Journal of Engineering and Applied Sciences 14 (6): 1840-1846, 2019
ISSN: 1816-949X

Development of the Engineering Mathematics Lab Module with Mathematica

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Abstract: The era of education transformation in the teaching of engineering mathematics aims to be consistent with the revolution of technology. Thus, the teaching methods in engineering mathematics needs to be aligned with this revolution. The objective of this study is to develop engineering mathematics lab module with Mathematica. Engineering Mathematics lab module comprises subjects such as Vector Calculus, Linear Algebra and differential equation. A preliminary test was conducted for Vector Calculus, Linear Algebra and differential equation subjects. Each of the test results was analyzed using Rasch Model. Difficult course outcomes from each test was identified as the topic of lab module. A total of 8 course outcomes were identified as the topics for the lab module. Questions were gathered for each topic and the mathematica solutions were derived. All the questions and mathematica solutions were verified by three experts. The course outcome for the third lab is to be able to apply green's theorem, stoke's theorem and gauss's theorem in solving engineering problems. Nine questions were developed based on the application level of bloom taxonomy. Mathematica captures student's interest in learning Engineering Mathematics as it is less time-consuming in solving mathematical problems.

Key words: Teaching, Engineering Mathematics, Mathematica, module, preliminary test, Linear Algebra

INTRODUCTION

Engineering Mathematics is the foundation of the mathematics subjects taken in the first and second years of an engineering degree in an undergraduate engineering degree course. The teaching method for Engineering Mathematics courses is via. lectures and in addition, students attended tutorial classes to enhance their procedural knowledge in the subject.

This traditional teaching method needs to be transformed in order to keep track with the advances of the technology. The objective of this study is to incorporate Mathematica Software in the teaching of Engineering Mathematics by developing an Engineering Mathematics module. Over the years many researchers have commented positively on the feedback received from the students who undergo teaching Enginnering Mathematics using software.

MATLAB classes were conducted for precalculus, calculus of one variable, calculus of many variables, linear algebra and ordinary differential equation subjects (Tonkes *et al*., 2005). Each of the teaching modules includes mathematical concepts, examples and exercises. Students started with running simple codes until they could run their own coding in MATLAB. Student's feedback was positive when asked about incorporating MATLAB in their curriculum.

Although, the underlying concept of three-dimensional calculus was taught for many hours, students still found it very tough to understand (Cook, 2006). Thus, maple graphing tool was used to teach the three-dimensional calculus subject with class projects. The project includes the topics from the functions of two variables, Lagrange multipliers, line integrals and plotting secant vectors. The project aimed to help students understand the mathematical concepts through visualization.

Students were weak in the matrix methods of the structural analysis course (Charney, 2008). This course needs the theory of structures and basics of linear algebra. Students could not cope with the subject as their knowledge in programming was insufficient. To overcome this situation, computer methods of structural analysis I subject was introduced and mathcad was incorporated in

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the course as it was simple to learn. Mathcad was used as a visual matrix manipulation tool to write structural analysis programs.

Differential calculus subject was explored using GeoGebra Software (Dikovic, 2009). After traditional classes were conducted, the lecturer conducted a pre-test. This was followed by experimental classes in a computer laboratory where group research, individual research and investigations were assigned to students. At the end of the session, a post-test was conducted. The post-test result was higher than the pre-test result and this proves that technological tool is a powerful tool for the simulation of important topics of differential calculus.

Sage, free open source software was used in teaching advanced calculus (Botana *et al.*, 2014). Thirty worksheets were developed using Sage in the form of a DVD. Students worked in pairs in the class activity. Students commented very positively that they avoided wasting time using the computation method. They preffered to use the time to study other mathematical concepts.

Mathematica was integrated in vector calculus and partial differential equation subjecta (Adair and jaeger, 2014). After a pre-test, students were divided into the control group and experimental group. Experimental group studied using six laboratory sessions while control group with six extra tutorial sessions. Later, both group had a post-test. Although, there is no difference in understanding simple concepts, mathematica helped in understanding difficult concepts in vector calculus and partial differential equation.

