Journal of Engineering and Applied Sciences 14 (7): 2321-2332, 2019
ISSN: 1816-949X

Calculate the Head Loss for the Circular Pipes using the Simulink/MATLAB

Ayad Kadhim Hussein
Department of Hydraulic Structures and Water Resources Engineering, Faculty of Engineering, University of Kufa, Najaf, Iraq

Abstract: The coefficients of Darcy-Weissbach, Chezy, Hazen-Williams and Manning equations are presented and calculated as well as the dissipation or loss of the head of each of the equations mentioned by Simulink/MATLAB. However, the main point is to find the friction factor for Colebrook-White equation which is easily solved by Simulink/MATLAB which means that it is solved without any complexity or effort. It has been shown that the Chezy constant is <50 and the Hazin coefficient is <20 and that the Manning coefficient is always lower than the special friction factor of Darcy-Weisbach for the circular pipe diameter smaller than or equal to 250 cm.

Key words: Darcy-Weissbach equation, Chezy equation, Hazen-Williams equation, Manning equation, Colebrook-White equation, head loss equation, Simulink/MATLAB

INTRODUCTION

It is clear that the distinction between the three flow forms based on Reynold's number which is presented by Eq. 1 are the laminar flow, the flow of transition and the turbulent flow in both smooth and rough pipes circular pipes depending on the relative roughness (Cengel and Cimbala, 2006). The amount of head loss or squandering resulting from friction during the flow of these different cases was calculated on the basis of the use of four famous equations, namely the Darcy-Weisbach, Chezy, Hazen-Williams and Manning equations are described in Eq. 2-5, respectively (White and Chul, 2016):

$$Re = \frac{V_{avg} D * 10^4}{\upsilon_{stoke}} \quad (1)$$

$$h_{f_{Darcy}} = f \frac{L}{D} \frac{V_{avg}^2}{2g}$$

$$R_{hydraulic} = \frac{D}{4} \text{ and } S = \frac{h_{f_{Chezy}}}{L} = \frac{h_{f_{Hazen}}}{L} = \frac{h_{f_{Manning}}}{L} \quad (2)$$

$$V_{avg} = C_{Chezy} \sqrt{R_{hydraulic} S} \xrightarrow{\text{yields}} h_{f_{Chezy}} = \frac{4LV_{avg}^2}{C_{Chezy}^2 D}$$

$$V_{avg} = 0.849 C_{Hazen} R_{hydraulic}^{0.63} S^{0.54} = 0.849 C_{Hazen} \left(\frac{D}{4}\right)^{0.63} \left(\frac{h_{f_{Hazen}}}{L}\right)^{0.54} \quad (3)$$

$$h_{f_{Hazen}} = \frac{6.824 L^{1.852} V_{avg}^{1.852}}{C_{Hazen}^{1.852} D^{1.1668}} \quad (4)$$

$$V_{avg} = \frac{1}{n} R_{hydraulic}^{0.6667} S^{0.5} = \frac{1}{n} \left(\frac{D}{4}\right)^{0.6667} \left(\frac{h_{f_{Manning}}}{L}\right)^{0.5}$$

$$h_{f_{Manning}} = \frac{6.3496 n^2 V_{avg}^2 L}{D^{4/3}} \quad (5)$$

where, h_f is the dispersion or loss of the head relative to the calculated equation by reference to the known name of the equation used, f is the friction factor = $4*f_{Fanning}$ where, $f_{fanning}$ is the coefficient of fiction or Fanning friction, D is the hydraulic diameter, L is length of circular pipe, V_{avg} is the average velocity of fluid, Re is the Reynolds number, $R_{hydraulic}$ is the hydraulic radius, υ_{stroke} is the kinematc viscosity in stoke units, L is the pipe length, g is the ground acceleration = 9.81 m/sec^2 and S is the energy slope or head loss gradient.

If the constants of the previous equations are obtained, they can be applied in Eq. 2-5, otherwise we will rely on the friction factor in the Darcy-Weisbach equation which it is fundamental to the calculation of other constants related to Eq. 3-5 are the Chezy's constant C_{Chezy}, roughness coefficient C_{Hazen} of the pipe for Hazen-Williams as shown in Eq. 7 and 8, respectively. I have reached an equation that links the Manning's roughness coefficient n with the coefficient C_{Hazen} of the Hazen-Williams equation (Eq. 9). If:

$$h_{f_{Darcy}} = h_{f_{Chezy}} = h_{f_{Hazen}} = h_{f_{Manning}} \quad (6)$$

$$f \frac{L}{D} \frac{V_{avg}^2}{2g} = \frac{4LV_{avg}^2}{C_{Chezy}^2 D} \xrightarrow{\text{yields}} C_{Chezy} = \sqrt{\frac{2g}{f}} = \sqrt{\frac{19.62}{f}} \quad (7)$$

$$f \frac{L}{D} \frac{V_{avg}^2}{2g} = \frac{6.824 L^{1.852} V_{avg}^{1.852}}{C_{Hazen}^{1.852} D^{1.1668}} \xrightarrow{\text{yields}} C_{Hazen} = \frac{14.0717}{f^{0.54} V_{avg}^{0.0799} D^{0.0901}} \quad (8)$$

$$\frac{6.824L^{1.852}V_{avg}^{1.852}}{C_{Hazen}^{1.852}D^{1.1668}} = \frac{6.3496n^{2}V_{avg}^{2}L}{D^{4/3}} \xrightarrow{\text{yields}} n = \frac{1.0367D^{0.0834}}{C_{Hazen}^{0.926}V_{avg}^{0.074}} \quad (9)$$

Since, Colebrook-White Eq. 10 has very accurate results, all researchers suggested using this equation. They proposed solving it in one of the following ways: iteration, trial and error, graphical or numerical analysis because the equation is within the implicit function category (Brkic, 2016, 2017) but solving it needs to spend some annoying time thinking about it, so, I decided to use Simulink/MATLAB (Karris, 2006a, b; Chapman, 2013) to solve this problem easily and very pleasantly:

$$\frac{1}{\sqrt{f}} = -2\log10\left(\frac{k}{3.7065D} + \frac{2.5226}{Re\sqrt{f}}\right) \quad (10)$$

where, k is a mm roughness and is similar to the Diameter D of the pipe mm. The Colebrook-White formula is applied in both the transition flow and turbulent flow in the rough flow is not limited to the relative roughness during the closed period (0.13, 2.6) but is used as an example for the purpose of illustration no more and smooth pipe flow within (0.0004, 0.050) (Flack and Schutlz, 2010; McKeon *et al*. 2005; Hultmark *et al*., 2013). For Laminar flow, Eq. 11 is used:

$$f = \frac{64}{Re} \quad \text{or} \quad f_{Finning} = \frac{16}{Re} \quad (11)$$

MATERIALS AND METHODS

Implementation of the program: Two programs are created. The first program presents one element to be read and entered into the relevant equations in which the numbers entered are not subject to any restrictions or conditions but according to the desire of the person who enters the numbers (Fig. 1 and 2) while the second program offers a set of elements that you specify with 15 elements per each variable (Fig. 3-5). The variables included in the equations are called “Input the data”, which includes each of “hydraulic Diameter (D in cm), average Velocity (V in m/sec), roughness height (k in mm), kinematic viscosity (υ in stoke), Length of pipe (L in m), Hazen-Williams Coefficient C in Hazen-Williams formula, Chezy Coefficient C in Chezy formula and Manning coefficient n in the Manning formula (Fig. 1-5). Equation 1 is applied after the input variables are defined. This appears after you double-click the Re icon (Fig. 1 and 3). The program then determines the flow type based on Reynolds number. If the flow is laminar, Eq. 11 (Fig. 6) applies and if otherwise, Eq. 10 applies (Fig. 7).

