

## Effect of the Movement Across a Long inclined, Buried, Creeping, Strike-Slip Fault in the Visco-Elastic Medium of Burger's Rheology

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**Abstract:** Fault movement across a fault is preceded by accumulation of stress near it over a considerable period of time. When this stress exceeds the total cohesive and frictional forces across the fault, a movement which may be sudden or creeping, across it set in. In this study, a creeping movement across a very long, buried, strike-slip fault inclined to the free surface and of finite width is considered in an isotropic, homogeneous, visco-elastic medium of Burger's rheology type. A mathematical model for such fault movement is developed during the period when there is no fault movement and also for the aseismic period which is restored after the sudden movement. The analytical expressions of displacement, stresses and strains for both the period are determined by the use of Green's function technique and correspondence principle. Finally, these displacement, stresses and strains are numerically computed with suitable values of the model parameters and the results thus obtained are presented graphically. Such theoretical models may be used for obtaining greater insight into the earthquake processes in seismically active regions.

**Key words:** Strike-slip fault, visco-elastic medium of Burger's rheology, Green's function technique, correspondence principle

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### INTRODUCTION

Recently there has been a growing interest in the use of static or quasistatic displacement, stresses and strains for the investigation of earthquake phenomena. Model of dynamic processes leading to an earthquake is one of the main concerns of seismologists and geological engineers. It is found that two consecutive major earthquakes in the seismically active region are usually separated by a long aseismic period during which slow and continuous aseismic surface movements are observed with the help of sophisticated measuring instruments like strainmeter, tilt meter, etc. Since, the aseismic period is almost static (quasi-static), the occurrence of foreshocks and aftershocks are neglected. Such aseismic surface movements indicate that slow aseismic change in stress and strain are occurring in the region which may eventually lead to sudden or creeping movements across the seismic faults. These faults may be strike-slip or dip-slip type, finite or long, surface breaking or buried situating in the region. To understand the mechanism of earthquake processes it is necessary to develop mathematical models to study the small ground deformation observed during the aseismic period in the seismically active regions.

**Literature review:** A pioneering work involving static ground deformation in elastic media was initiated by Steketee (1958a, b), Chinnery and Dushan (1972), Chinnery (1961, 1964) and Maruyama (1964, 1966). Andrews (1974), Turcotte and Spence (1974) and Rybicki (1971) did remarkable research in analysing the displacement, stress and strain for strike-slip movement of the fault in the elastic medium. Later some theoretical models have been developed by Rybicki (1971), Mukhopadhyay and Mukherji (1979), Mukhopadhyay *et al.* (1980), Sen *et al.* (1993), Sen and Debnath (2012) and Savage (1975).

Ghosh *et al.* (1992), Segall (2010) and Sen and Debnath (2013) did wonderful works in analysing the displacement, stresses and strains in the layered medium. Sen *et al.* (2012) and Debnath and Sen (2014), discussed about long interacting strike-slip faults in the viscoelastic half space. There after a model for a finite strike-slip fault under tectonic forces was developed by Debnath and Sen (2015).

In most of the cases elastic or viscoelastic half space of maxwell type and standard linear solid or layered medium were considered to represent the lithosphere-asthenosphere system. To the best of our

knowledge, no theoretical model has still been developed in the viscoelastic half space of four element (Burger's element) type to represent earthquake faults. The research work of Hu *et al.* (2016) and observations in the seismically active regions during aseismic period suggest that Burger's type viscoelastic material may be a suitable representation of the lithosphere-asthenosphere system. In this study, we represent the lithosphere-asthenosphere system in the viscoelastic medium of Burger's rheology type and a strike-slip fault inclined to the free surface at a certain depth from the free surface is situated in the half-space. The movement of the fault taken to be creeping in nature and the tectonic force that, we have considered in our calculation is linearly increasing function of time.

## MATERIALS AND METHODS

**Formulation:** A two-dimensional theoretical model with a long vertical buried strike-slip fault F of width D is taken in the lithosphere-asthenosphere system consisting of a viscoelastic half-space of Burger's rheology type. To represent this, we introduce a rectangular cartesian coordinate system ( $y_1$ - $y_3$ ) with  $y_3 = 0$  as the plane free surface,  $y_3$  axis pointing downwards, so that, the viscoelastic half-space can be described by  $y_3 \geq 0$ . Let d be the depth of the upper edge of the fault below the free surface. Suppose ( $y_1$ ,  $y_3$ ) and ( $\xi_1$ ,  $\xi_3$ ) indicate the coordinates of observational points and dislocation source. We introduce new coordinates ( $\xi_1'$ ,  $\xi_3'$ ) as shown in Fig. 1a, so that ( $\xi_1$ ,  $\xi_3$ ) can be associated ( $\xi_1'$ ,  $\xi_3'$ ) by the relation  $\xi_1' = \xi_1$ ,  $\xi_2' = \xi_2 \sin \theta - (\xi_3 - d) \cos \theta$  and  $\xi_3' = \xi_2 \cos \theta + (\xi_3 - d) \sin \theta$ . Thus, the fault is given by F: ( $\xi_2' = 0$ ,  $0 \leq \xi_3' \leq D$ ). The length of the fault is assumed to be very long compared to its width D, so that, the components of displacement ( $u$ ,  $v$ ,  $w$ ), stresses  $\tau_{ij}$  and strain  $E_{ij}$ ,  $i, j = 1, 2, 3$  are independent of  $y_1$  and are functions of  $y_2$ ,  $y_3$  and time  $t$  only and they separate out into two distinct and mutually independent groups-one group containing the components  $u$  ( $\tau_{12}$ ,  $\tau_{13}$ ), ( $E_{12}$ ,  $E_{13}$ ) associated with the strike-slip movement and the other group containing the remaining components associated with a possible dip-slip movement of the fault. We here consider the strike-slip movement across the fault. The Burger's Model which is the combination of the Kelvin-Voigt and the Maxwell type material as shown in Fig. 1b gives a comprehensive model for deriving time-dependent solutions for displacement, stresses and strains.

