

## Algorithms Heuristic for Solving the of Open Vehicle Routing Problem

Byron Campo Zuniga, Adel Mendoza Mendoza and Daniel Mendoza Casseres  
Universidad del Atlantico, Barranquilla, Colombia

**Abstract:** This study describes and compares the performance of two heuristics (PST-Prim and modified nearest neighbor algorithm) with three extended use heuristic algorithms (nearest neighbor, random solution and trivial solution). The five techniques are used to solve the problem of open routing of OVRP vehicles. Seventeen comparative problems of extended use were used. The technique that obtained the best performance in terms of objective function (in 82% of the solutions) and route with the least travel (in 71% of the solutions) was the PST-Prim algorithm. The trivial solution algorithm was the one that consumed the least execution time.

**Key words:** PST-Prim algorithm, performance, heuristics, techniques, trivial solution, consumed

### INTRODUCTION

In every vehicle routing problem, the aim is to design the routes of a transport fleet that serves a set of customers. In the classic problem or VRP (Vehicle Routing Problem), each vehicle starts from a single main depot, runs through a certain number of pre-assigned customers and finally, returns to the starting point. Customers or nodes can be visited only once by a single vehicle and the aggregate demand of all nodes assigned to a vehicle must not exceed its capacity (Braekers *et al.*, 2016). An Open Vehicle Routing Problem (OVRP) is a trivial variant of the classic VRP and the difference lies in that once each vehicle serves each and every node that has been assigned to it is not required to return to the main depot in other words, every route in the OVRP departs from the initial depot and necessarily ends in a customer. From this perspective, the OVRP has great appeal to organizations that choose to hire, lease or outsource a fleet of vehicles instead of acquiring their own (Ngueveu *et al.*, 2010; Brito *et al.*, 2015; Hosseinabadi *et al.*, 2018).

The OVRP was formally formulated for the first time in the research of Sariklis and Powell (2000) and is defined as a non-directed graph  $G = (V; A)$  where, the zero node  $v_0$  is the deposit and  $N = \{v_1, \dots, v_n\}$  is the set of clients. In this way,  $V = \{v_0, \dots, v_n\}$  is the set of all the nodes of the graph  $G$  with  $v_0 \cap N = \emptyset$ . Every node  $v_i$  in  $N$  ( $1 \leq i \leq n$ ) has an associated constant demand  $q_i > 0$ , the deposit or zero node has null demand ( $q_0 = 0$ ). The set of arcs is denoted by  $A$ . Each arc  $(i; j)$  in  $A$ , con  $i \neq j$  y  $j \neq 0$  has associated a constant cost or weight  $d_{ij} > 0$  which is the distance that a vehicle must travel to go from node  $i$  to  $j$ . In the distance matrix  $D$ , the  $i$ th row is the starting node and the  $j$ th row is the finishing node:

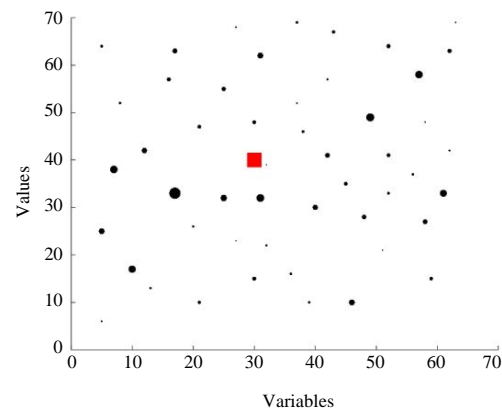


Fig. 1: Graphic representation of a vehicle routing problem

$$D = \begin{pmatrix} d_{11} & d_{12} & \dots & d_{1n} \\ d_{21} & d_{22} & \dots & d_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ d_{n1} & d_{n2} & \dots & d_{nn} \end{pmatrix}$$

Figure 1 shows a set of points in the Euclidean plane that model a vehicle routing problem with a deposit or zero node (red box), customers are located at the black dots. The size of these points is a relative measure of their demand.

To try to give solution to the OVRP, different methods have been proposed over time: exact, heuristic, metaheuristic and recently hyperheuristic (Fu *et al.*, 2005; Yousefikhoshbakht *et al.*, 2014; Tyasnurita *et al.*, 2017). However, this research is limited to the comparison of the performance of five constructive heuristic techniques that are not only capable of providing solutions in computational times lower than those of high-level

techniques (such as metaheuristics and hyperheuristics) but also serve as a start-up phase in perturbative algorithms, providing them with an initial solution. These techniques are PST-Prim algorithm, nearest neighbor algorithm, modified nearest neighbor algorithm, random solution algorithm and trivial solution algorithm. The relative performance of these five algorithms will be evaluated by comparing the value of the objective function of the solution proposed by each algorithm.