Three areas in linear algebra namely system-matrix-determinant, vector space and linear map were focused upon, to create intelligent educational software to solve problems in linear algebra (Do Van and Kim, 2011). Coding was implemented in Maple and the solution was derived in C programming. Students who tested this intelligent education software commented very positively about the inclusion of the software whereby the software was very useful to study linear algebra.

Students faced problems in absorbing the knowledge and applied the theory of linear algebra due to its highly abstract theories (Chen, 2013). Thus they ended up bored as most time was wasted in solving complex questions through handwriting. MATLAB was introduced to students and students managed to obtain fast results with high accuracy using MATLAB. The software boost student's interest towards learning linear algebra and enhanced student's learning methods in theory and application.

MATLAB and CoNum were incorporated in the syllabus of differential equation (Carneiro *et al.*, 2010). Students had no option, either to study in the university campus or in their industrial working place via the video conference. MATLAB were chosen to solve matrix operations while CoNum was chosen due to its simplicity. CoNum was used to explore the first order differential equation while the output was presented via. MATLAB. Students were exposed in solving real problems using CoNum and MATLAB.

Maxima was introduced to students to enhance the mathematical thinking in learning differential equations (Zeynivannezhad, 2014). A preliminary test was conducted prior to the teaching experiments and the results showed that students were weak in plotting graph. Later maxima was used to solve first order differential equations, second order differential equations and laplace transforms. The step by step method of drawing graphs in maxima helped students to visualize the graphs as well as enhance student's conceptual understanding on the subject.

This study aimed to develop a lab module in order to aid the teaching of Engineering Mathematics using software. In the next section, the method of developing the lab module will be discussed. The lab topic for a particular topic will be illustrated. This will be followed by some recommendations for implementing the lab module.

MATERIALS AND METHODS

This study derived eight course outcomes that are needed to develop a lab module. The course outcomes were chosen from three different preliminary tests. As the initial study, a preliminary test was conducted for vector calculus for first year electrical, electronics and system department students in the first semester of study in the year 2016. Then another preliminary test was conducted for the same batch of students in 2017 for linear algebra subject. This continued to the third preliminary test for differential equation subject.

The output from the preliminary tests was analyzed against the Rasch Model. For each of the preliminary tests, the difficult course outcomes were identified. The summary statistics for item and the person-item distribution map from the Rasch Model were able to categorize the questions for each preliminary test according to the difficulty level.

All the difficult course outcomes for vector calculus, linear algebra and differential equation subjects are selected as the course outcomes for the lab session. Three course outcomes were selected for vector calculus subject, two course outcomes were selected for linear algebra subject while three course outcomes were selected for differential equation subject. Table 1 shows the course for the lab session.

Table 1: Course outcome for lab

Weeks	Course outcomes
1	Able to apply the basic concept of partial derivatives
2	Able to apply the concepts of line integral, double integral and triple integral in solving engineeringproblems
3	Able to apply Green's theorem, Stokes' theorem, Gaus's theorem in solving engineering problems
4	Able to use concepts of vector space, linear independent in space dimension and matrix transformation
5	Able to understand the concepts of power series
6	Able to solve first and second orders of differential equations
7	Able to perform the step-by-step analysis to model the simple engineering problem using the second order non-homogeneous differential equations using an appropriate technique
8	Able to use fourier series to solve partial differential equations