After obtaining the f value, the head dispersal or loss is calculated by the Darcy-Wesbach equation by Eq. 2 (Fig. 8). On the basis of the value f, both the Chezy’s constant C_{Chezy} with a Chezy formula are obtained with Eq. 7 (Fig. 9). And roughness coefficient C_{Hazen} of the pipe for Hazen-Williams is fixed with a formula using the Eq. 8

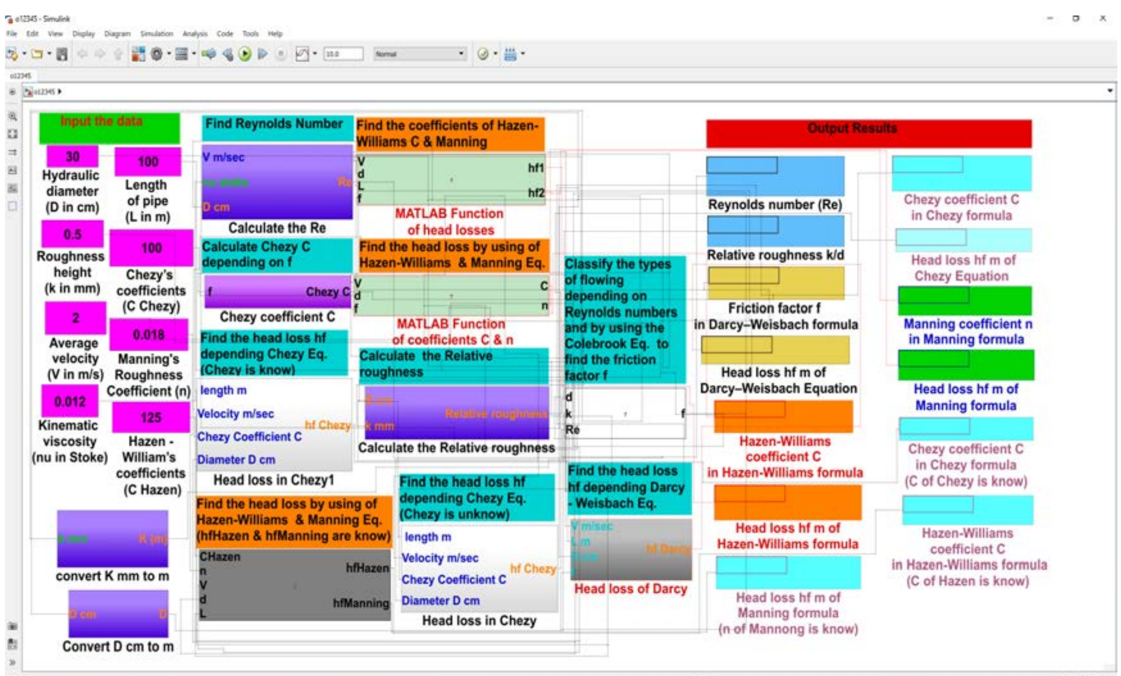

Fig. 1: Single data entry before implementation

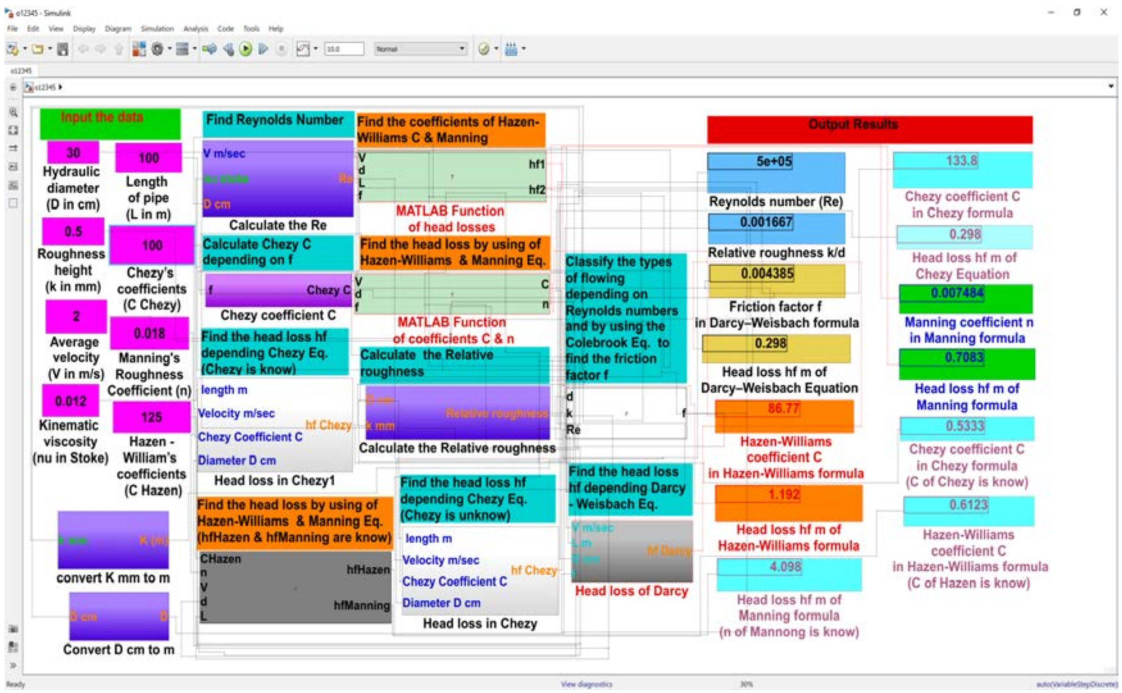

Fig. 2: Single data entry after implementation

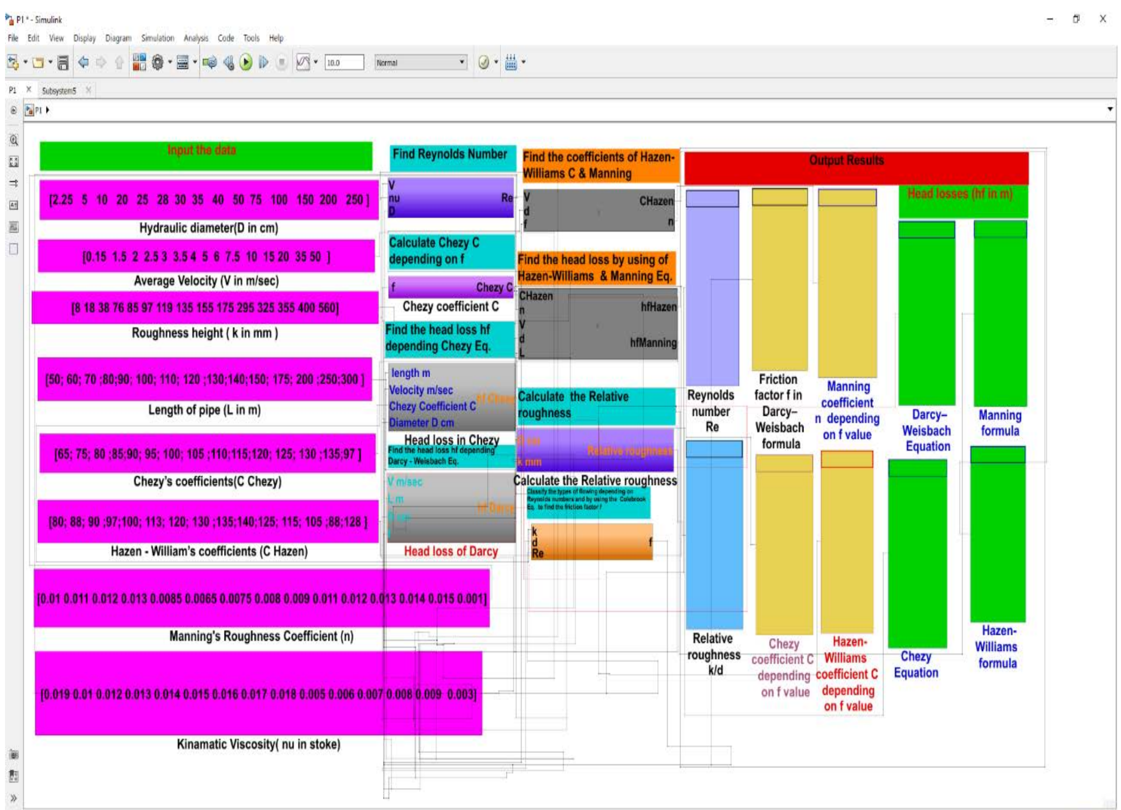

Fig. 3: Group data entry before implementation

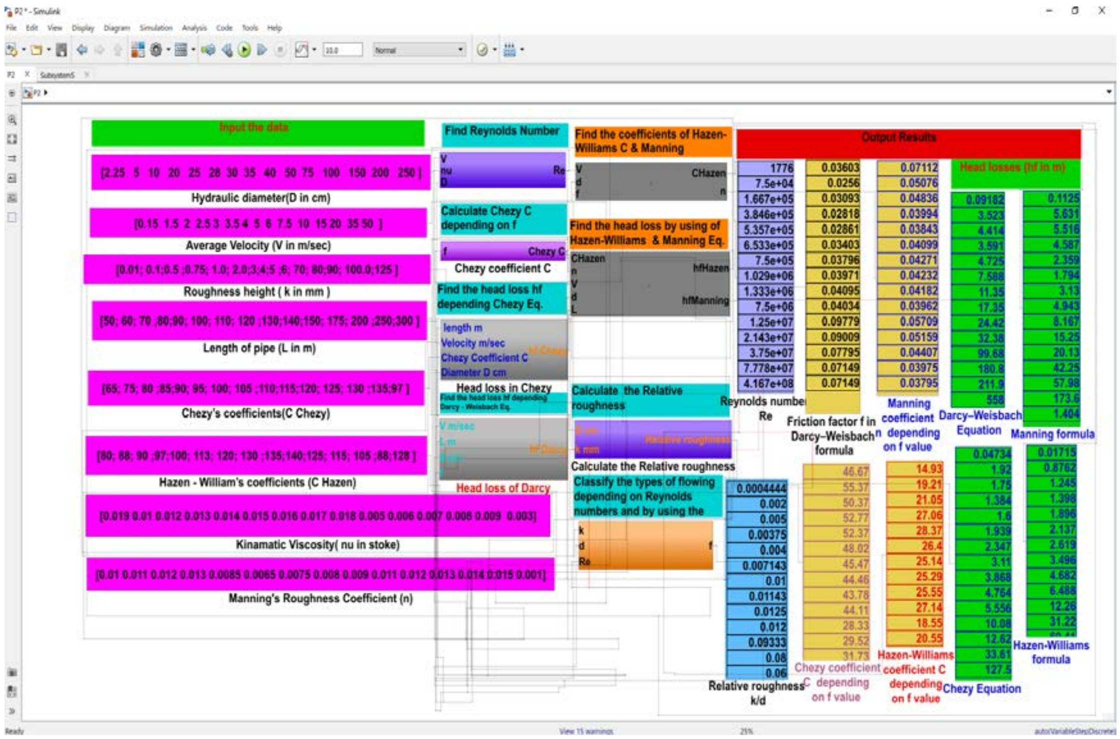

Fig. 4: Group data entry after implementation of smooth circular pipes