**Constitutive equations:** The constitutive laws provide the relation between stress and strain possibly including time

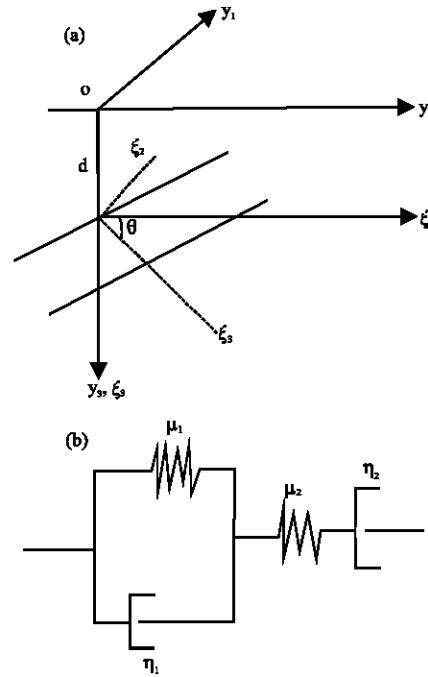


Fig. 1: a) Section of the model by the plane  $\xi_2' = 0$  and b) Burger's Model

derivatives. We here consider strike-slip movement across the fault when the medium is in aseismic state ( $t = 0$ ) for which the displacement  $u$ , stresses  $\tau_{12}$ ,  $\tau_{13}$  and strains  $E_{12}$ ,  $E_{13}$  are present. The stress-strain relations for Burger's rheology model of viscoelastic material are taken as follows (Segall, 2010):

$$\tau_{12} + p_1 \frac{\partial}{\partial t} (\tau_{12}) + p_2 \frac{\partial^2}{\partial t^2} (\tau_{12}) = 2q_1 \frac{\partial}{\partial t} (E_{12}) + 2q_2 \frac{\partial^2}{\partial t^2} (E_{12}) \quad (1)$$

$$\tau_{13} + p_1 \frac{\partial}{\partial t} (\tau_{13}) + p_2 \frac{\partial^2}{\partial t^2} (\tau_{13}) = 2q_1 \frac{\partial}{\partial t} (E_{13}) + 2q_2 \frac{\partial^2}{\partial t^2} (E_{13}) \quad (2)$$

Where:

$$p_1 = \frac{\eta_1}{\mu_1} + \frac{\eta_2}{\mu_2} + \frac{\eta_1}{\mu_2}, \quad p_2 = \frac{\eta_1 \eta_2}{\mu_1 \mu_2}, \quad q_1 = \eta_1, \quad q_2 = \frac{\eta_1 \eta_2}{\mu_2}$$

Where:

$\eta_1, \eta_2$  = The respective effective viscosities  
 $\mu_1, \mu_2$  = The respective effective rigidities of the materials

**Stress equation of motion:** For the small deformations, if the inertial forces are very small, so that, the acceleration can be taken to be negligible and if there are no body forces acting in the system during our consideration, then, the quasistatic equilibrium equation is:

$$\frac{\partial}{\partial y_2}(\tau_{12}) + \frac{\partial}{\partial y_3}(\tau_{13}) = 0 \quad (3)$$

where  $(-\infty < y_2 < \infty, y_3 \geq 0, t \geq 0)$ :

**Boundary conditions:** For the Fault F:

$$\tau_{12}(y_2, y_3, t) \rightarrow \tau_{\infty}(t) = \tau_{\infty}(0)(1+Kt), K > 0 \quad (4)$$

as  $|y_2| \rightarrow \infty, (y_3 \geq 0, t \geq 0)$

$$\tau_{13}(y_2, y_3, t) = 0 \text{ on } y_3 = 0, (|y_2| \rightarrow \infty, t \geq 0) \quad (5)$$

Also:

$$\tau_{13} \rightarrow 0 \text{ as } y_3 \rightarrow \infty, (|y_2| \rightarrow \infty, t \geq 0) \quad (6)$$

where,  $\tau_{\infty}(t)$  is the shear stress of maintained by tectonic forces far away from the fault which may or may not vary with time but is taken to be independent of  $y_3$ . In our case, we assume that  $\tau_{\infty}(t) = \tau_{\infty}(0)(1+Kt)$ ,  $K > 0$  is a increasing function of time.

**Initial conditions:** We assume the time  $t$  from a suitable instant when the model is in an aseismic state and there is no seismic disturbance in it. Let,  $u = u_0$  at the time  $t = 0$  and  $u_t = \partial/\partial t u = 0$  at time  $t = 0$ . We also, assume that:

$$\tau_{12} = (\tau_{12})_0, \frac{\partial}{\partial t}(\tau_{12}) = 0, \tau_{13} = (\tau_{13})_0, \frac{\partial}{\partial t}(\tau_{13}) = 0 \text{ at}$$

time  $t = 0$  and  $E_{12} = (E_{12})_0, E_{13} = (E_{13})_0$  at time  $t = 0$

The above initial values satisfy all the relations given in Eq. 1-6.

**Solution:** Differentiating (1) partially with respect to  $y_2$  and (2) with respect to  $y_3$ , then adding and finally using the relation (3) and initial condition, we get:

$$\nabla^2 U = 0 \text{ where } U = u - u_0 \quad (7)$$

Taking Laplace transform of the resulting equation with respect to time  $t$ , we get:

$$\nabla^2 \bar{U} = 0 \text{ where } \bar{U} = \bar{u} - \frac{u_0}{s} \quad (8)$$

$s$  is the Laplace transform variable.

