

## Design of PID Controller Using Internal Model Control Based Lambda Tuning for an Industrial Blending Process

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**Abstract:** The proportional-integral-derivative controller is widely employed in process industries due to its simplicity, reliability and ability to achieve major control objectives. The Internal Model Control (IMC) design is a top-notch technique used for tuning these controllers for Single Input Single Output (SISO) systems with dead-time. Filter constant,  $\Lambda$  is a very important concept in the IMC design and proper estimation of the constant is fundamental. The following text studies the effect of varying the filter constant  $\Lambda$  from its ideal value to extreme values and understands the system behaviour for the differing filter constants for first order plus dead time model representing blending process. It was observed that improper estimation of  $\Lambda$  lead to unstable systems, large overshoots and highly oscillatory response which are completely undesirable in any control process. The inferences have been achieved with the aid of time domain analysis using MATLAB.

**Key words:** FOPDT, dead-time, PID, SISO, IMC, integrating time, derivative time, proportional gain, process time constant, process gain, lambda, blending system

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### INTRODUCTION

Blending systems are very common in process industries such as oil and gas, waste water treatment, paper, food, pharmaceutical, chemical and many more (Smuts, 2011). The real time industrial processes are highly non-linear in nature and exhibit dead time (Smith and Corripio, 2006). This dead time occurs majorly due to the following factors (Bequette, 2002):

- External factors such as transportation lag due to long pipelines or large travel distances
- Internal factors such as non linearities of the final control element, i.e., blunt use of conventional actuator sizing for valves and excessive tuning of the controller
- Uncertainties like noisy data, erroneous assumptions of important parameters and incorrect modelling of the systems

Presence of dead time element complicates the analysis and design of control systems and makes satisfactory control more difficult as the performance might endure instability, high sensitivity to parametric uncertainties and poor disturbance rejection (Altmann, 2005). Any industrial process is mathematically represented in the form of nonlinear

differential equations (continuous domain) or difference equations (discrete domain). Using analytical methods such as state space analysis, initial-final value theorems, etc., to solve these equations become a challenge with the increasing non linearities, orders of the transfer functions and dead time (Smuts, 2011). The FOPDT Model is often an equitable approximation to such process behaviours as it has the efficacy for controller tuning rules and can be used as a computationally surrogate model in simulations for training and optimization (Korsane *et al.*, 2014). Higher order industrial processes can be modelled as FOPDT as the simulations become much easier (Juneja *et al.*, 2010). The FOPDT Model has the continuous transfer function (Eq. 1) (Bequette, 2002):

$$\frac{K_p}{\tau_s + 1} e^{-\theta s} \quad (1)$$

Where:

$K_p$  = Process gain

$\tau$  = Process time constant

$\theta$  = Process dead time

A Proportional Integral Derivative controller (PID controller) is a control loop feedback mechanism used in industrial control systems to lower the degree

of deviation (error) of the process variable from the set-point (Babu and Swarnalatha, 2017). The PID controller has three principal control effects. The Proportional (P) action when used alone always exhibits some offset to the system. To minimise the offset, one can tune the system by changing the proportional gain, however, beyond a certain limit, the response becomes heavily oscillatory and unstable. In addition, one can never completely eliminate offset by using P controller alone. With the integral action in picture, the offset can be completely eliminated as it gets integrated till it becomes null. However, this happens at the cost of increased process settling time and occurrence of more oscillations. With the Derivative (D) action in addition to the P-I action, the oscillations can be dampened and smoothened out. This reduces the settling time thereby speeding up the response and stabilizing the system (Korsane *et al.*, 2014). However, the derivative action is also known to amplify noise present in the system as it takes the derivative of the error (de/dt) and causes faster wear and tear of the equipment. Thus, industrial processes with high measurement noise tend to avoid PID controllers (Shahrokhi and Zomorodi, 2013). The measurement noise in a system arises from the sensors in the transducers (Bequette, 2002). If the sensor accuracy is the problem, then the entire automation becomes a failure (Altmann, 2005). In the following text, a PID controller is used where the overall controller output is the sum of the contributions from the above mentioned three actions. The three adjustable PID parameters are controller gain  $K_c$ , integral time  $T_i$  and derivative time  $T_d$  (Phu *et al.*, 2017). The transfer function of PID controller in parallel form is (Eq. 2) (Bequette, 2002):

$$G_c(s) = K_c \left( 1 + \frac{1}{T_i * s} + T_d * s \right) \quad (2)$$

Where:

$K_c$  = Controller gain

$T_i$  = Integral time

$T_d$  = Derivative time

Internal model control is a model based control technique developed by Garcia and Morari (1982) which provides an appropriate trade-off between robustness and performance of the system and accounts for model uncertainty and disturbances. The basis of IMC is pole-zero cancellation with controller zeros being used to cancel process poles and Q parameterization structure (Fig. 1). This enables IMC with good set point tracking ability, optimum compensation for disturbance and parametric uncertainty (Morari and Zafiriou, 1989). It

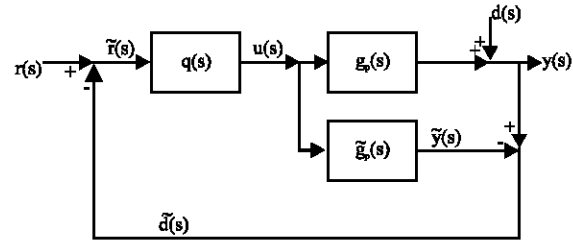


Fig. 1: IMC structure

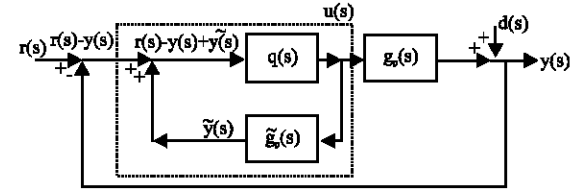


Fig. 2: Rearranged IMC structure

can be employed for Single Input Single Output (SISO) processes, Multi Input Multi Output (MIMO) systems, continuous and discrete designs, unstable open loop systems, systems with feed-forward control, feedback control and so forth (Rao *et al.*, 2015).

The filter constant  $\lambda$  also known as closed time constant  $\tau_c$  is a very important concept in IMC and proper design of the filter is fundamental. The optimum filter constant reduces process variability and aids in achieving a non-oscillatory loop with desired dynamics of the process (Rivera *et al.*, 1986). The rivera guidelines for determining  $\lambda$  are used (Eq. 3 and 4) (Rivera *et al.*, 1986):

$$\frac{\lambda}{\theta} > 0.8 \quad (3)$$

$$\lambda > 0.1\tau \quad (4)$$

The value of  $\lambda$  is obtained using the above criteria for proper tuning of the controller parameters (Juneja *et al.*, 2010). The process dynamics are identified from the response by fitting an appropriate transfer function model to the results (Kala *et al.*, 2014):

Where:

$r(s)$  = Set point

$r'(s)$  = Modified set point =  $r(s) - d'(s)$

$u(s)$  = Manipulated input

$g_p(s)$  = Process

$g_{\tilde{p}}(s)$  = Process model

$d(s)$  = Disturbance

$d'(s)$  = Estimated disturbance

$y(s) = y'(s)$   $y(s)$  = Measured process output

$q(s)$  = Internal model controller

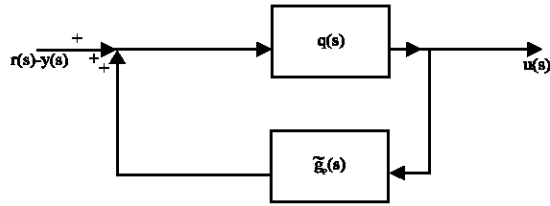


Fig. 3: Inner loop of rearranged IMC structure

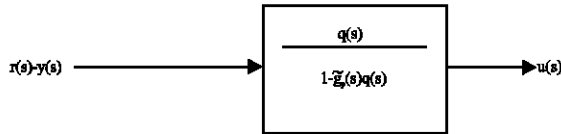


Fig. 4: Equivalent block diagram of the inner loop

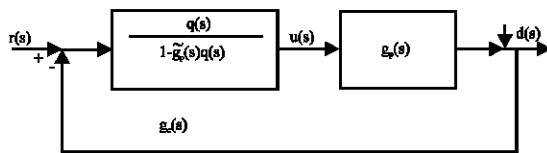


Fig. 5: Block diagram representing standard IMC structure

Figure 1 shows the Q-parameterization structure. It consists of the IMC controller  $q(s)$  and internal process model  $g'_p(s)$  (Fig. 2-4). From Fig. 5, the feedback controller is given by Eq. 5:

$$g_c(s) = \frac{q(s)}{1 - g'_p(s)q(s)} \quad (5)$$

The IMC controller in Fig. 2 is given by Eq. 6:

$$q(s) = \frac{g_c(s)}{1 + g'_p(s)g_c(s)} \quad (6)$$

## MATERIALS AND METHODS

Fourth order transfer function (Eq. 7) mimicking Blending processes has been used for experimentation. The response of the transfer function is given in Fig. 6:

$$G(s) = \frac{1}{(10s+1)(0.1s+1)(0.05s+1)} \quad (7)$$

Using the two point method of approximation, the first order plus dead time model is obtained (Eq. 8):