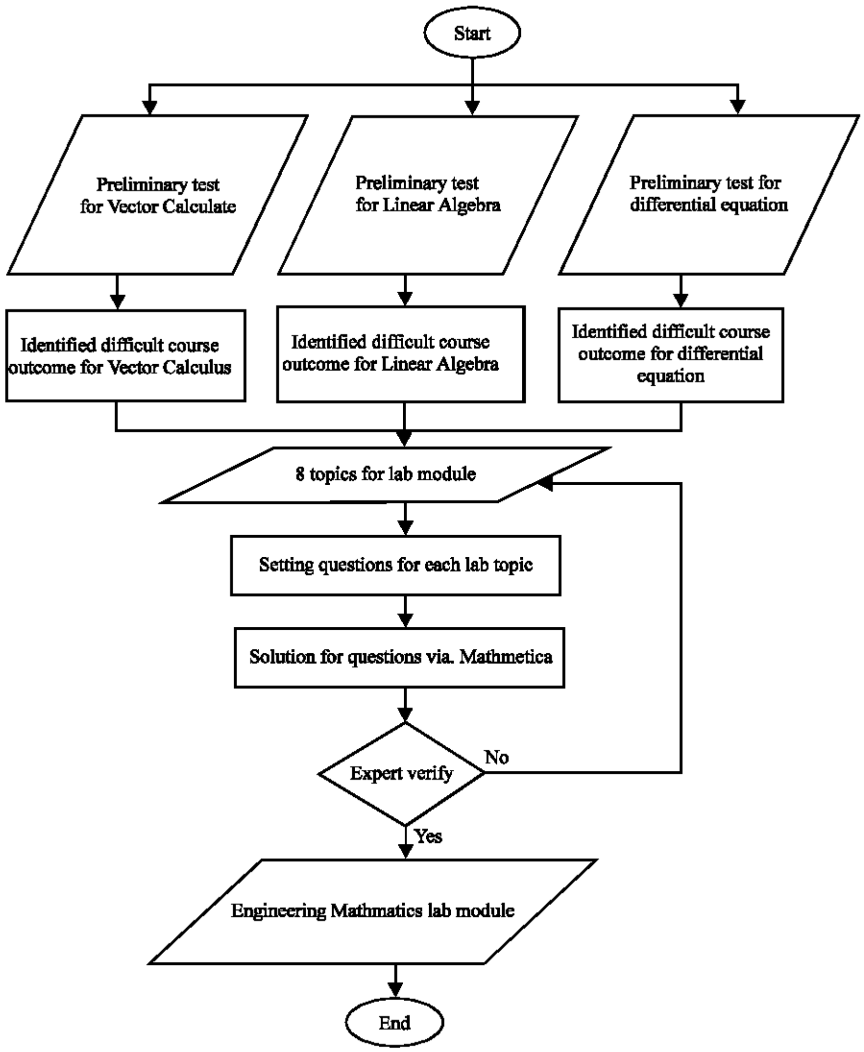

Fig. 1: Development of lab module

Development of lab module: The development of lab module is shown in Fig. 1. The difficult course outcomes which were identified from the preliminary test for vector calculus, linear algebra and differential equation are selected as the eight topics for the lab module.

For each of the lab topics, questions were gathered to fulfil the course outcomes. Then, the question was keyed in into Mathematica software. Mathematica software was run and it gave the output. Both the questions and the solutions for all the topics were given to three experts to verify the content and the solutions. If the experts are not satisfied, the questions need to be changed and the solution will be obtained using Mathematica. Once, the experts are satisfied, the lab module will then be complete.

Mathematica was selected to run a laboratory session because the function and structure of the language were easy to implement. The programming language is a highly structured functional programming and it is tailored to various theoretical and technical applications. Mathematica has helped students to understand difficult math concepts in no time with the use of the built-in functionality. Representatives of mathematical concepts in the form of graphs and simulations also facilitate a holistic understanding of students.

One of the labs which is lab 3 is further discussed. The course outcome for lab 3 is for students to be able to apply Green's theorem, Stoke's theorem, Gaus's theorem in solving engineering problems. This course outcomes relates Green's theorem to a double integral over a plane region in terms of a line around its boundary. Stoke's theorem is a generalization of Green's theorem to three dimensions. Gaus's theorem concerns with flux across surfaces such as spheres, that "enclose" a region of space.

Nine questions were selected for this course outcome. There is no question on plotting graphs. All the questions targeted at the problem-solving method.

Table 2 shows the questions for lab 3. The Mathematica solutions for lab 3 is given in the Appendix. Question 1 require to use the Green theorem to evaluate a function for given coordinates. In Mathematica, firstly a function, P is defined for the first function. This is followed by defining a second function, Q. In the third line, the formula to calculate the Green theorem id define. Once, all these are written clearly, Mathematica will calculate and give the answer in 2 lines. There is no working need to be shown in Mathematica. The program will calculate the working and give the final answer.

Question 7 requires to use divergence theorem to evaluate the function. By changing to polar coordinates and substituting the limits, Mathematica able to give the final answer. Mathematica able to give the answer within 2 lines. A lot of time can be saved for computation using Mathematica software.