**Displacement, stresses and strains in the absence of any fault movement:**

The displacement, stresses and strains are all continuous throughout the system and all the equations and boundary conditions given in Eq. 1-6 are valid. The exact solutions for the displacement, stresses and strains can be found by taking Laplace transform of Eq. 1-6 with respect to time  $t$  which can be solved as given in the study. If  $\tau_{\infty}(t)$  does not have a significant change over the period of time we are considering, then it will be reasonable to take  $\tau_{\infty}(t) = \text{constant}$ . On taking inverse Laplace transform, the solutions are obtained as:

$$\left. \begin{aligned} (u)_{\text{absence of fault movement}} &= u_0 + \frac{\tau_{\infty}(0)}{q_1} \left[ t - \frac{q_2}{q_1} \left( 1 - e^{-\frac{q_1 t}{q_2}} \right) + K \left( \frac{t^2}{2} + \frac{q_2}{q_1} \left( 1 - t - e^{-\frac{q_1 t}{q_2}} \right) \right) + p_1 \left( t - \frac{q_2}{q_1} \left( 1 - e^{-\frac{q_1 t}{q_2}} \right) \right) + p_2 \left( 1 - t - e^{-\frac{q_1 t}{q_2}} \right) \right] y_2 \\ (\tau_{12})_{\text{absence of fault movement}} &= \frac{(\tau_{12})_0}{A} \left[ (p_1 - p_2 r_1) e^{-r_1 t} - (p_1 - p_2 r_2) e^{-r_2 t} \right] + \left[ \tau_{\infty}(t) - \frac{\tau_{\infty}(0)}{A} \left[ (p_1 - p_2 r_1) e^{-r_1 t} - (p_1 - p_2 r_2) e^{-r_2 t} \right] \right] \\ (\tau_{13})_{\text{absence of fault movement}} &= \frac{(\tau_{13})_0}{A} \left[ (p_1 - p_2 r_1) e^{-r_1 t} - (p_1 - p_2 r_2) e^{-r_2 t} \right] \\ (E_{12})_{\text{absence of fault movement}} &= \frac{1}{2} (E_{12})_0 + \frac{1}{2} \frac{\tau_{\infty}(0)}{q_1} \left[ t - \frac{q_2}{q_1} \left( 1 - e^{-\frac{q_1 t}{q_2}} \right) + K \left( \frac{t^2}{2} + \frac{q_2}{q_1} \left( 1 - t - e^{-\frac{q_1 t}{q_2}} \right) \right) + p_1 \left( t - \frac{q_2}{q_1} \left( 1 - e^{-\frac{q_1 t}{q_2}} \right) \right) + p_2 \left( 1 - t - e^{-\frac{q_1 t}{q_2}} \right) \right] \\ (E_{13})_{\text{absence of fault movement}} &= \frac{1}{2} (E_{13})_0 \end{aligned} \right\} \quad (9)$$

Where:

$$r_1 = \frac{(p_1 - A)}{2p_2}, r_2 = \frac{(p_1 + A)}{2p_2}, A = (p_1^2 - 4p_2)^{\frac{1}{2}}$$

and  $p_1, p_2$  are same as given in Eq. 1 and 2. Now,  $\tau'_{12} =$  The stress across the fault:

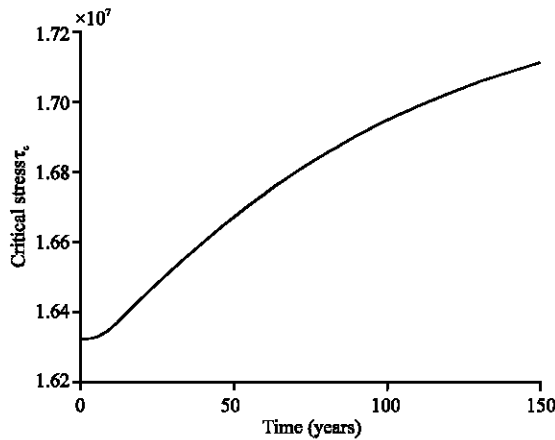


Fig. 2: Critical stress with time

$$F = \tau_{12} \sin \theta - \tau_{13} \cos \theta = \frac{(\tau'_{12})_0}{A} \left[ (p_1 - p_2 r_1) e^{-\eta_1 t} - (p_1 - p_2 r_2) e^{-\eta_2 t} \right] + \left[ \tau_{\infty}(t) \sin \theta - \frac{\tau_{\infty}(0) \sin \theta}{A} \left[ (p_1 - p_2 r_1) e^{-\eta_1 t} - (p_1 - p_2 r_2) e^{-\eta_2 t} \right] \right]$$

where  $(\tau'_{12})_0 = (\tau_{12})_0 \sin \theta - (\tau_{13})_0 \cos \theta$ .

From the above solution, we find that for the Fault F,  $\tau'_{12}$  increases gradually with time and finally tends to  $\tau_{\infty}(t) \sin \theta$ , i.e.,  $\tau_{\infty}(0) (1 + Kt) \sin \theta$  but we assume that the geological condition as well as the characteristic of the Fault F is such that it starts creeping when the magnitude of stress  $\tau'_{12}$  reaches some critical value  $\tau_c$  (say)  $< \tau_{\infty}(t) \sin \theta$ . We consider different inclination  $\theta = \pi/6, \pi/4, \pi/3, \pi/2$ . It is noted that, the smaller values of  $\theta$  are not considerable due to the fact that such situations are not occurs in reality. Here, we consider  $\tau_c = 170$  bar, i.e.,  $17 \times 10^6$  N/m<sup>2</sup> (Pascal) and it is found in Fig. 2 that  $\tau'_{12}$  reaches the value 170 bar after time  $T = 114$  years for  $\theta = \pi/3$ .