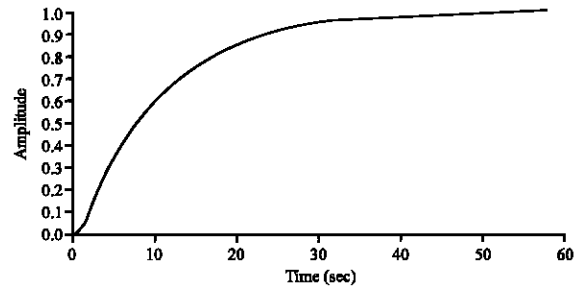


Fig. 6: Response of 4th order transfer function

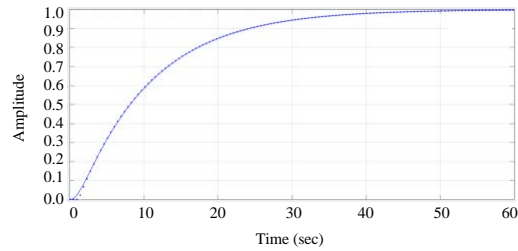


Fig. 7: Response of 4th order transfer function and dead time approximation

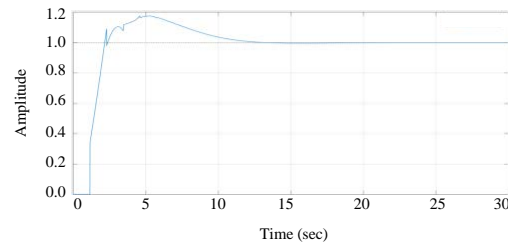


Fig. 8: Response for a step unit using IMC technique for 4th order blending process

Table 1: Tuning formulae for IMC tuning of PID controller

Tuning technique	$K_c$	$T_i$	$T_d$
IMC	$\tau_p + 0.5\theta_p/k_p (\lambda + 0.5\theta_p)$	$\tau_p + 0.5\theta_p$	$\tau_p \cdot \theta_p / 2\tau_p + \theta_p$

Table 2: Tuning parameters of the PID controller

Tuning method	Proportional gain $K_c$	Integrating time $T_i$ (sec)	Derivative time $T_d$ (sec)
IMC ( $\lambda = 1.1$ )	6.3134	10.575	0.5437

$$P(s) = \frac{e^{-1.15s}}{10s+1} \quad (8)$$

Equation 7 and 8 are simulated in MATLAB. Fig. 7 shows the response of the 4th order transfer function model and FOPDT Model (Skogestad, 2003). From Fig. 7 both the transfer function graphs and the FOPDT graphs are approximately the same, therefore, the approximation done using two-point method is correct (Hussain *et al.*, 2014). Table 1 shows the IMC formulations used to obtain the numeric values in Table 2.

Table 3: Time domain characteristics

Tuning method	Overshoot $M_p$ (%)	Settling time $T_s$ (sec)	Rising time $T_r$ (sec)	Peak time $T_p$ (sec)	Amplitude at peak time
IMC ( $\lambda = 1.1$ )	2.577	13.02	3.249	3.302	1.18

Table 4: Time domain characteristics with varying lambda

Lambda $\lambda$	Proportional gain $K_p$	Peak time $T_p$ (sec)	Amplitude at $T_p$	Rise time $T_r$ (sec)	Settling time $T_s$ (sec)	Overshoot $M_p$ (%)
0.1	15.6667	Unstable	Unstable	Unstable	Unstable	234.365
0.5	9.8372	3.291	1.596	2.649	19.3	60.484
0.7	8.2941	3.292	1.347	2.850	18.6	34.459
0.9	7.1694	3.298	1.67	3.049	17.8	15.698
1.1	6.3134	3.302	1.18	3.249	13	2.577
1.5	5.0960	5	1.36	3.248	33	0.483
1.7	4.6484	5.3	1.41	3.25	35.6	0.485
2.2	3.8110	5.6	1.41	3.254	38	0.495
3.5	2.5951	Unstable	Unstable	Unstable	Unstable	244.873
4.5	2.0870	Unstable	Unstable	Unstable	Unstable	254.850