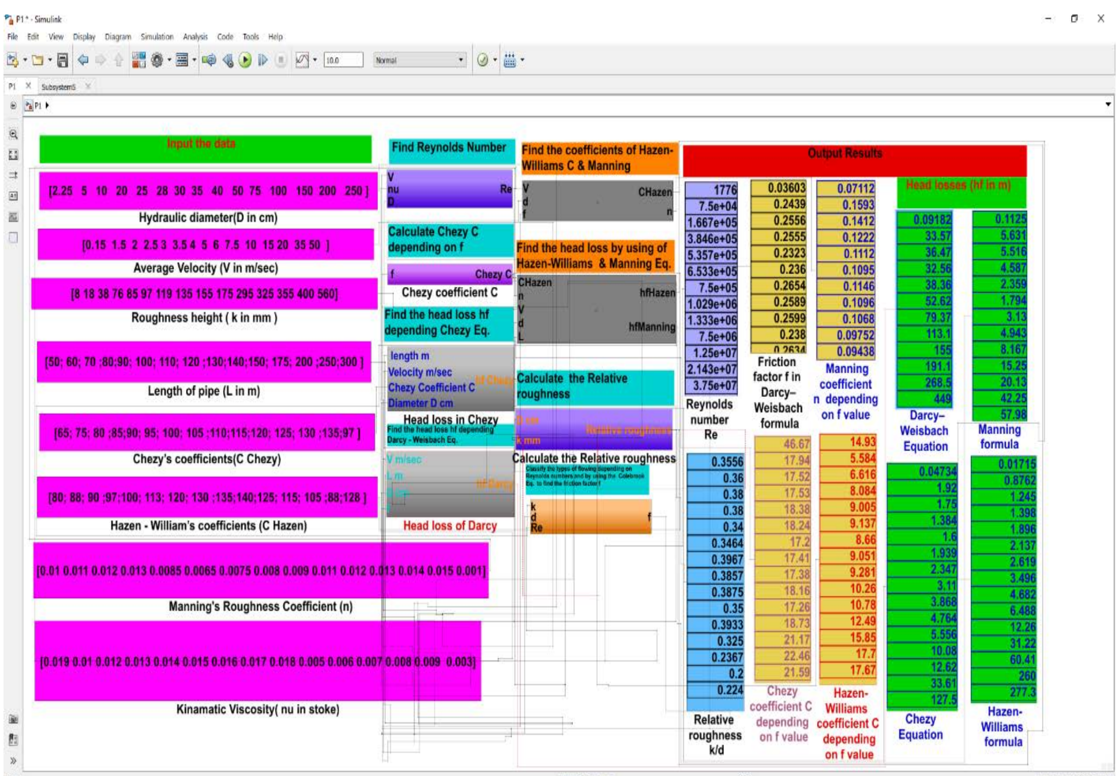

Fig. 5: Group data entry after implementation of rough circular pipes

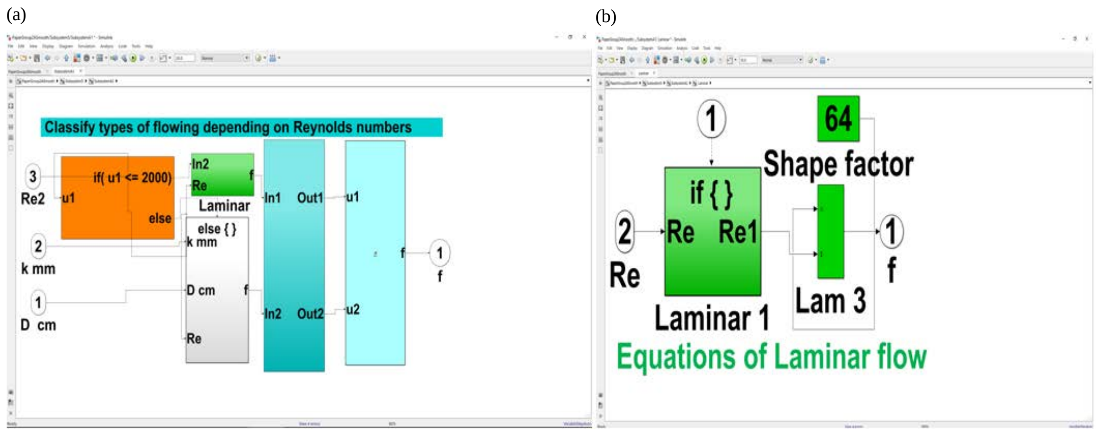

Fig. 6: a, b) Implementation of the program in case the flow is Laminar

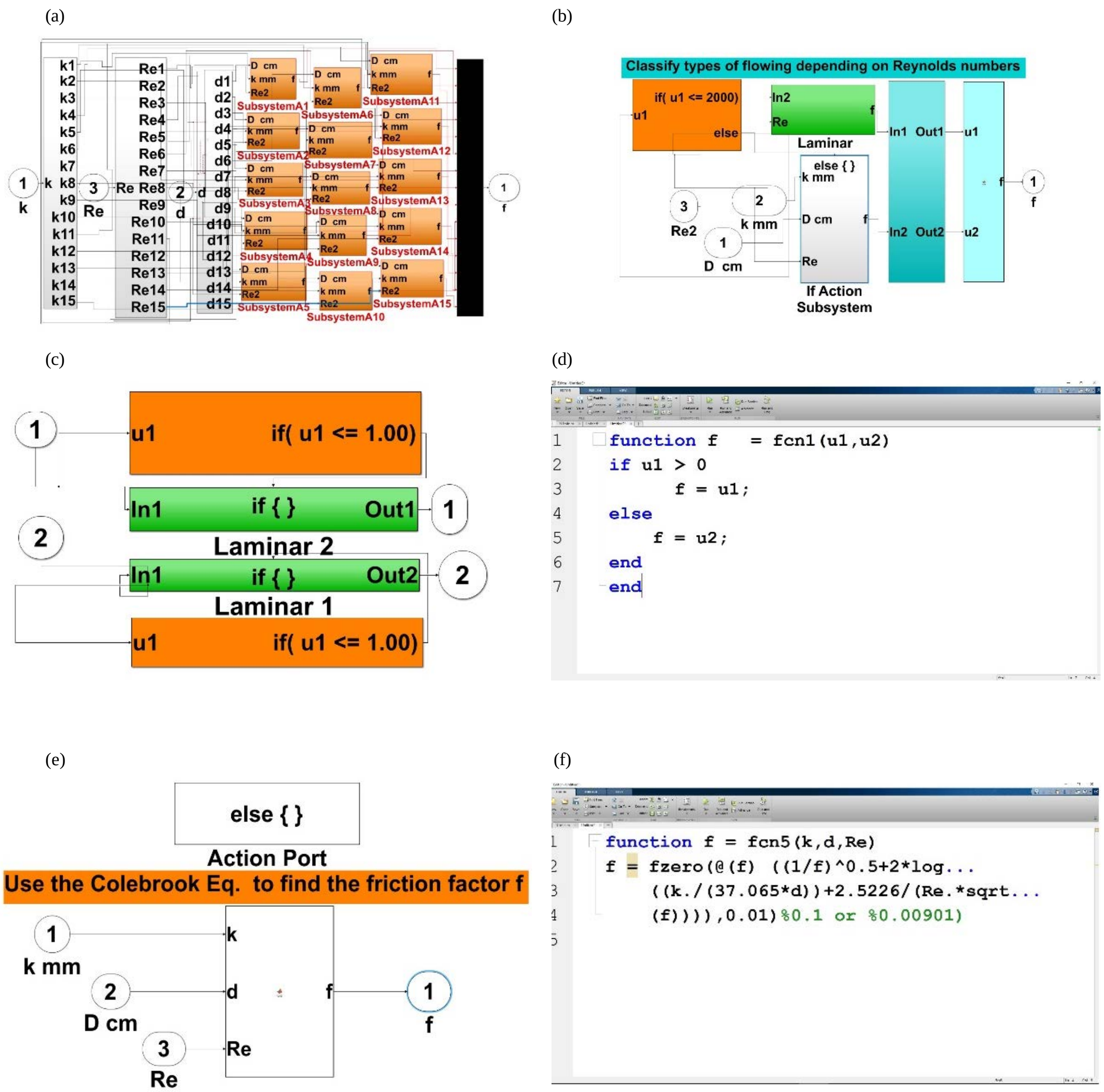

Fig. 7: a-f) The stages of implementation of program 1-6 in case of turbulent flow