**Displacement, stresses and strain after the commencement of the fault movement:** We assume that after a time  $t = T$ , the accumulated stress  $\tau'_{12}$  (which is the main driving force for the strike-slip motion of the Fault F) near F exceeds the critical level  $\tau_c$  and the fault starts creeping due to which the accumulated stress will release at least to some extent. We leave out this short period of time during and immediately after creeping movement and consider the model after the restoration of the aseismic state which happens when the seismic disturbances near the fault gradually disappear. For  $t > T$ , all the basic Eq. 1-8 remain valid and are continuous everywhere except for the Fault F across which the displacement component  $u$  has a discontinuity which characterizes the creeping fault movement given by:

$$[u]_F = U(t_1) f(\xi'_3) H(t_1) \text{ across } F \left( \begin{matrix} \xi'_3 = 0, 0 \leq \xi'_3 \leq D \\ D, t_1 = t - T > 0 \end{matrix} \right)$$

where,  $[u]_F$  is the discontinuity in displacement across F and  $H(t_1)$  is the Heaviside unit step function. The solutions for displacement ( $u$ )<sub>after fault movement</sub>, stresses ( $(\tau_{12})$ <sub>after fault movement</sub>,  $(\tau_{13})$ <sub>after fault movement</sub>) and strains ( $(E_{12})$ <sub>after fault movement</sub>,  $(E_{13})$ <sub>after fault movement</sub>) during this aseismic period restored after major seismic event are derived from Eq. 17-22 as given in this study and the final solutions can be represented in the following forms:

$$\left. \begin{aligned} u &= (u)_{\text{absence of fault movement}} + (u)_{\text{after fault movement}} \\ \tau_{12} &= (\tau_{12})_{\text{absence of fault movement}} + (\tau_{12})_{\text{after fault movement}} \\ \tau_{13} &= (\tau_{13})_{\text{absence of fault movement}} + (\tau_{13})_{\text{after fault movement}} \\ E_{12} &= (E_{12})_{\text{absence of fault movement}} + (E_{12})_{\text{after fault movement}} \\ E_{13} &= (E_{13})_{\text{absence of fault movement}} + (E_{13})_{\text{after fault movement}} \end{aligned} \right\} \quad (10)$$

For the  $U(t_1) = Vt_1$  where,  $V$  is constant creep velocity, we have the final solution as:

$$\left. \begin{aligned} u &= u_0 + \frac{\tau_{\infty}(0)}{q_1} \left[ t - \frac{q_2}{q_1} \left( 1 - e^{-\frac{q_1 t}{q_2}} \right) + K \left( \frac{t^2}{2} + \frac{q^2}{q_1} \left( 1 - t - e^{-\frac{q_1 t}{q_2}} \right) + p_1 \left( t - \frac{q_2}{q_1} \left( 1 - e^{-\frac{q_1 t}{q_2}} \right) \right) + p_2 \left( 1 - t - e^{-\frac{q_1 t}{q_2}} \right) \right) \right] y_2 \\ (\tau_{12}) &= \frac{(\tau_{12})_0}{A} \left[ (p_1 - p_2 r_1) e^{-\eta_1 t} - (p_1 - p_2 r_2) e^{-\eta_2 t} \right] + \left[ \tau_{\infty}(t) - \frac{\tau_{\infty}(0)}{A} \left[ (p_1 - p_2 r_1) e^{-\eta_1 t} - (p_1 - p_2 r_2) e^{-\eta_2 t} \right] \right] + \\ (\tau_{13}) &= \frac{(\tau_{13})_0}{A} \left[ (p_1 - p_2 r_1) e^{-\eta_1 t} - (p_1 - p_2 r_2) e^{-\eta_2 t} \right] + \frac{V}{2\pi A} H(t_1) \Psi_3(y_2, y_3) \left[ \frac{(q_1 - q_2 r_1)}{r_1} (1 - e^{-\eta_1 t_1}) - \frac{(q_1 - q_2 r_2)}{r_2} (1 - e^{-\eta_2 t_1}) \right] \end{aligned} \right\}$$

$$(E_{12}) = \frac{1}{2}(E_{12})_0 + \frac{1}{2} \frac{\tau_\infty(0)}{q_1} \left[ t - \frac{q_2}{q_1} \left( 1 - e^{-\frac{q_1 t}{q_2}} \right) + K \left( \frac{t^2}{2} + \frac{q_2}{q_1} \left( 1 - t - e^{-\frac{q_1 t}{q_2}} \right) + P_1 \left( t - \frac{q_2}{q_1} \left( 1 - e^{-\frac{q_1 t}{q_2}} \right) \right) + P_2 \left( 1 - e^{-\frac{q_1 t}{q_2}} \right) \right) \right] + \left. \frac{V t_1}{4\pi} H(t_1) \psi_3(y_2, y_3)(E_{13}) = \frac{1}{2}(E_{13})_0 + \frac{V t_1}{4\pi} H(t_1) \psi_2(y_2, y_3) \right\} \quad (11)$$

where,  $\psi_1$ - $\psi_3$  are given in the Appendix. It has been observed that the displacement, strains and stresses are unique and will remain bounded everywhere in the model including the upper and lower edges of the fault. The conditions for bounded stresses and strains are that the function  $f(\xi'_3)$ ,  $f'(\xi'_3)$  are continuous in  $0 \leq \xi'_3 \leq D$  and either  $f''(\xi'_3)$  is continuous in  $0 \leq \xi'_3 \leq D$  or  $f''(\xi'_3)$  is continuous in  $0 < \xi'_3 < D$ , except for a finite number of points of finite discontinuity in  $0 \leq \xi'_3 \leq D$  or  $f''(\xi'_3)$  is continuous in  $0 < \xi'_3 < D$  and there exists real constants  $m < 1$  and  $n < 1$  such that  $\xi'_3{}^m f'(\xi'_3{}^m) \rightarrow 0$  or to a finite limit as  $\xi'_3 \rightarrow 0^+$  and that  $(D - \xi'_3)^n f'(\xi'_3) \rightarrow 0$  or to a finite limit as  $\xi'_3 \rightarrow D^-$  and  $f(D) = 0 = f'(D)$ ,  $f(0) = 0$ .