**PID controller tuning:** Table 2 gives the parameters for tuning the PID obtained using the IMC formulations (Table 1). Figure 8 gives the response of the system with controller parameters obtained using the IMC formulations in MATLAB (Table 2). Table 3 gives the time domain characteristics of the response obtained in Fig. 8. With  $\lambda = 1.1$ , the system has a minimal percentage overshoot of 2.577%, settling time of 13.02 sec, rise time 3.249 sec and peak time of 3.302 sec. Table 3 shows the optimum readings for the FOPDT Model.

**IMC study by changing lambda:** The filter constant is now varied from its ideal value of 1.1 and time domain analysis has been done. The following Table 4 shows the numerical figures for the responses observed by varying the filter constant for a step input wit set point at 1, integrating time 10.575 sec and derivative time 0.543735 sec. Note that the responses and the time domain characteristics have been observed for over a time period of 50 sec.

## RESULTS AND DISCUSSION

At  $\lambda = 0.1$  which is an extremely low value compared to 1.1 which is obtained using the Rivera criteria for choosing the filter constant (Eq. 3 and 4), unstable system response with infinitely increasing oscillations is observed. The following Fig. 9 shows the response.

The following value of lambda that is lambda = 0.5, Fig. 10 shows severe overshoots at the initial stages of the response the response shoots up to 1.6 amplitude for the set point of 1 as well as takes 19 sec to settle down.

Lambda = 0.9 is a very close value to 1.1, however, more optimum results are obtained for the ideal value. Response with filter constant = 0.9 (Fig. 9 and 10) takes more time to settle down to the

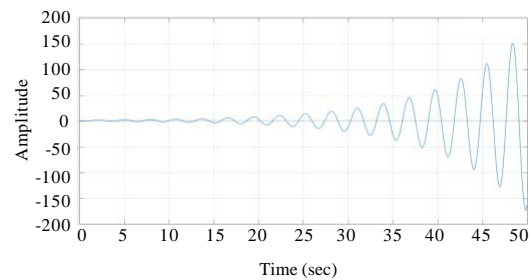


Fig. 9: System response with filter constant = 0.1

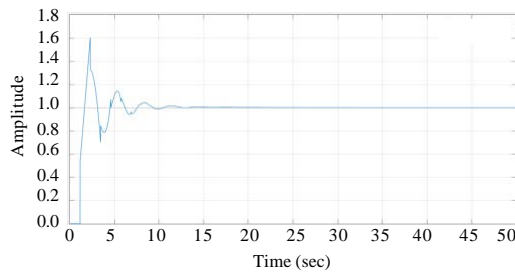


Fig. 10: System response with filter constant = 0.5

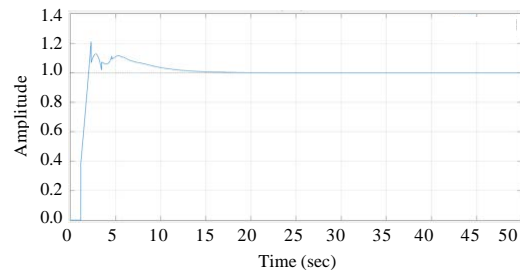


Fig. 11: System response with filter constant = 0.9

desired set point of 1 that is the settling time is 17.8 sec whereas for filter constant 1.1 is 13 sec. In addition, the percentage amplitude and amplitude at peak time are also more (Table 4). With increasing lambda values, the settling time also increases. Here, for lambda = 2.2 (Fig. 12), the settling time is 38.2 sec, almost three times the ideal settling time 13 sec. In addition the am. At lambda = 3.5 (Fig. 13) we observe

