525
-4	5068
5	5676
6	6357
7	7120
8	7974
9	8931
10	10003

Table 10: Projected time series of medicine student numbers

Years (t)	f-values
1	1438
2	1525
3	1616
4	1713
5	1816
6	1925
7	2040
8	2163
9	2295
10	2430

Table 11: Projected time series of the health sciences student numbers

Years (t)	f-values
1	1116
2	1216
3	1326
4	1446
5	1576
6	1717
7	1871
8	2040
9	2224
10	2424

Table 12: Projected time series of the dental student numbers

Years (t)	f-values
1	317
2	400
3	504
4	635
5	799
6	1007
7	1269
8	1599
9	2014
10	2538

Table 13: Projected time series of science and technology student numbers

Years (t)	f-values
1	736
2	847
3	974
4	1120
5	1288
6	2481
7	1703
8	1958
9	2252
10	2590

Students numbers in medicine:

$$ar^{n-1} = 1438r^9 = 2500$$
$$r^9 = 1.73852573$$
$$r = \sqrt[9]{1.73852573} = 1.063375846$$

The study uses r = 1.06. Subsequently, using the geometric ratio method, the anticipated time series of the student numbers for the next 10 years follow Table 10.

Student numbers in health sciences:

$$ar^{n-1} = 2500$$
$$1116r^9 = 2500$$
$$r^9 = 2.240143369$$
$$r = \sqrt[9]{2.240143369} = 1.093753699$$

The study uses r = 1.09. Thus, using the geometric ratio, the anticipated time series of the student numbers on campus for the next 10 years would be Table 11:

Students numbers in dentistry:

$$ar^{n-1}$$
$$317r^9 = 2500$$
$$r^9 = 7.886435331$$
$$r = \sqrt[9]{7.88643533} = 1.257921142$$

The study uses r = 1.26. As a result, when using the geometric ratio, the projected time series of the student numbers on campus for the next 10 years would be Table 12.

Student numbers in science and technology:

$$ar^{n-1} = 736r^9 = 2500$$
$$r^9 = 3.39673913$$
$$r = \sqrt[9]{3.39673913} = 1.145531169$$

The section of the paper uses r = 1.15. As a result, if the geometric ratio were to be used, the anticipated time series of the student numbers enrolled in SMU for the next 10 years would be (Table 13). Table 9 displays the projected annual time series counts of all the students. Table 14-28, then, split these counts according to their

Table 14: Expected student numbers for SMU

Years (t)	d	r
1	3607	3607
2	4318	4040
3	5029	4525
4	5740	5068
5	6451	5676
6	7162	6357
7	7873	7120
8	8584	7974
9	9295	8931
10	10006	10003

Table 15: Required annual increases of student numbers

Years (t)	d	r
1	-	-
2	711	433
3	711	485
4	711	543
5	711	608
6	711	681
7	711	763
8	711	854
9	711	967
10	711	1072

Table 16: Increases in student numbers relative to the 1st year

Years (t)	d	r
1	-	-
2	711	433
3	1422	918
4	2133	1461
5	2844	2505
6	3555	2750
7	4266	3513
8	4977	4567
9	5688	5324
10	6399	6396

Table 17: Projected time series of medical student numbers

Years (t)	d	r
1	1438	1438
2	1556	1525
3	1674	1616
4	1792	1713
5	1910	1816
6	2028	1925
7	2146	2040
8	2264	2163
9	2382	2292
10	2500	2430

Table 18: Required annual increases of medical student numbers

Years (t)	d	r
1	-	-
2	118	87
3	118	91
4	118	97
5	118	103
6	118	109
7	118	115
8	118	123
9	118	129
10	118	138

Table 19: Increases in medical student numbers relative to the first year

Years (t)	d	r
1	-	-
2	118	87
3	236	178
4	354	275
5	472	378
6	590	487
7	708	602
8	826	725
9	944	854
10	1062	992

Table 20: Projected time series of health science student numbers

Years (t)	d	r
1	1116	1116
2	1270	1216
3	1424	1326
4	1578	1446
5	1732	1576
6	1886	1717
7	2040	1871
8	2194	2040
9	2348	2224
10	2505	2424

Table 21: Required annual increases of health science student numbers

Years (t)	d	r
1	-	-
2	154	100
3	154	110
4	154	120
5	154	130
6	154	141
7	154	154
8	154	169
9	154	184
10	154	200

Table 22: Increases in health science student numbers relative to the 1st year

Years (t)	d	r
1	-	-
2	154	100
3	308	210
4	462	330
5	616	460
6	770	601
7	924	755
8	1078	924
9	1232	1108
10	1386	1303

Table 23: Projected time series of the health science student numbers

Years (t)	d	r
1	317	317
2	560	400
3	803	504
4	1046	635
5	1289	799
6	1532	1007
7	1775	1269
8	2018	1599
9	2261	2014
10	2504	2538

Table 24: Required annual increases of medical student numbers

Years (t)	d	r
1	-	-
2	243	83
3	243	104
4	243	131
5	243	164
6	243	208
7	243	262
8	243	330
9	243	415
10	243	524

Table 25: Increases in medical student numbers relative to the first year

Years (t)	d	r
1	-	-
2	243	83
3	486	187
4	729	318
5	972	482
6	1215	690
7	1458	952
8	1701	1282
9	1944	1697
10	2187	2221

Table 26: Projected time series of the science and technology student numbers

Years (t)	d	r
1	736	736
2	932	847
3	1128	974
4	1324	1120
5	1520	1288
6	1716	1481
7	1912	1703
8	2108	1958
9	2304	2252
10	2500	2590

Table 27: Required annual increases of science and technology student numbers

Years (t)	d	r
1	-	-
2	196	111
3	196	127
4	196	146
5	196	168
6	196	193
7	196	222
8	196	255
9	196	294
10	196	338

Table 28: Increases in science and technology student numbers relative to the 1st year

Years (t)	d	r
1	-	-
2	196	111
3	392	238
4	588	384
5	784	552
6	980	745
7	1176	967
8	1372	1222
9	1568	1516
10	1764	1854

respective schools. Table 10-13 provide the breakdown of Table 9 as the annual student count time series according to the different schools to which the learners belong. In both cases, the rates of algebraic and geometric learner count are displayed next to each other.