## RESULTS AND DISCUSSION

**Numerical computations:** The numerical solutions for displacement, stresses and strains are computed by assigning suitable values of model parameters (Cathles III 1975; Aki and Richards, 1980) and considering the recent studies on rheological behaviour of the crust and upper mantle by Karato (2010) and Clift *et al.* (2002):

- $\mu_1 = 3.5 \times 10^{10}$  N/m<sup>2</sup>(Pascal)
- $\mu_2 = 3 \times 10^{10}$  N/m<sup>2</sup>(Pascal)
- $\eta_1 = 3.5 \times 10^{19}$  Pas
- $\eta_2 = 3 \times 10^{19}$  Pas
- $\tau_\infty(t) = 200 \times 10^5$  N/m<sup>2</sup> (Pascal)
- $D = 5 \times 10^3$  m
- $(\tau_{12})_0 = 20 \times 10^5$  N/m<sup>2</sup> (Pascal)
- $(\tau_{13})_0 = 20 \times 10^5$  N/m<sup>2</sup> (Pascal)
- $\tau_\infty(0) = 20 \times 10^5$  N/m<sup>2</sup> (Pascal)
- $K = 10^{-9}$
- $V =$  Constant creep velocity = 0, 0.01, 0.02, 0.03 m
- $\theta = \pi/2, \pi/3, \pi/4, \pi/6$

We take  $f(y_3)$  as given as (Ghosh *et al.*, 1992):

$$f(\xi'_3) = \left( 1 - 3 \frac{\xi'^2_3}{D^2} + 2 \frac{\xi'^3_3}{D^3} \right)$$

Which satisfies all the conditions for bounded stress and strain. We have computed displacement, stresses and strains taking above value of the parameters with new time origin  $t_1 = t - T$ , where,  $T = 114$  years using MATLAB.

First, we consider the rate of change of surface displacement per year due to fault creep with  $y_2$  on the free surface  $y_3 = 0$  and  $d = 0$ , taking constant creep velocity  $V = 0.02$  m/year with this assumption, Fig. 3a is almost same as obtained by Sen *et al.* (1993) with out the magnitude of displacement. Figure 3a shows that nature of displacement depends significantly on the inclination of the fault to the horizontal. However, some similarities are found for the faults of different inclination.

The maximum magnitude of the rate of change of surface displacement due to the fault creep is attained near the fault for both  $y_2 > 0$  and  $y_2 < 0$ . This rate of change of surface displacement decreases rapidly as we move

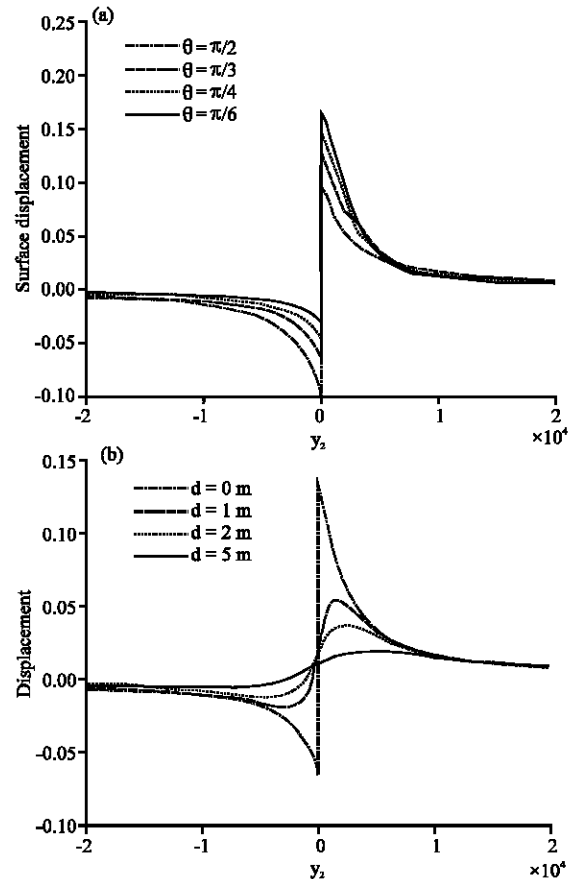


Fig. 3a: Rate of change of displacement due to creeping fault movement with  $y_2$  for various inclination of the fault and b) Displacement after creeping movement with  $y_2$  for different depth from the free surface  $y_3 = 0$

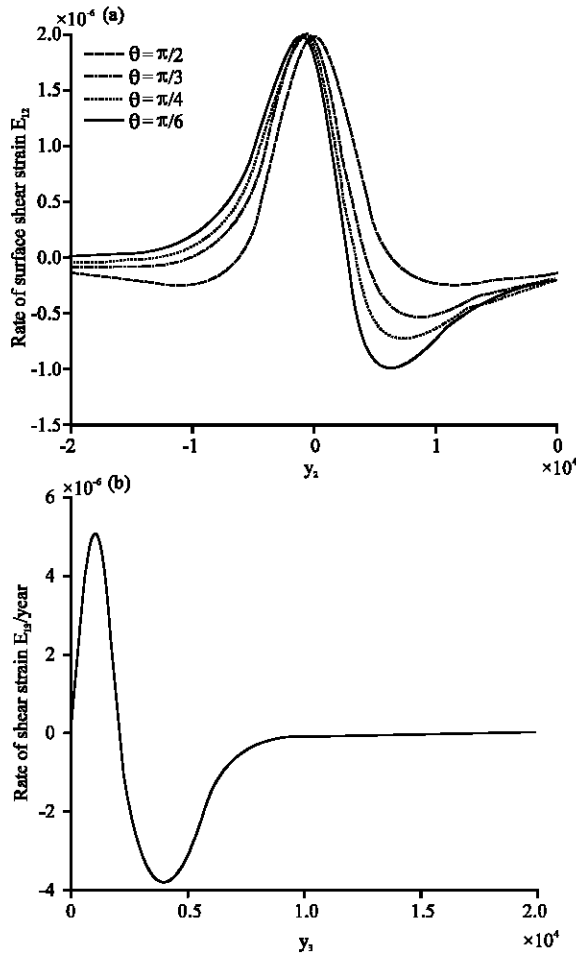


Fig. 4a: Rate of surface shear strain  $E_{12}$  after creeping movement with  $y_2$  for various angle and b) Rate of shear strain  $E_{13}$  due to creeping movement with  $y_3$