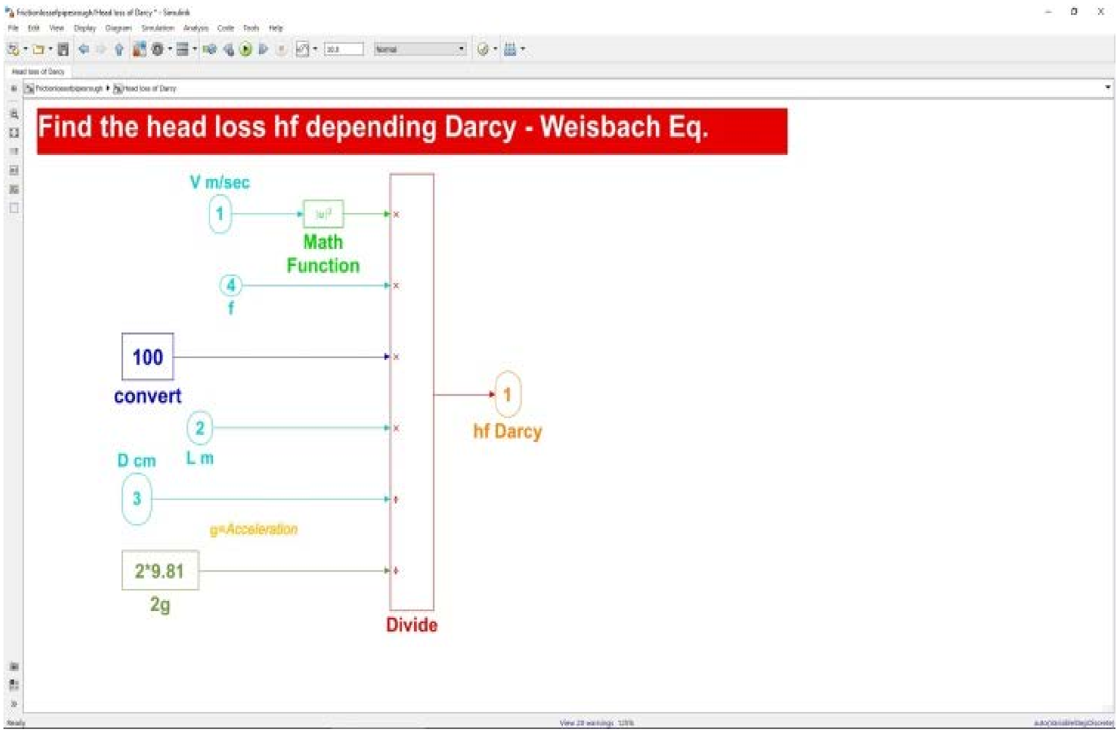

Fig. 8: Calculate head or friction loss based on Darcy-Weisbach formula

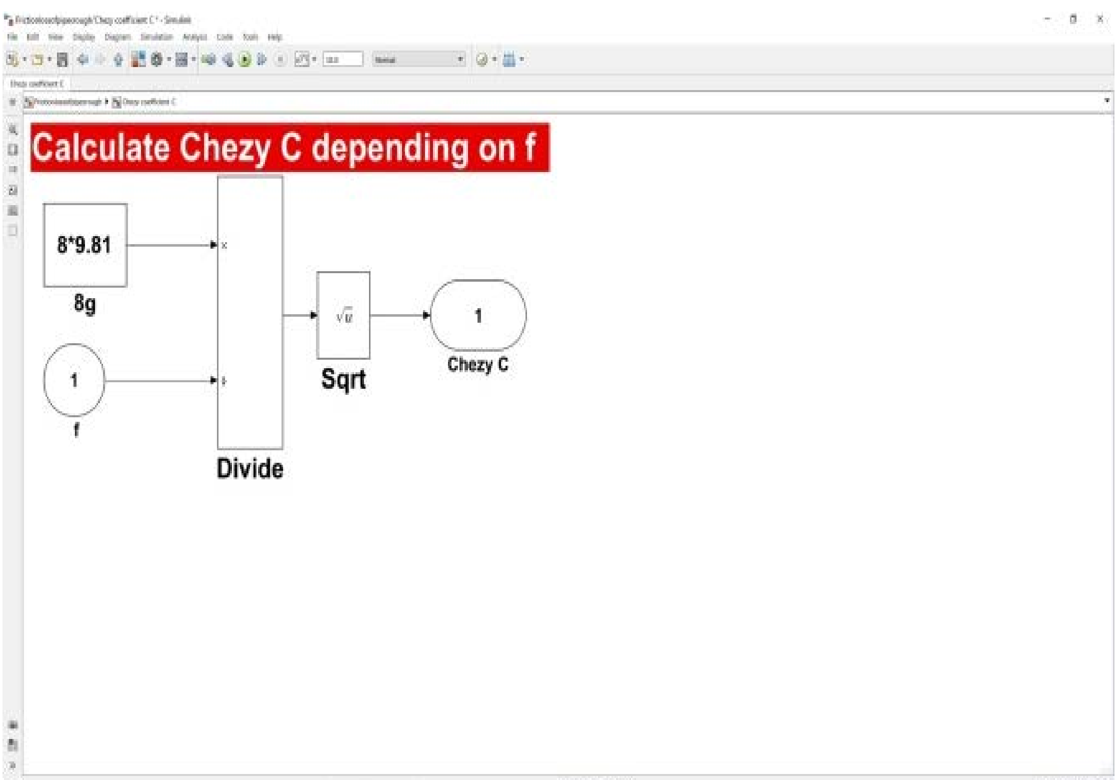

Fig. 9: Calculate C_{Chezy} based on f

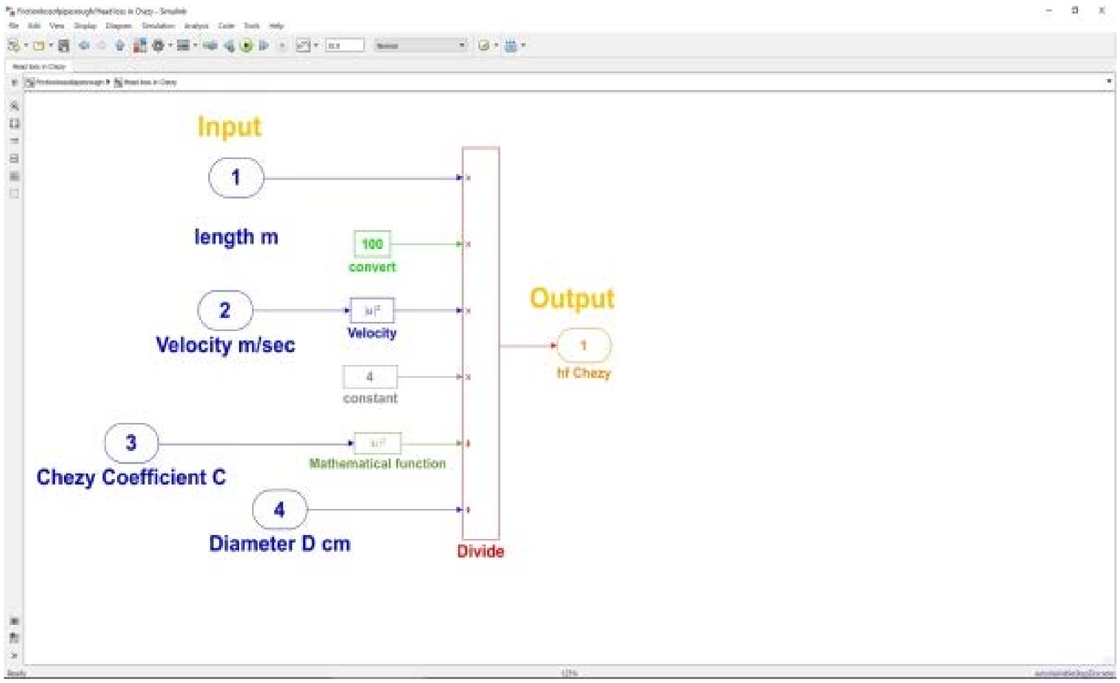

Fig. 10: Calculate head or friction loss based on Chezy formula

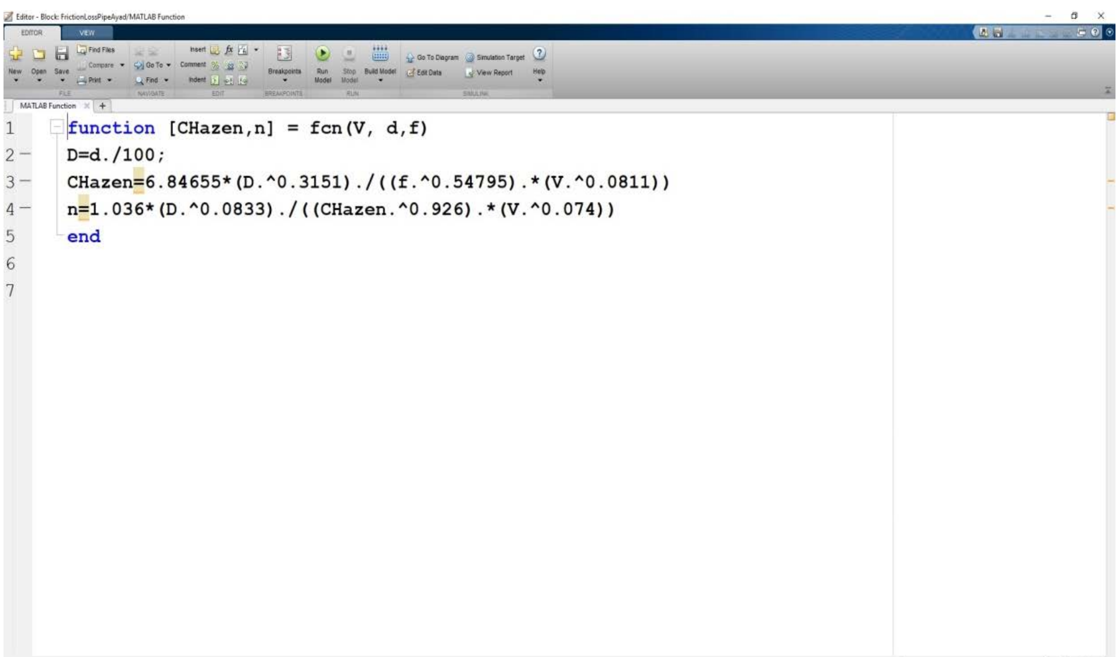

Fig. 11: Calculate the C_{Hazen} and n depending on the F-value

(Fig. 10). It is also possible to find both head or friction losses for each Chezy formula calculated by Eq. 3 (Fig. 11) and Hazen-Williams formula by Eq. 4 (Fig. 12). After obtaining the Hazen Constant C_{Hazen}, I calculate the Manning's roughness coefficient by applying Eq. 9 (Fig. 12). Then calculate the removal or loss of head by Eq. 5 (Fig. 12). All results are shown in Fig. 2, 4 and 5. The above can be summed up in Table 1.