## MATERIALS AND METHODS

The methodological design of this research is of quantitative type supported in the application of the five algorithms. These algorithms can have as restrictions, separately or simultaneously, the capacity of the vehicles ( $Q$ ) and the total distance travelled by route ( $L_m$ ), among others. In particular, the algorithms PST-Prim, the nearest neighbor and the modified nearest neighbor algorithm are sequential insertion heuristics. These are constructive methods in which a solution is created by successive insertions of nodes in the routes. In each iteration, there is a partial solution whose routes only visit a subset of customers, selecting in each iteration an unvisited customer to be inserted.

**PST-Prim algorithm:** This algorithm is based on Prim's algorithm (Prim, 1957; Jarnik, 1930; Dijkstra, 1959). With Prim's algorithm it is possible to find a Minimum Spanning Tree (MST) in any non-directed and related graph. These result to be subgraphs of the original and for its construction, the nodes that constitute it dispose of unrestricted degree of freedom ( $df_0$ ). In order to research, the Prim algorithm starts from any vertex and in each iteration it looks for the edge that has the least weight of all those that start from the current MST and end in some node of the original graph that does not belong to the current MST, ties are broken arbitrarily. This is how Prim's algorithm prevents the appearance of subcycles. This process ends when all the nodes of the original network have been inserted into the MST. As expected in the context of the OVRP, the nodes in a solution will not have degrees of freedom  $>2$ , so, it is necessary to define a tree coating solution with characteristics that fit the particular conditions of the OVRP. An example of this type of construction is the one cited by Brandao (2004). This research makes use of the  $k$ -Degree Centre Tree ( $k$ -DCT) proposed by Christofides *et al.* (1981) which is a tree that starts from a Degree-Constrained Minimum Spanning Tree (DCMST) to give solution to the VRP. Regarding the degrees of freedom of the nodes, a  $k$ -DCT has the following restrictions:

- The deposit or zero node has a degree of freedom between one and  $k$  ( $1 \leq df_0 \leq k$ )
- The client nodes have degree of freedom equal to two ( $df_c \leq 2; \forall c \in N$ )

Brandao (2004) proposes as a solution to the OVRP to make use of the  $k$ -DCT and once built all the arcs ( $i, 0$ ) in  $A$  that have been generated in the solution are eliminated. With this proposal the closed routes of the  $k$ -DCT become open.

To apply then the heuristic PST-Prim it is required of the construction of a covering tree from the characteristics of the OVRP and it is achieved adopting the first restriction in degrees of freedom of the  $k$ -DCT but admitting a relaxation in the second one:

- The deposit or zero node has degree of freedom between one and  $k$  ( $1 \leq df_0 \leq k$ )
- The client nodes have degree of freedom between one and two ( $df_c = 2; \forall c \in N$ )

With this relaxation, every algorithm used to give solution to the OVRP is allowed to establish, according to criteria, the nodes in which the routes will end. These nodes will have  $df = 1$ . The covering tree that arises from admitting this relaxation has the name Path-Spanning Tree (PST). In the context of the PST-Prim heuristic, to give solution to the OVRP is to try to construct a minimum PST in which the routes surpass the restrictions of capacity and/or of total distance travelled of the routes.

Contrary to Prim's algorithm, PST-Prim heuristics requires that the starting node is always  $v_0$  (the deposit). This heuristic searches in each iteration the arc with less weight than part of the tank or a node "branch front" and ends in a node of the original graph not included in the PST under construction. A branch front node is the one that until the current iteration has been entered into the PST (under construction) and has a degree of freedom equal to one ( $df = 1$ ). Similar to Prim's proposal, this procedure ensures that no subcycles are given. Figure 2a shows the solution to the OVRP problem using this technique.