away from the fault on the free surface and for  $|y_2| \rightarrow \infty$ , it becomes very small and tending to zero. This rate of change of surface displacement has opposite for  $y_2 > 0$  and  $y_2 < 0$ .

It is found from Fig. 3a that for  $y_2 > 0$ , the rate of surface displacement increases as  $\theta$  decreases and this rate for  $\theta = \pi/6$  is about 0.15 m and when  $\theta = \pi/2$ , it is about 0.1 m. However, for  $y_2 < 0$ , the opposite scenario occurs. For  $\theta = \pi/2$  this rate is anti-symmetrical with respect to  $y_2 = 0$ . But for  $\theta \neq \pi/2$  there is no such anti symmetric.

In Fig. 3b, it has been shown that the displacement not only varies with the various inclination of the fault which is shown in Fig. 3a but also, depends on depth  $d$  from the free surface  $y_3 = 0$ . With the values of  $\theta = \pi/3$  and creep velocity 0.02 m, this displacement is maximum near the fault for  $d = 0$  and as  $d$  increases this

displacement becomes negligible. If, we move far away from the fault this displacement vanishes which is true from geophysical observational fact.

Figure 4a shows the rate of accumulation/release (per year) of the surface shear strain  $E_{12}$  due to the fault creep near the fault ( $y_3 = 0$ ) and away from the fault for different inclination with  $d = 5$  km and  $V = 0.02$  m. It is found that, the accumulation/release of the surface shear strain  $E_{12}$  depend on various angle but this effect falls off rapidly as we move far away from the fault trace on the free surface. For  $\theta \neq \pi/2$ , the surface shear strain  $E_{12}$  accumulation due to the creep is greatest near the fault trace and is symmetric. For  $\theta = \pi/2$ , this effect is not symmetrical about the fault trace and maximum rate of accumulation/release of the surface shear strain  $E_{12}$  occurs a little away from the fault trace. The maximum rate of release of surface shear strain increases as  $\theta$  increases. The effect of the fault creep on the surface shear strain depends significantly on the inclination of the fault to the horizontal.

Figure 4b shows that the rate of shear strain  $E_{13}$  accumulation/release with different depth after the fault movement for  $y_2 = 1$  km,  $d = 1$  km and  $\theta = \pi/2$ ,  $V = 0.01$  m. It is found that there is a sharp accumulation of the rate of shear strain from free surface upto the depth of 0.14 m (approx). As the depth increases, there is a decrease in the rate of shear strain accumulation upto 0.18 m (approx). As the depth increases further, the rate of shear strain release upto the depth 0.4 m (approx) and this shear strain release decreases as we move upto the depth 0.7 m (approx) beyond which the rate of shear strain approaches to zero. This events is true from real observational fact. The order of release of shear strain is  $10^{-6}$  which is conformity with the observational fact in the seismically active regions.

In Fig. 5a, the accumulation of surface share strain under the action of  $\tau_{12}(t)$  against time has been plotted. We find that shear strain in the absence of fault movement  $E_{12}$  first slowly decreases, thereafter it increases rapidly with the initial  $(E_{12})_0 = 0$  and attain a value of  $15 \times 10^{-5}$  on average which is in near conformity with the observational value during the aseismic period. Figure 5b shows variation of release of  $\tau'_{12}$  with time for various constant creep velocity per year near mid point of the fault for a fixed inclination  $\theta = \pi/3$ , taking  $y_3 = 0$ ,  $d = 5$  km and  $y_2 = 5$  km. It is found from Fig. 2, there is a steady accumulation of shear stress  $\tau'_{12}$  near the fault with gradually decreasing rate of accumulation but after the commencement of the fault creep at time  $T = 114$  years, the accumulated stress starts to release. This rate of release of shear stress near the mid point of the fault  $\xi'_{12} = 0$ ,  $\xi'_3 = D/2$  due to the creeping movement is affected by the creep velocities  $V$ . For sufficiently large creep velocities, there is a gradual

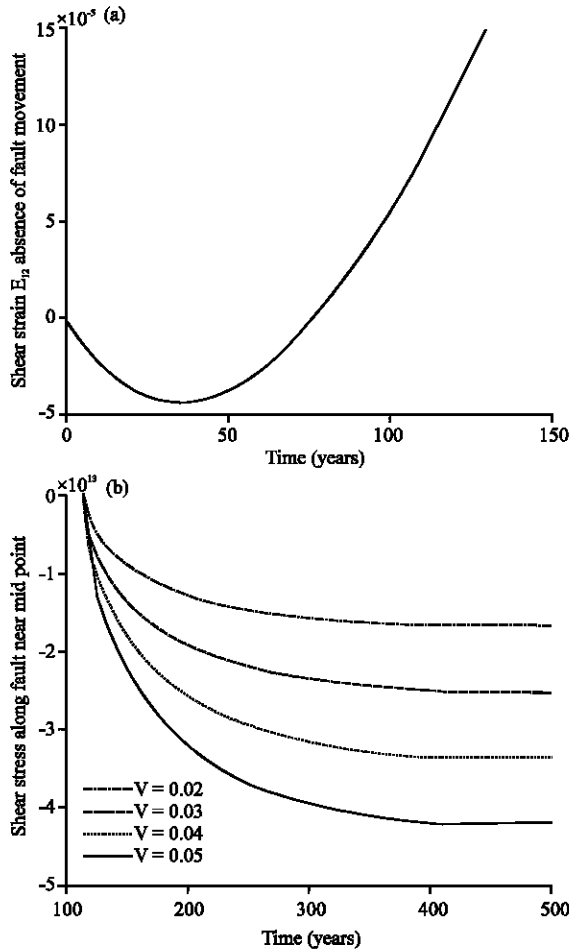


Fig. 5: a) Strain before fault movement and b) Shear stress  $\tau'_{12}$  after creeping movement with time near the mid point of the fault

release of shear stress near the fault after fault creep instead of accumulation, this feature is observed from the geophysical perspective.