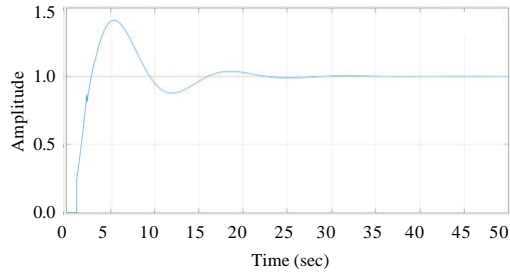


Fig. 12: System response with filter constant = 2.2

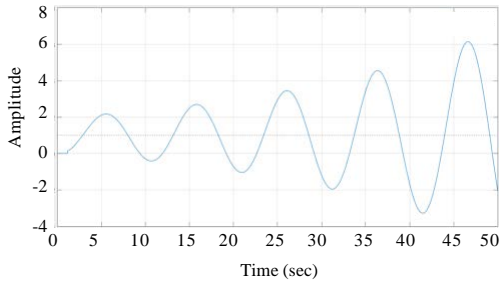


Fig. 13: System response with filter constant = 3.5

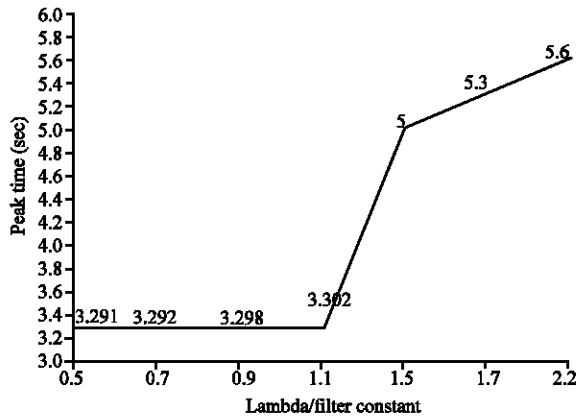


Fig. 14: Peak time versus lambda characteristics

system instability which continues, hereafter. For the FOPDT Model used for experimentation, the ideal value of lambda used is 1.1 and this value is used as a standard to compare the time domain characteristics obtained for other lambda. Using the rivera criteria for selection of proper filter constant, it is mandatory that the lambda be  $>0.8 \times \text{process dead time}$  and simultaneously be  $<0.1 \times \text{process time constant}$ . The process dead time and process time constant are calculated using the two point method of approximation and their values are 1.15 and 10 sec, respectively,  $0.8 \times 1.15 = 0.92$ .

Therefore, ideally lambda value should be  $>0.92$ , hence, the value chosen, here is 1.1 and good results have been obtained (Table 4). The minimum 2.577%

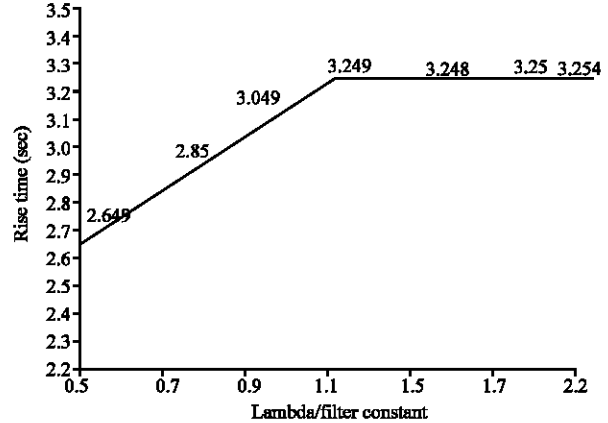


Fig. 15: Rise time versus lambda characteristics

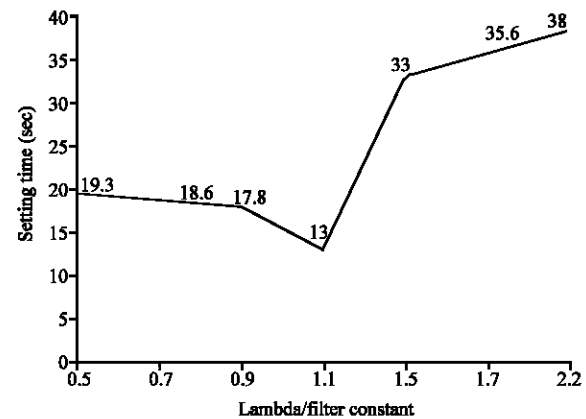


Fig. 16: Settling time versus lambda characteristics

Overshoot, minimum peak time of 3.202 sec for the minimum amplitude at peak time of 1.18 and minimum settling time of 13 sec is observed. The following Fig. 14 shows the trend of peak time with increasing filter constant. Till the ideal value of 1.1, we observe a minimal peak time of 3.2 sec, however, after 1.1 values, the peak time increase. Note that the unstable cases haven't been involved for observing the time domain characteristics trends for peak time, rise time and settling time. Figure 15 and 16 shows the trend of rise time with increasing values of filter constant. The rise time doesn't show significant variations with varying filter constant Fig. 16.