Use the green theorem to evaluate $\int_C xy\,dx+x^2y^3dy$ where C is a triangle with vertices (0, 0), (1, 0), (1, 3). Evaluate $\int_C y^3\,dx-x^3dy$ where C is positively oriented circle of radius 2 centered at the origin. Use Green's theorem to evaluate $\int_C x^2\,ydx+xdy$ along the triangular path shown in Fig. 2. Use Green's Theorem to find the area of a disk of radius a. There are two circles in a disk; the inner one and the outer one. The area of circle is defined $A=\frac{1}{2}\int_C xdy-ydx$ Use Stoke's theorem to evaluate $\iint_C \text{curl}\vec{F}.d\vec{s}$ where $\vec{F}=z^2\vec{i}-3xy\vec{j}+x^3y^3\vec{k}$ and S is the part of $z = 5-x^2-y^2$ above the plane z = 1. Assume that S is oriented upwards.

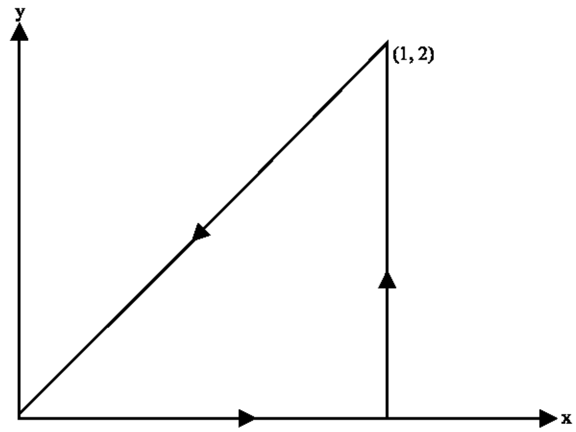

Fig. 2: Triangular path

Use Stoke's theorem to evaluate $\int_C \vec{F}.d\vec{r}$ where $\vec{F}=z^2\vec{i}-y^2\vec{j}+x\vec{k}$ and C is the triangle with vertices (1, 0, 0), (0, 1, 0) and (0, 0, 1) with counter-clockwise rotation. Use the divergence theorem to evaluate $\iint_S \vec{F}.d\vec{S}$ where $\vec{F}=xy\vec{i}-\frac{1}{2}y^2\vec{j}+z\vec{k}$ and the surface consists of the three surfaces, $z = 4-3x^2-3y^2$, $1\leq z\leq 4$ on the top, $x^2+y^2 = 1$, $0\leq z\leq 1$ on the sides and z = 0 on the bottom. Evaluate $\iint_S (3xi+2yi).dA$. where, S is the sphere $x^2+y^2+z^2 = 9$. Evaluate $\iint_S (y^2zi+y^3j+xzk).dA$ where, S is the boundary of the cube defined by $-1\leq x\leq 1$, $-1\leq y\leq 1$ and $0\leq z\leq 2$.

RESULTS AND DISCUSSION

The lab module focused on teaching Engineering Mathematics using computational methods. The lab module consists of course outcome from three different subjects namely vector calculus, linear algebra and differential equation.

Mathematica has the largest number of built-in mathematical functions. Mathematica also has a massive built-in library. Mathematica is widely used in functional programming. Mathematica manage to give answer quickly as it needs few lines of codes to solve a problem.

The lab module was developed for 6 months. Initially, the process of learning the software had lasted for 3 months. Then, the development of the lab module took another 3 months.

Mathematica Software was aimed at plotting graphs and reducing the steps in handwritten calculations. Lab three is aimed at reducing the numberof steps in computation. Students can solve more questions using Mathematica compared to handwritten solutions.

Referring to the literature, technological tool will boost student's interest in learning because of obtaining quick solution using the tools. Teaching using computational tools will help the students to remember well the underlying concepts of a topic. Teaching via. Mathematica can be seen as an alternative method to teaching Engineering Mathematics.