**Nearest neighbor algorithm:** The Nearest Neighbor algorithm (kNN) generates routes joining vertices, points or nodes, taking into account the corner with the least distance or cost of a point to the last point entered. This sequence of insertion of points, starts from the main node and then incorporates the nearest point or the least expensive. Then, the search for the next point begins which will be inserted if it has the shortest distance or the lowest cost of the rest, ties are broken arbitrarily. Unlike

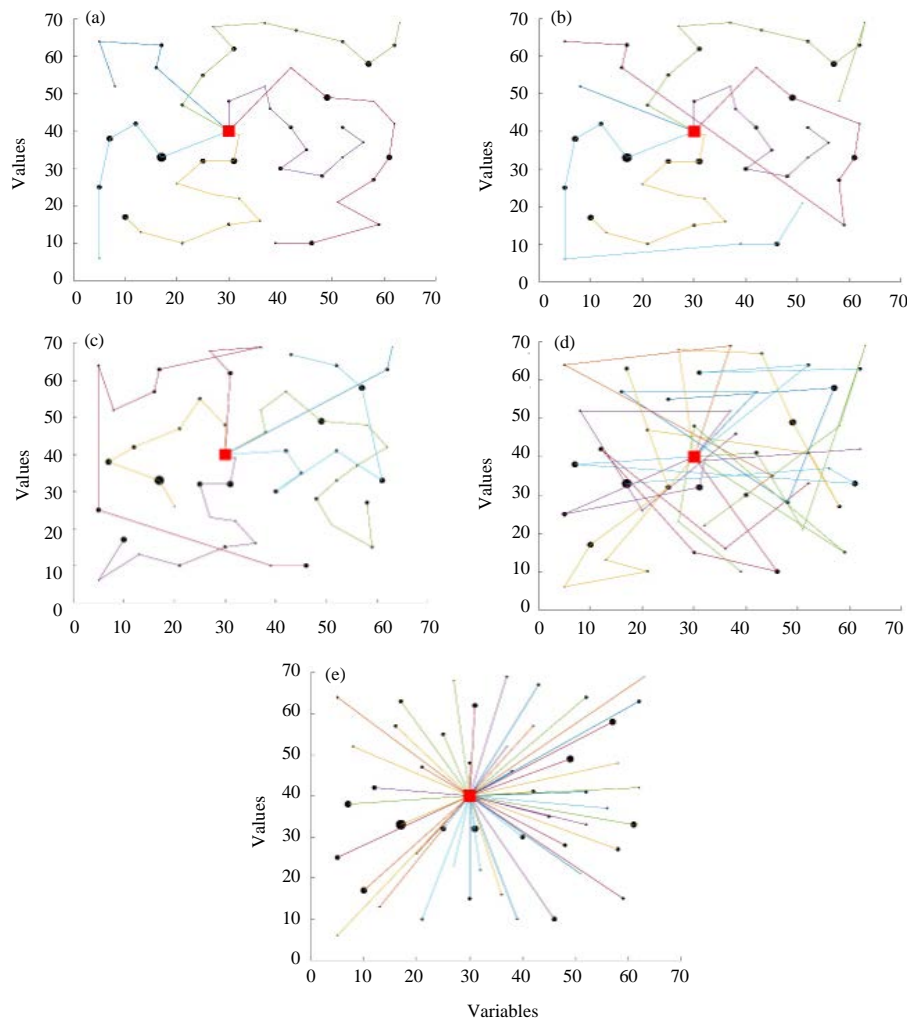


Fig. 2: a-e) The solution to the OVRP problem using this technique

PST Prim heuristics, kNN only considers inserting clients in the last route created. Figure 2b shows the solution to the OVRP problem using this technique.

**Modified nearest neighbor algorithm:** The Modified Nearest Neighbor algorithm (MKNN) is the result of a modification of the kNN inspired by Eppen (2000) 's proposal for the distance matrix  $D$  which once determined, requires obtaining the maximum per column  $m_j (m_j = \max(d_j), \forall_j$  where,  $d_j$  is the  $j$ th vector column of  $D$ ). The modified distance matrix  $D^*$  is obtained as shown as:

$$D^* = \begin{pmatrix} m_1 - d_{11} & m_2 - d_{12} & \dots & m_n - d_{1n} \\ m_1 - d_{21} & m_2 - d_{22} & \dots & m_n - d_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ m_1 - d_{n1} & m_2 - d_{n2} & \dots & m_n - d_{nn} \end{pmatrix}$$

This modification is supplied to the nearest neighbor algorithm. Figure 2c shows the solution to the OVRP problem using this technique.