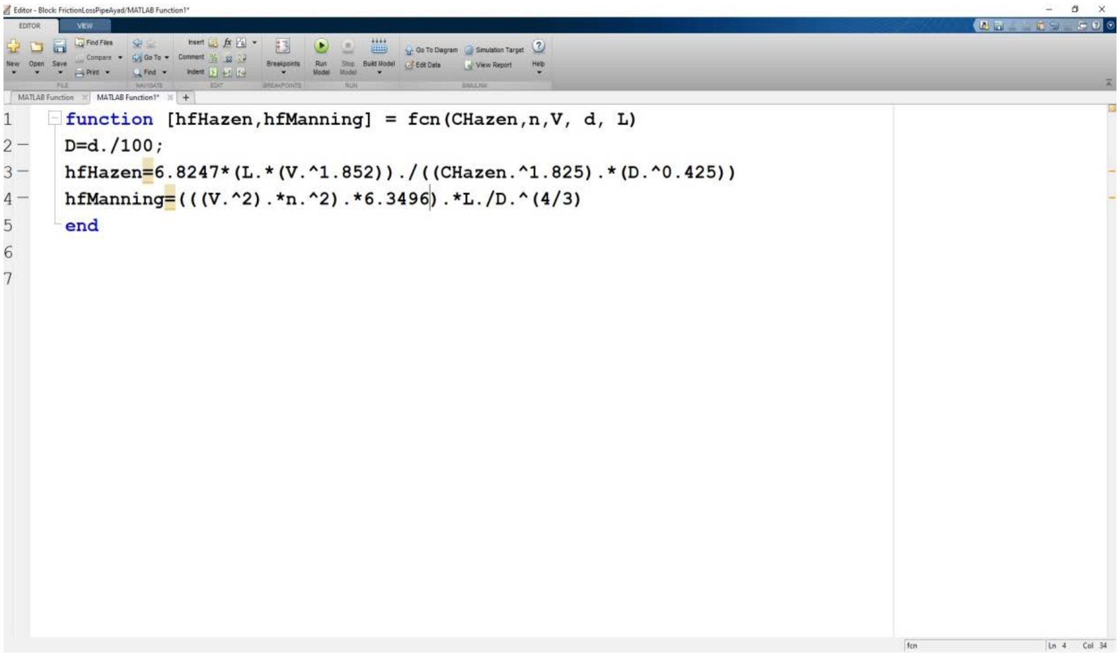

Fig. 12: Calculate head loss based on Hazen-Williams formula and Manning formula

Table 1: Loss of energy head and coefficients of the four equations

	The coefficients			Head loss h_f	
Equations name	Symbols	Equation No.	Figure No.	Equation No.	Figure No.
Darcy-Weisbach	f	10 or 11	6 or 7	2	8
Chezy	C_{Chezy}	7	9	3	10
Hazen-Williams	C_{Hazen}	8	11	4	12
Manning	n	9	11	5	12

Computations implemented in the program: As previously explained in the "Program execution", data are entered into the program, for example, any data you want can be entered or more accurately any data you may need, In the beginning, as shown in the Fig. 1 and 2, after entering individual data into the program under the heading "Input the data" and the results of that individual data under the heading "Output results". Moreover, you can also want to calculate any variables in the individual results. It is usually difficult to obtain coefficients or constants of the three equations (Eq. 3-5) and we can only infer the value of friction factor using Eq. 10 or 11. Therefore, we decided to extract the friction factor f for the Darcy equation and other coefficients were extracted based on f. This value is equal to all other equations because of the hypothesis of Eq. 6. If the coefficients are available, we calculate the head loss for each equation as shown in "Output the single results". If you want to enter a group of cells instead of one for each variable, enter the data set in the same way as we did and this is shown in Fig. 3-5. The difference between them is the existence of one box for each variable while in this field a data set of fifteen digits per variable as an example within the program and any number that may need or want to use it.

Input the single data:

- Hydraulic Diameter (D in cm) = 30
- Roughness height (k in mm) = 0.5
- Average Velocity (V in m/sec) = 2
- Kinematic viscosity (nu in Stoke) = 0.012
- Length of pipe (L in m) = 100
- Hazen-Williams Coefficient C in Hazen-Williams formula = 125
- Chezy Coefficient C in Chezy formula = 100
- Manning's roughness coefficient n in Manning formula = 0.018

Output the single results

- Reynolds number (Re) = 5e+05
- Relative roughness k/d = 0.001667
- Head loss h_f m of Darcy-Weisbach Equation = 1.545
- Head loss h_f m of Chezy equation = 0.5333
- Head loss h_f m of Hazen-Williams formula = 0.6123
- Head loss h_f m of Manning formula = 4.098
- Friction factor f in Darcy-Weisbach formula = 0.022736588938142
- Fanning friction factor or coefficient of friction f = 0.0056841472345355

- Chezy Constant C in Chezy formula is based on the friction factor in Darcy-Weisbach formula = 58.751213606587
- Hazen-Williams Coefficient C in Hazen-Williams formula is based on the friction factor in Darcy-Weisbach formula = 35.215310757159
- Manning's roughness coefficient n in Manning formula is based on the friction factor in Darcy-Weisbach formula = 0.032904427876689

Input the data set of smooth pipe results:

- Hydraulic Diameter (D in cm) = [2.25, 5, 10, 20, 25, 28, 30, 35, 40, 50, 75, 100, 150, 200, 250]
- Average Velocity (V in m/sec) = [0.15, 1.5, 2, 2.5, 3, 3.5, 4, 5, 6, 7.5, 10, 15, 20, 35, 50]
- Roughness height (k in mm) = [0.01, 0.1, 0.5, 0.75, 1.0, 2.0, 3, 4, 5, 6, 70, 80, 90, 100.0, 125]
- Kinematic viscosity (nu in stoke) = [0.019, 0.01, 0.012, 0.013, 0.014, 0.015, 0.016, 0.017, 0.018, 0.005, 0.006, 0.007, 0.008, 0.009, 0.003]
- Length of pipe (L in m) = [50, 60, 70, 80, 90, 100, 110, 120, 130, 140, 150, 175, 200, 250, 300]
- Chezy's Coefficients (C_{Chezy}) = [65, 75, 80, 85, 90, 95, 100, 105, 110, 115, 120, 125, 130, 135, 97]
- Hazen-William's Coefficients (C_{Hazen}) = [80, 88, 90, 97, 100, 113, 120, 130, 135, 140, 125, 115, 105, 88, 128]
- Manning's roughness coefficient (n) = [0.01, 0.011, 0.012, 0.013, 0.0085, 0.0065, 0.0075, 0.008, 0.009, 0.011, 0.012, 0.013, 0.014, 0.015, 0.001]

Output the data set of smooth pipe results:

- Reynolds number (Re) = [1776, 7.5e+04, 1.667e+05, 3.846e+05, 5.357e+05, 6.533e+05, 7.5e+05, 1.029e+06, 1.333e+06, 7.5e+06, 1.25e+07, 2.143e+07, 3.75e+07, 7.778e+07, 4.167e+08]
- Relative roughness k/d = [0.0004, 0.0020, 0.0050, 0.0037, 0.0040, 0.0071, 0.0100, 0.0114, 0.0125, 0.0120, 0.0933, 0.0800, 0.0600, 0.0500, 0.0500]
- Head loss h_f m of Darcy-Weisbach equation = [0.1, 3.5, 4.4, 3.6, 4.7, 7.6, 11.4, 17.3, 24.4, 32.4, 99.7, 180.8, 211.9, 558.0, 1093.2]
- Head loss h_f m of Chezy equation = [0.0473, 1.9200, 1.7500, 1.3841, 1.6000, 1.9391, 2.3467, 3.1098, 3.8678, 4.7637, 5.5556, 10.0800, 12.6233, 33.6077, 127.5375]
- Head loss h_f m of Hazen-Williams formula = [0.0172, 0.8762, 1.2451, 1.3975, 1.8960, 2.1369, 2.6194, 3.4961, 4.6820, 6.4875, 12.2578, 31.2226, 60.4104, 260.0273, 277.2775]
- Head loss h_f m of Manning formula = [0.1125, 5.6308, 5.5157, 4.5874, 2.3595, 1.7940, 3.1301, 4.9426, 8.1670, 15.2460, 20.1273, 42.2526, 57.9835, 173.6329, 1.4035]
- Friction factor f in Darcy-Weisbach formula = [0.0360, 0.0256, 0.0309, 0.0282, 0.0286, 0.0340, 0.0380, 0.0397, 0.0409, 0.0403, 0.0978, 0.0901, 0.0780, 0.0715, 0.0715]
- C(Chezy(f)) = Chezy constant C in Chezy formula is based on the friction factor in Darcy-Weisbach formula = [46.6713, 55.3689, 50.3740, 52.7720, 52.3710, 48.0223 45.4673, 44.4582, 43.7780, 44.1095, 28.3293, 29.5155, 31.7291, 33.1321, 33.1321]
- C(Hazen (f)) = Hazen-Williams Coefficient C in Hazen-Williams formula is based on the friction factor in Darcy-Weisbach formula = [14.9259, 19.2065, 21.0462, 27.0584, 28.3653, 26.4002, 25.1377, 25.2859, 25.5501, 27.1431, 18.5471, 20.5535, 24.6979, 27.0963, 28.2413]
- n(Manning (f)) = Manning coefficient n in Manning formula is based on the friction factor in Darcy-Weisbach formula = [0.0711, 0.0508, 0.0484, 0.0399, 0.0384, 0.0410, 0.0427, 0.0423, 0.0418, 0.0396, 0.0571, 0.0516 , 0.0441, 0.0397, 0.0380]