A significant observation is made in the trend of settling time with increasing filter constant. The least settling time is 13 sec and observed for the ideal value 1.1 only. For all the other values (with stable responses) the settling time is more than 13 sec. The settling time shows a decreasing trend as lambda approaches its ideal value, a minimum 13 sec

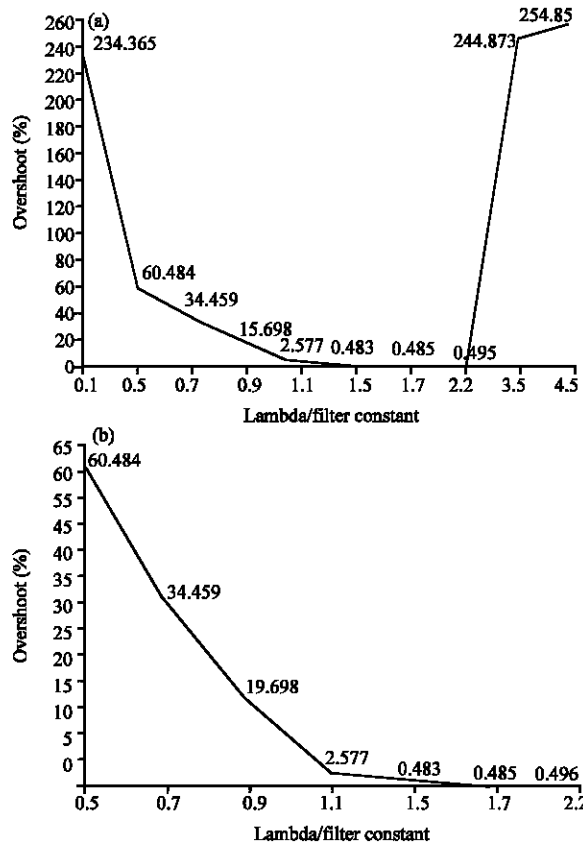


Fig. 17: a) Percentage overshoot versus lambda characteristics for stable and unstable responses and b) Percentage overshoot versus lambda characteristics for stable responses

settling time is achieved at 1.1 and hereafter the trend only goes higher. The settling time after the ideal value of 1.1 only keeps increasing and the maximum value of 38 sec to settle down is observed at  $\lambda = 2.2$ . Figure 17 shows the above observations graphically. The unstable responses are observed for very low  $\lambda$  values (here, 0.1) and very high values (3.5). For these unstable responses the percentage overshoot are large in the ranges of 200-260%. Figure 17a shows the trend of percentage overshoot with increasing  $\lambda$  for both stable and unstable responses. Stable responses have been observed in the range of 0.5-2.2  $\lambda$ . Figure 17b shows the trend of percentage overshoot with increasing  $\lambda$  for stable responses only. It is very evident from the trend that the overshoot decreases with the increasing  $\lambda$ . Even after the ideal value of 1.1, the overshoot decreases which is very fundamental to a good

process control. Minimum % overshoot is 0.495% for  $\lambda = 2.2$ . However, looking at the other trend of settling time, we observe the settling time is maximum for  $\lambda = 2.2$  that is 38 sec. Hence, at  $\lambda = 1.1$ , optimum results are obtained with respect to all the parameters peak time, settling time, rise time and percentage overshoot.

## CONCLUSION

Real time industrial processes possess dead time inherently due to many reasons and the presence of dead time is highly undesirable in the process as it makes the control much more difficult and challenging. A 4th order blending process was approximated as first order plus dead time model using the two point method of approximation and this model was used for all the experimentation purpose. Internal model control tuning technique was implemented using a conventional PID controller to the above FOPDT Model. Filter constant,  $\Lambda$  defines the IMC design and proper estimation of the constant is fundamental. Ideal value of  $\lambda$  using the riveria criteria was calculated as 1.1 and was used as a standard to compare the trends with varying values of  $\lambda$  ranging from 0.1-4.5. It was observed that improper estimation of  $\Lambda$  that is very low value (0.1) and very high value (3.5) lead to unstable systems, large overshoots and highly oscillatory response which are completely undesirable in any control process. In the range of 0.5-2.2, large settling time was a direct observation and decreasing percentage overshoot. Hence, it is very important to design the IMC using a proper value of  $\lambda$ . Only an appropriate value of  $\lambda$  will aid in reducing process variability and effect of disturbances in an active system.

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