**Random solution algorithm:** The random solution algorithm randomly builds without any given recursive procedure, a solution for the OVRP that may or may not eventually satisfy all the model's constraints. Initially it prescribes the number of routes and randomly places all customers on them. Once this is done it evaluates on the resulting solution the restrictions of the model. If the solution complies with the restrictions it is accepted otherwise, a new attempt is made to randomly assign customers. As the algorithm may employ an unreasonable number of attempts at customer allocation, a number of allowed attempts is set and if no viable solution is found, a relaxation is made on the preset number of vehicles.

Once this is done, a new cycle of attempts is carried out. This process is repeated until the algorithm achieves a solution that complies with each and every one of the restrictions of the problem. Figure 2d shows the solution to the OVRP problem using this technique. It is observed that from the node zero or deposit a random number of routes that do not conserve a preconceived construction criterion.

**Trivial solution algorithm:** This algorithm simply consists of assigning a vehicle to each client. Figure 2e shows a graphical example of the behavior of this algorithm over the example problem in Fig. 1. It shows that from the zero node or deposit (red box) starts a number of routes equal to the number of points, each of these routes assigns only the point they visit.

## RESULTS AND DISCUSSION

The algorithms described above were programmed in a script.m and ran in the numerical analysis Software GNU Octave 4.2.1 on Lubuntu Linux 17.04 as operating system. A computer with Intel Core i3-2330 processor and 6 GB of RAM was used. The algorithms were tested on a selection of widely used problems for comparative purposes (Hosseiniabadi *et al.*, 2018). This selection is based on the proposals of Christofides (1979) and Fisher (1994) which are datasets of problems provided by the researchers. As a general characteristic, all the problems are euclidean and symmetrical. There are a total of 17 comparative problems, 14 proposed by Christofides identified as CMT

and 3 by Fisher identified as F. In Fig. 3-5, the comparison of the performance of each algorithm in each problem is shown graphically.

Figure 3 shows that in 14 of the 17 problems (82% of the problems), the PST-Prim heuristic is the one that proposes solutions with lower objective function values. The heuristics of the nearest neighbor and the heuristics of the modified nearest neighbor are close to each other above the PST-Prim. However, it is remarkable the difference in performance that these three have with the remaining two algorithms.

Excluding proposals for trivial solution, Fig. 4 shows how in 12 of the 17 problems (71% of the problems), the PST-Prim heuristic is the one that delivers routes with the shortest distance travelled possible. The comparison of this algorithm with the rest is made without including the heuristic proposal of trivial solution because in the latter case, the vehicles are obliged to visit a single client which goes against what is expected in the management of real transport logistics systems. However, this algorithm is particularly important due to the ease with which it can be programmed in comparison with other comparative algorithms which makes it highly desirable in metaheuristic and hyperheuristic implementations.

As shown in Fig. 5, the trivial solution algorithm is the least time consuming to deliver a viable solution. One can see how the heuristic of random solution is the one that takes the most time and in particular the problems CMT6-CMT10. This is explained by the fact that these problems would be identical to the problems CMT1-CMT5 but it is because the latter only have a capacity restriction