## CONCLUSION

In this model of the lithosphere-asthenosphere system represented by Burger's rheology type material with long inclined, plane strike-slip fault, our study provides overview of some physical phenomena due to the creeping fault movement. Then, the model is validated by numerical results which are computed by using suitable values of the model parameters. The nature of the displacement, stresses and strains are analysed by considering their graphical representation. The movement of fault causes stress accumulation/release near the fault which essentially depends on not only on various

inclination of the fault and creep velocity but also on the different depth from the free surface and observational point in the medium.

**Appendix:** Using constitutive Eq. 1 and 2, stress equation of motion (Eq. 3) and initial conditions, the governing equation can be written as:

$$\nabla^2 U = 0 \text{ where } U = u - u_0 \quad (12)$$

Taking the Laplace transform of Eq. 9, then the above equation in the transform domain is:

$$\nabla^2 \bar{U} = 0 \text{ where } \bar{U} = \bar{u} - \frac{u_0}{s} \quad (13)$$

Taking the Laplace transform of all constitutive equation and boundary conditions:

$$\begin{aligned} \bar{\tau}_{12} = & \frac{(p_1 + p_2 s)(\tau_{12})_0}{1 + p_1 s + p_2 s^2} + \frac{(q_1 s + q_2 s^2) \frac{\partial \bar{u}}{\partial y_2}}{1 + p_1 s + p_2 s^2} - \\ & \frac{(q_1 + q_2 s) \left( \frac{\partial \bar{u}}{\partial y_2} \right)_0}{1 + p_1 s + p_2 s^2} \end{aligned} \quad (14)$$

where,  $p_1, p_2, q_1, q_2$  are given by Eq. 1 and 2 and similar other equation for  $\tau_{13}$ . Also, we have the boundary conditions in transform domain as:

$$\begin{aligned} \bar{\tau}_{12}(y_2, y_3, s) & \rightarrow \tau_\infty(0) \left( \frac{1}{s} + \frac{K}{s^2} \right) \text{ as } \\ |y_2| & \rightarrow \infty, (y_3 \geq 0, t \geq 0) \end{aligned} \quad (15)$$

And on the free surface,  $y_3 = 0$ :

$$\bar{\tau}_{13}(y_2, y_3, s) = 0, (-\infty < y_2 < \infty, t \geq 0) \quad (16)$$

Also:

$$\bar{\tau}_{13} \rightarrow 0 \text{ as } y_3 \rightarrow \infty (|y_2| \rightarrow \infty, t \geq 0) \quad (17)$$

where,  $\bar{\tau}_{12} = \int_0^\infty \tau_{12} e^{-st} dt$ ,  $s$  being Laplace transform variable. We solve the governing Laplace's equation with the boundary conditions Eq. 4-6 and 15-17. To solve the boundary value problem, it is customary to assume that  $u$  has the form:

$$\bar{u}(y_2, y_3) = \frac{u_0}{s} + Ay_2 + By_3$$

where,  $A, B$  are constants. Using the initial and boundary conditions and then taking the inverse Laplace transform,

the solution before any fault movement which is given by Eq. 9. After the fault movement, an additional uniform dislocation condition which characterise the creeping movement across F given by:

$$[u]_F = U(t_1) f(\xi_3^1) H(t_1) \text{ across } F \quad (18)$$

$$(\xi_3^1 = 0, 0 \leq \xi_3^1 \leq D, t_1 = t - T > 0)$$

where,  $[u]_F$  is the discontinuity in displacement across F:

$$[u]_F = \lim_{\xi_3^1 \rightarrow 0^+} u - \lim_{\xi_3^1 \rightarrow 0^-} u, (0 \leq \xi_3^1 \leq D) \quad (19)$$

and  $H(t_1)$  is the Heaviside unit step function. Taking Laplace transform of Eq. 18, then:

$$[\bar{u}] = U(s) f(\xi_3^1) \quad (20)$$

All the basic equations initial and boundary conditions are same after the fault movement. The only modified boundary condition is:

$$\bar{\tau}_{12}(y_2, y_3) \rightarrow 0 \text{ as } |y_2| \rightarrow \infty, (y_3 \geq 0, t \geq 0)$$