CONCLUSION

This study was aimed at developing a lab module that will aid the teaching of Engineering Mathematics via. Mathematica. The contribution of this study is the development of one module that comprises three different Engineering Mathematics subjects namely vector calculus, linear algebra and differential equation. The module was developed using the analysis of Rasch Model in the preliminary test for vector calculus, linear algebra and differential equation.

The Rasch Model has identified the course outcomes for the lab topic. The questions were selected for each topics and the Mathematica solution was derived carefully. After validation of experts, this module is completed. The lab module which consists of eight lab topics will be tested on students. There will be a pre-test conducted at the beginning of the Engineering Mathematics topics. The students will undergo lab sessions for 8 weeks. After the 'treatment' sessions the students will do a post-test. The pre-post test results will be analyzed against the Rasch Model to prove that there is improvement in teaching using the computational tools.

ACKNOWLEDGEMENT

The researchers wish to express gratitude towards Universiti Kebangsaan Malaysia for supporting the research.

Appendix:

Q1;

```
ClearAll["Global`*"]
P = x*y;
Q = x^2 y^3;
Z = D[Q, x] - D[P, y];
```

$$\int_0^1 \int_0^{3x} Z \, dy \, dx$$

$$\frac{23}{4}$$

Q2;

```
ClearAll["Global`*"]
P1 = y^3;
Q1 = -x^3;
Z1 = D[Q1, x] - D[P1, y]
```

$$-3x^2 - 3y^2$$

```
ZD1 = Factor[Z1] / -3;
r1 = Sqrt[ZD1];
dr = D[r1, y];
dtheta = D[r1, y];
dx = Denominator[dr];
dy = Denominator[dtheta];
Rnew = r1*dx*dy
```

$$(x^2 + y^2)^{3/2}$$

$$3 * \int_0^{2\pi} \int_0^2 r^3 \, dr \, d\theta$$

$$24\pi$$

Q3;

```
ClearAll["Global`*"]
P = x^2 y;
Q = D[Q, x] - D[P, y];
```

$$\int_0^1 \int_0^{2x} Z \, dy \, dx$$

$$\frac{1}{2}$$

Q4;

```
Clear[all]
xnew = a*Cos[t];
ynew = a*Sin[t];
dxnew = D[xnew, t]
dynew = D[ynew, t]
```

$$\frac{1}{2} \int_0^{2\pi} xnew * dynew \, dt - \frac{1}{2} \int_0^{2\pi} ynew * dxnew \, dt$$

$$-a \, Sin[t]$$

$$a \, Cos[t]$$

$$a^2 \pi$$

Q5;

```
ClearAll["Global`*"]
var1 = Sin[t]
var2 = Cos[t]
```

$$\int_0^{2\pi} \left(\text{var1} - 24\, \text{var1} * \text{var2}^2\right) dt$$

```
Sin[t]

Cos[t]

0
```

Q6;

```
ClearAll["Global`*"]
Needs["VectorAnalysis`"]
(*Needs["VectorAnalysis`"]*)
```

$C1 = \text{Curl}\left[\{z^2, y^2, x\}, \text{Cartesian}[x, y, z]\right]$

```
{0, -1 + 2 z, 0}
```

```
G = z - 1 + x + y;
G1 = Grad[G, Cartesian[x, y, z]]

{1, 1, 1}
```

$$\int_0^1 \int_0^{-x+1} (2\,(1 - x - y) - 1)\, dy\, dx$$

$$-\frac{1}{6}$$

Q7;

```
ClearAll["Global`*"]
```

$$\int_0^{2\pi} \int_0^1 \int_0^{4-3r^2} r\, dz\, dr\, d\theta$$

$$\frac{5\pi}{2}$$

Q8;

```
ClearAll["Global`*"]
F = Div[{3 x, 2 y, 0}, Cartesian[x, y, z]]
r = 3;
```

$DS1 = 5 * \frac{4}{3} * \pi * r^3$

```
5

180 π
```

Q9;

```
ClearAll["Global`*`"]
```

$F1 = \text{Div}\left[\{y^2, y^3, xz\}, \text{Cartesian}[x, y, z]\right]$

$$DS1 = \int_0^2 \int_{-1}^1 \int_{-1}^1 F1\, dx\, dy\, dz$$

$3y^2$

8

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