In order to obtain a clear difference between smoothness and roughness, it was decided to maintain all inputs under the heading "Input data set of smooth pipe results" unchanged. The roughness height has been changed only, so, the previous results under "Output data set of smooth pipe results" are the same and only the different results are written here.

Input the data set of rough pipe results:

- Roughness height (k in mm) = [8, 18, 38, 76, 85, 97, 119, 135, 155, 175, 295, 325, 355, 400, 560]

Output the data set of rough pipe results:

- Relative roughness k/d = [0.3556, 0.3600, 0.3800, 0.3800, 0.3400, 0.3464, 0.3967, 0.3857, 0.3875, 0.3500, 0.3933, 0.3250, 0.2367, 0.2000, 0.2240]
- Friction factor f in Darcy-Weisbach formula f = [0.0360, 0.2439, 0.2556, 0.2555, 0.2323, 0.2360, 0.2654, 0.2589, 0.2599, 0.2380, 0.2634, 0.2237, 0.1751, 0.1555, 0.1683]
- n(Manning (f)) = Manning coefficient n depending on f value = [0.0711, 0.1593, 0.1412, 0.1222, 0.1112, 0.1095, 0.1146, 0.1096, 0.1068, 0.0975 , 0.0944, 0.0819, 0.0665, 0.0590, 0.0586]
- C(Chezy(f)) = Chezy coefficient C depends on F value = [46.6713, 17.9367, 17.5238, 17.5251, 18.3811, 18.2372, 17.1953, 17.4109, 17.3754, 18.1588, 17.2607, 18.7291, 21.1697, 22.4650, 21.5929]
- C(Hazen (f)) = Hazen-Williams coefficient C depends on f value = [14.9259, 5.5844, 6.6163, 8.0844, 9.0045, 9.1369, 8.6604, 9.0511, 9.2808, 10.2624, 10.7761, 12.4856, 15.8512, 17.7005, 17.6651]

The total data entered as a data set can be seen in the smooth and rough pipes in Fig. 13 and the output data in the smooth pipe in Fig. 14 and in the rough pipe in Fig. 15.

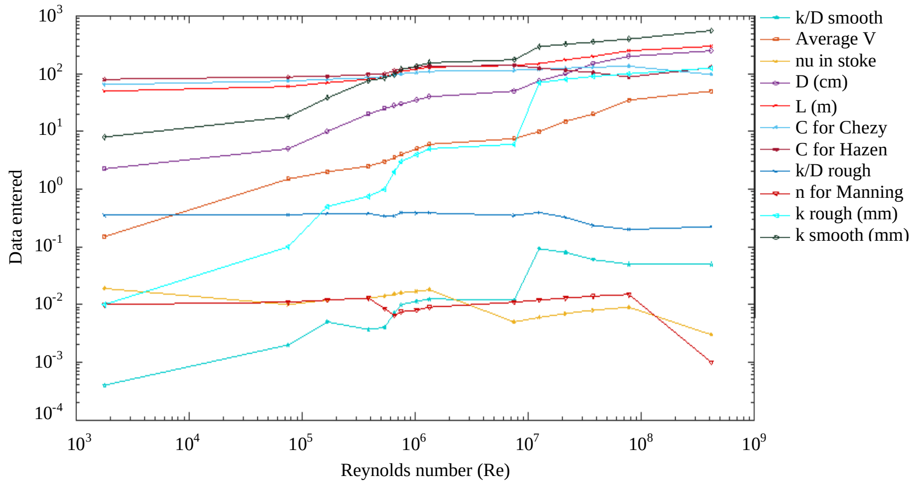

Fig. 13: Data entered against Reynolds number for both smooth and rough pipes

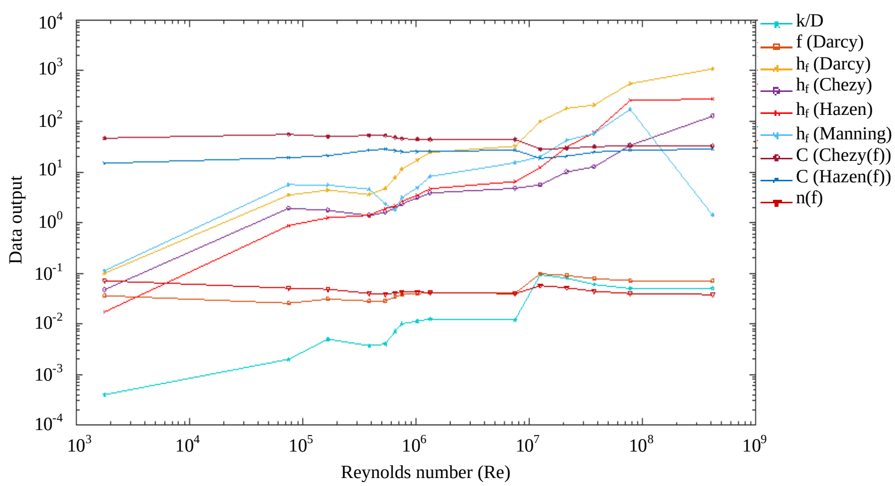

Fig. 14: Data output against Reynolds number for smooth pipe

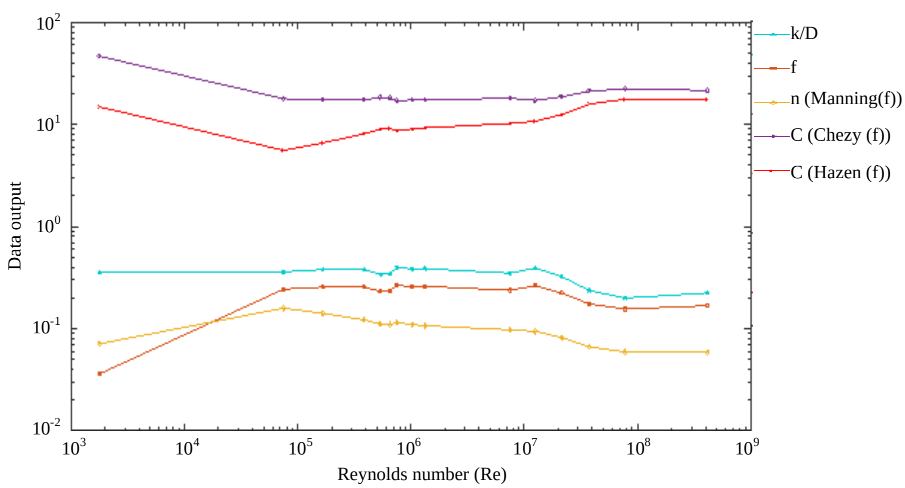

Fig. 15: Data output against Reynolds number for rough pipe

RESULTS AND DISCUSSION

Discussion of the program results: The values of C_{Chezy}, C_{Hazen} and n within a closed interval [65, 135], [80, 140] and [0.001, 0.01], respectively while the program is based on the Colebrook White equation which is characterized by its accuracy in f extraction which is used to obtain C_{Chezy}, C_{Hazen} and n [21.5929, 46.6713], [14.9259, 17.6651] and [0.0586, 0.0711], respectively, so, we recommend adopting these closed intervals when calculating head or friction loss in the circular pipes (Fig. 11-13).