We solved the resulting boundary value problem by modified Green's function method developed by Maruyama (1964, 1966) and Rybicki (1971) and correspondence principle. Let,  $Q(y_1, y_3)$  be observational point in the medium and  $P(\xi_1, \xi_3)$  be dislocation point on the Fault F, then, we have:

$$\bar{u}(Q) = \int_F \bar{u}(P) G(P, Q) \quad (21)$$

where,  $G(P, Q) = G_{12}(P, Q) d\xi_3 - G_{13}(P, Q) d\xi_2$  and  $G_{12}(P, Q)$ ,  $G_{13}(P, Q)$  are given by:

$$G_{12}(P, Q) = \frac{1}{2\pi} \left[ \frac{y_2 - \xi_2}{L^2} + \frac{y_2 - \xi_2}{M^2} \right], G_{13}(P, Q) =$$

$$\frac{1}{2\pi} \left[ \frac{y_3 - \xi_3}{L^2} + \frac{y_3 - \xi_3}{M^2} \right], L^2 = (y_2 - \xi_2)^2 + (y_3 + \xi_3)^2$$

$$M^2 = (y_2 - \xi_2)^2 + (y_3 - \xi_3)^2$$

The two coordinate axes  $(\xi_1, \xi_3)$  and  $(\xi_1', \xi_3')$  connected by the relation  $\xi_1 = \xi_1'$ ,  $\xi_2 = \xi_2' \sin \theta + \xi_3' \cos \theta$ ,  $\xi_3 = d - \xi_2' \cos \theta + \xi_3' \sin \theta$ , so that, on the fault  $\xi_2' = 0$  and  $0 \leq \xi_3' \leq D$ . From Eq. 20 and 21, one can write:

$$\bar{u}(Q) = \frac{\bar{U}(s)}{2\pi} \int_0^D \left[ \frac{y_2 \sin \theta - (y_3 - d) \cos \theta}{L^2} + \frac{y_2 \sin \theta + (y_3 + d) \cos \theta}{M^2} \right] f(\xi_3') d\xi_3' =$$

$$\frac{\bar{U}(s)}{2\pi} \psi_1(y_2, y_3)$$

$$L^2 = (y_2 - \xi_3' \cos \theta)^2 + ((y_3 - d) - \xi_3' \sin \theta)^2 =$$

$$\xi_3'^2 - 2\xi_3' [y_2 \cos \theta + (y_3 - d) \sin \theta] + y_2^2 + (y_3 - d)^2$$

$$M^2 = (y_2 - \xi_3' \cos \theta)^2 + ((y_3 + d) + \xi_3' \sin \theta)^2 =$$

$$\xi_3'^2 - 2\xi_3' [y_2 \cos \theta - (y_3 + d) \sin \theta] + y_2^2 + (y_3 + d)^2$$

Taking inverse laplace transform with respect to time  $t_1 = t - T$ ,  $u(Q)_{\text{after fault movement}} = U(t_1)/2\pi \psi_1(y_2, y_3) H(t_1)$ . The creep velocity across F is given by  $\partial/\partial t(u) = V(t_1) f(\xi_3')$  where,  $V(t_1) = dU(t_1)/dt_1$  which is assume to be finite for all  $t_1 \geq 0$ . If, we assume  $U(t_1) = Vt_1$  where,  $V$  is constant creep velocity, then:

$$u(Q) \text{ after fault movement} = \frac{Vt_1}{2\pi} \psi_1(y_2, y_3) H(t_1) \quad (22)$$

Where:

$$\psi_1(y_2, y_3) = \int_0^D \left[ \frac{y_2 \sin \theta - (y_3 - d) \cos \theta}{L^2} + \frac{y_2 \sin \theta + (y_3 + d) \cos \theta}{M^2} \right] f(\xi_3') d\xi_3'$$

It is to be noted that  $u = 0$  for  $t_1 = t - T \leq 0$ . From the Eq. 14, 21 and assuming displacement, stress and strain to be zero for  $t_1 = t - T \leq 0$ , thus:

$$\bar{\tau}_{12} = \frac{V}{2\pi s(1+p_1s+p_2s)} \psi_2(y_2, y_3)$$

Taking inverse Laplace transform, we get:

$$(\tau_{12}) \text{ after fault movement} = \frac{V}{2\pi A} H(t_1) \psi_2$$

$$(y_2, y_3) \left[ \frac{(q_1 - q_2 r_1)}{r_1} (1 - e^{-q_1 t_1}) - \frac{(q_1 - q_2 r_2)}{r_2} (1 - e^{-q_2 t_1}) \right] \quad (23)$$

Where:



$$\psi_2(y_2, y_3) = \frac{\partial \psi_1}{\partial y_2} = \int_0^D f(\xi'_3) \left[ \frac{\xi_3'^2 \sin \theta - 2\xi'_3(y_3-d) - (y_2^2 - (y_3-d)^2) \sin \theta + 2y_2(y_3-d) \cos \theta}{L^4} \right] + \left[ \frac{\xi_3'^2 \sin \theta + 2\xi'_3(y_3+d) - (y_2^2 - (y_3+d)^2) \sin \theta - 2y_2(y_3+d) \cos \theta}{M^4} \right] d\xi'_3$$

Similarly one can obtain:

$$(\tau_{13})_{\text{after fault movement}} = \frac{V}{2\pi A} H(t_1) \psi_3(y_2, y_3) \quad (24)$$

$$(y_2, y_3) \left[ \frac{(q_1 - q_2 r_1)}{r_1} (1 - e^{-q_1 t_1}) - \frac{(q_1 - q_2 r_2)}{r_2} (1 - e^{-q_2 t_1}) \right]$$

Where:

$$\psi_3(y_2, y_3) = \frac{\partial \psi_1}{\partial y_3} = \int_0^D f(\xi'_3) \left[ \frac{\xi_3'^2 \sin \theta - 2\xi'_3 y_2 + (y_2^2 - (y_3-d)^2) \cos \theta + 2y_2(y_3-d) \sin \theta}{L^4} \right] - \left[ \frac{\xi_3'^2 \cos \theta - 2\xi'_3 y_2 + (y_2^2 - (y_3+d)^2) \cos \theta - 2y_2(y_3+d) \sin \theta}{M^4} \right] d\xi'_3$$

Using the relation between displacement field and strain field, we can obtain strains as follows:

$$(E_{12})_{\text{after fault movement}} = \frac{V t_1}{4\pi} H(t_1) \psi_2(y_2, y_3) \quad (25)$$

$$(E_{13})_{\text{after fault movement}} = \frac{V t_1}{4\pi} H(t_1) \psi_3(y_2, y_3) \quad (26)$$

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