Yearly increases and increases relative to origin: The anticipated increases in the student numbers from current year to the next are given below. The following tables present projections in terms of algebraic against geometric series rule. The first table presents annual increases in student numbers and then tailed immediately by numbers of students who will be enrolled in the years in question.

Medical students: The increases, relative to the first years are as follows:

Health science students: The increases, relative to the first year are as follows:

Dental students: The increases, relative to the first year appear below.

Science and technology: The increases, relative to the first year are as follows: Table 28. Increases in science and technology student numbers relative to the 1st year.

Table 15-28 presents the increases in student numbers according to the two standard methods, algebraic and geometric series. The student numbers are stated relative to the numbers in the 1st year of the institution.

It is generally, expected that the geometric series would yield larger expect numbers due to being multiplicative. However, the smaller starting values of student numbers allowed the additive algebraic series expectations to be larger. There is a possibility of separating each school to follow a method that suits its capabilities and needs.

RESULTS AND DISCUSSION

Entire student body: The projected student numbers increase for the initial year t = 1 to final year t = 10 is 277.24%. The annual increase is constant with the algebraic increase and increases gradually with the geometric ratio. However, for the 10 years this is an annual average increase of about 28%. The campus as a whole requires a large increase in order to realise the targeted numbers of students in the tenth year as expected. Medicine requires the least increase while dental faculty requires an overwhelmingly large increase in student numbers. The percentage increases required in the faculties of health sciences and science and technology are also high with the latter requiring more compared to the former.

Medical students: The anticipated increase for the initial year t = 1 to final year t = 10 is 173.85%. The annual increase is constant with the algebraic increase and increases gradually with the geometric ratio. This, for the 10 years in question is an average increase of about 17%.

Dental students: The anticipated increase for the initial year t = 1 to final year t = 10 is 788.64%. The annual increase is constant with the algebraic increase and increases gradually with the geometric ratio. On average, the annual increase in the 10 years is about 79%.

Health science students: The anticipated increase for the initial year t = 1 to final year t = 10 is 224.01%. The annual increase is constant with the algebraic increase and increases gradually with the geometric ratio. The annual average increase in the 10 years is about 22%.

Science and technology: The anticipated increase for the initial year t = 1 to final year t = 10 is 339.67%. The annual increase is constant with the algebraic increase and increases gradually with the geometric ratio. The average annual increase in the 10 years is about 38%.

Interpretations: Differences show slower growth when algebraic series is used and quicker growth when the geometric series is used. These disparities are important when a decision is made by SMU management to plan its future student growth.

The geometric predictions are always higher than the algebraic predictions by the nature. The choice of which method to adopt by the institution should be based on the capability of SMU to handle growing student trends. If the resources allow quicker student number growth, the geometric pattern should be adopted. Where there are no resources to handle quick growth, algebraic series should be used for student number expansions. Thus, the factors affecting student growth are the resources of space, staffing and finances for increased dynamics.

CONCLUSION

Both the algebraic and geometric series give student growths for every student group. If SMU can manage to grow quicker, then the student number growths to use are the ones given by the geometric series. Where slow growth can be the only option, the algebraic series values should be used for student number growths.

RECOMMENDATIONS

It is recommended that the spaces and staffing on SMU should be increased gradually, over the years to accommodate the anticipated student numbers.

REFERENCES

01. Plessis, S. D. and B. Smit, 2007. South Africa's growth revival after 1994. J. Afr. Econ., 16: 668-704.
02. Granger, C.W.J., 1981. Some properties of time series data and their use in econometric model specification. J. Econometrics, 16: 121-130.
03. Bowerman, B.L. and R.T. O'Connell, 1993. Forecasting and Time Series: An Applied Approach. 3rd Edn., Duxbury Press, New York, USA., ISBN: 9780534379698, Pages: 726.
04. Box, G.E.P. and G.M. Jenkins, 1976. Time Series Analysis: Forecasting and Control. Holden-Day Publisher, Oakland, CA., USA., ISBN-13: 9780816211043, Pages: 575.
05. Chatfield, C., 1993. Calculating interval forecasts. J. Bus. Econ. Stat., 11: 121-135.
06. Durbin, J. and S.J. Koopman, 2001. Time Series Analysis by State Space Methods. Oxford University Press, Oxford, UK., ISBN: 9780198523543, Pages: 253.
07. Percival, D.B. and A.T. Walden, 1993. Spectral Analysis for Physical Applications. Cambridge University Press, Cambridge, USA., ISBN: 9780521435413, Pages: 583.
08. Bless, C. and R. Kathuria, 1993. Fundamentals of Social Statistics: An African Perspective. Juta and Co., Johannesburg.
09. Butcher, D.F. and W.A. Muth, 1985. Predicting performance in an introductory computer science course. Commun. ACM., 28: 263-268.
10. Campbell, P.F. and G.P. McCabe, 1984. Predicting the success of freshmen in a computer science major. Commun. ACM., 27: 1108-1113.