Based on the Reynolds number within the range [1776, 4.2E08] for the different diameters of the hypothetical circular pipes between [2.25, 250] cm and the mean velocity [0.15, 50] m /sec and the relative roughness of the smooth circular pipes [4.73E-2, 0.05] and rough circular pipes [0.224, 0.3556]. It is clear that the C_{Chezy} does not exceed 50 which is more than the C_{Hazen} which is <20 and the f friction factor is higher than the Manning's roughness coefficient n as shown in Fig. 16-18.

Based on the above, we must choose a C_{Chezy} to be <50 and a C_{Hazen} <20 and also a Manning coefficient n should be less than the friction factor f.

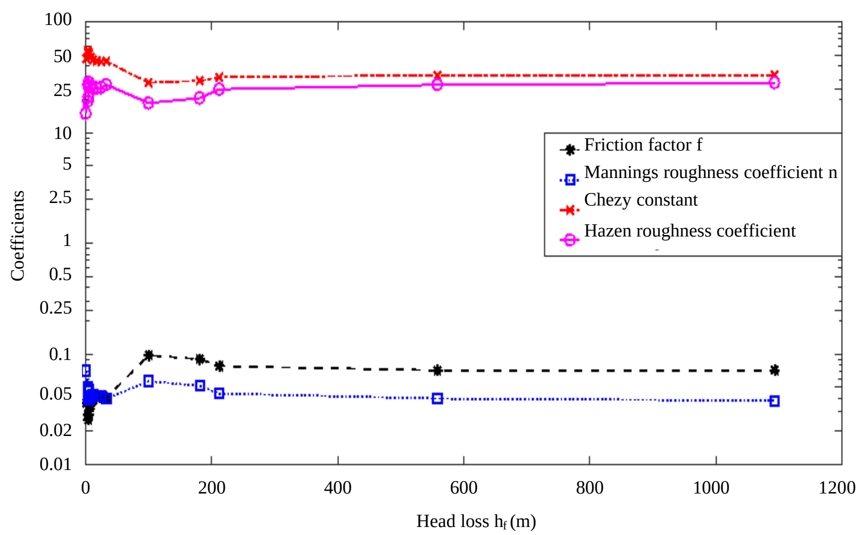

Fig. 16: The relationship between the loss of the rough pipe head and friction factor f and other variables that depend on f

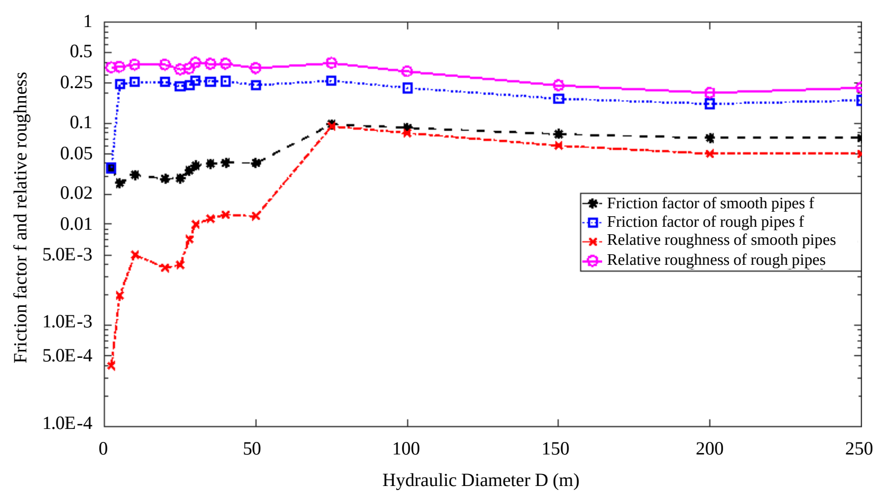

Fig. 17: The relationship between the loss of the smooth pipe head and friction factor f and other variables that depend on f

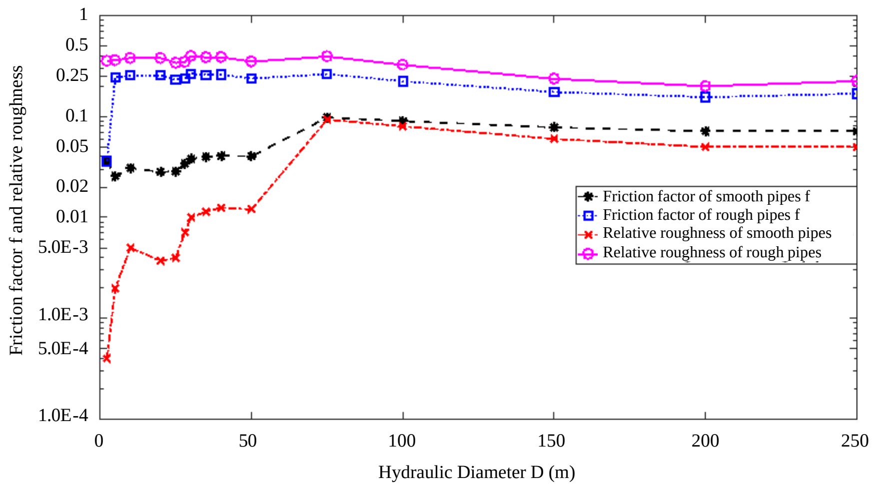

Fig. 18: The relationship between the hydraulic pipe of the rough and smooth pipe and the friction factor f and the relative roughness

CONCLUSION

After watching the implementation of Simulink/ MATLAB, it is clear that its use is simple and its results are very accurate because it eliminates the inherent difficulty of the Colebrook-White equation without any problem. Based on the implementation of the calculation steps within the program, it was confirmed that C_{Chezy} must be chosen to be <50 and C_{Hazen} is <20 and also should be Manning coefficient n<f.

REFERENCES

Brkic, D., 2016. A note on explicit approximations to Colebrook's friction factor in rough pipes under highly turbulent cases. Intl. J. Heat Mass Transfer, 93: 513-515.

Brkic, D., 2017. Solution of the implicit Colebrook equation for flow friction using Excel. Spreadsheets Educ., 10: 1-12.

Cengel, Y.A. and J.M. Cimbala, 2006. Fluid Mechanics: Fundamentals and Applications. McGraw-Hill Education, New York, USA., ISBN:9780071117203, Pages: 956.

Chapman, S.J., 2013. MATLAB Programming with Applications for Engineers. Cengage, Boston, Massachusetts, USA., ISBN:9781408094228, Pages: 569.

Flack, K.A. and M.P. Schultz, 2010. Review of hydraulic roughness scales in the fully rough regime. J. Fluids Eng., 132: 041203-1-041203-10.

Hultmark, M., M. Vallikivi, S.C.C. Bailey and A.J. Smits, 2013. Logarithmic scaling of turbulence in smooth-and rough-wall pipe flow. J. Fluid Mech., 728: 376-395.

Karris, S.T., 2006b. Signals and Systems with MATLAB Computing and Simulink Modeling. 3rd Edn., Orchard Publications, London, England, UK., ISBN-13:978-0-9744239-9-9, Pages: 206.

Karris, T.S., 2006a. Introduction to Simulink with Engineering Applications. Orchard Publication, ISBN: 0974423971.

McKeon, B.J., M.V. Zagarola and A.J. Smits, 2005. A new friction factor relationship for fully developed pipe flow. J. Fluid Mech., 538: 429-443.

White, F.M. and R.Y. Chul, 2016. Fluid Mechanics. 8th Edn., McGraw-Hill Education, New York, USA., ISBN:9789814720175, Pages: 848.