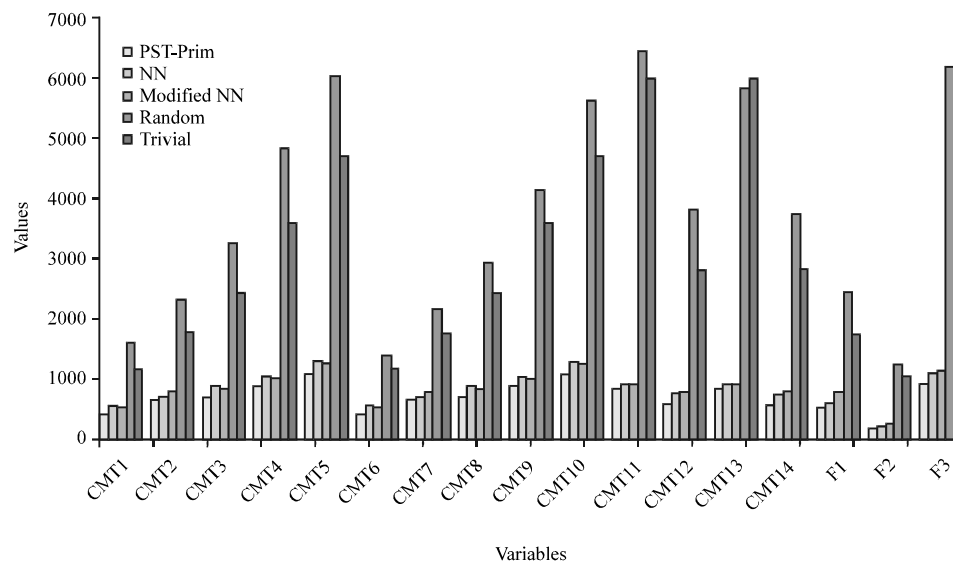


Fig. 3: Objective function values of the solutions provided by the algorithms

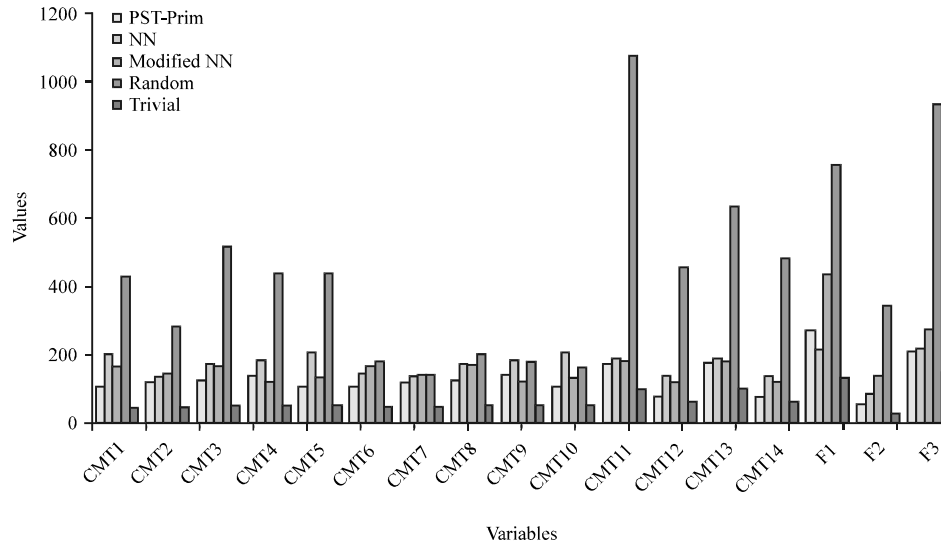


Fig. 4: Values of the total distance travelled by the longest route of the solutions provided by the algorithms

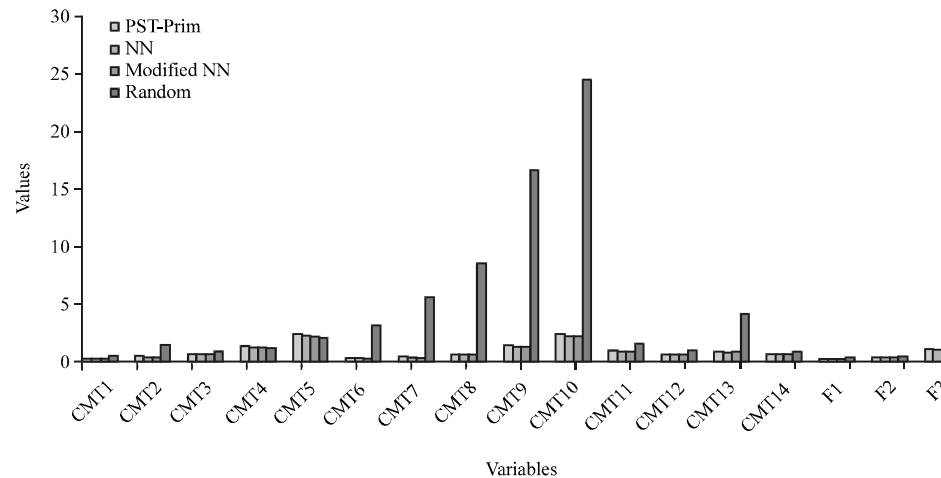


Fig. 5: Values of resource consumption time used by algorithms to provide a solution to each problem

(Q) while the former in addition to this restriction have the total distance travelled per route ( $L_m$ ), an additional restriction requires a greater computational effort.

## CONCLUSION

In addition, the total distance travelled by the longest route of the solution proposed by each algorithm will be compared it is desired that the longest route of a solution be the shortest possible. The third and last comparison variable is the time resource consumed by each algorithm to provide a solution to a given problem.

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