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An Estimate of the Reliability of the Rayleigh Distribution in the Reliability Stress Strength Using Bayesian Method and Robustfit Method

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Abstract: In this study using two random variables follows Rayleigh distribution such that X is stress and Y is strength with parameters α and β using different size (n, m). Reliability is estimated using the Bayesian method and Robustfit method of the estimate in samples taken from stress and strength distributions. statistical metrics are used to compare between two methods and to get the best method and from MSE, TD and R^2. A small sample of reliability estimates is compared through the Monte Carlo Simulation.

Key words: Rayleigh distribution, stress, strength, reliability is estimated, Bayesian method, Robustfit method

INTRODUCTION

Rayleigh distribution is a special case for weibull two-parameter distribution suitable for life test studies. (Rao, 2012a, b; Kundu and Gupta, 2006; Lee *et al*., 1980; Kundu and Raqab, 2009). This distribution has many required properties and physical explanations and has a nice increase in rate. There are devices (each material element possesses inherent strength) that survive because of its power. These devices receive a certain level of stress and maintain them. But if a higher level of stress is applied then their strength is unable to maintain and they collapse. Suppose Y represents the 'stress' which is applied to a specific device and X represents "strength" to maintain stress, then the reliability of stress strength is indicated by R = P(Y<X) if X, Y are assumed to be random. We defended the study of the relativity of stress strength based on Rayleigh distribution. Rayleigh distribution has the following density function:

$$f(x,\alpha)=\frac{x}{\alpha^2}\frac{-x}{e^2\alpha^2},\quad x>0,\quad \alpha>0$$

And the distribution function:

$$F(x,\alpha)=1-\frac{-x}{e^2\alpha^2},\quad x>0,\quad \alpha>0$$

Let X and Y be two random variables such that X represents "strength" and Y, represents "stress" and X, Y follow a joint pdf f(x, y), then reliability of the component is:

$$R\text{-}P(Y<X)\text{-}\int_{-\infty}^{\infty}\int_{-\infty}^{x} f(x,y)\,dydx$$

Where, f (x, y) = f (x) g (y), so that:

$$R=\int_{-\infty}^{\infty}\int_{-\infty}^{x} f(x)g(y)\,dydx$$

where, f (x) and g(y) are pdf's of X and Y, respectively. The purpose of this study is to examine the reliability of the power of stress on the X, Y are two independent random variables where X and Y follows a Rayleigh distribution with parameters α and β such that:

$$f(x,\alpha)=\frac{x}{\alpha^2}\frac{x}{e^2\alpha^2}\ \text{and}\ g(y,\beta)=\frac{y}{\beta^2}\frac{-y}{e^2\beta^2}\ \text{then}$$

$$R=P(Y<X)=\int_0^{\infty}\int_0^{x}\frac{x}{\alpha^2}\frac{-x}{e^2\alpha^2}\ \frac{y}{\beta^2}\frac{-y}{e^2\beta^2}\,dydx \tag{1}$$

$$R=\frac{\alpha^2}{\alpha^2+\beta^2}$$

Bayesian estimation: Let $x_1, x_2, ..., x_n$ and $y_1, y_2, ..., y_m$ have two random samples of the order of size n, m, respectively on the strength and stress distribution of all Rayleigh changes with parameters α and β scale there are several steps to calculate the Bayes estimators of the Rayleigh distribution with one parameter, so, to do this must we must to know the prior distribution and

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posterior distribution as follows (Pandey and Upadhyay, 1986; Rao, 2012b; Kotz and Pensky, 2003):

$$\text{Posterion distribution} = \frac{\text{Prior distribution} \times \text{Likelihood}}{\text{Marginal distribution}}$$

The conditional probability density function of α and β given the data is given by:

$$\pi(\alpha, \beta|y, x) = \pi_1(\alpha|x) * \pi_2(\beta|y)$$

Where:

$$\pi_1(\alpha|x) = \frac{2\left(\frac{s^2}{2}\right)n+c-\frac{1}{2}}{\Gamma\left(n+c-\frac{1}{2}\right)\left(\alpha^2\right)^{m+c}} e^{\left[\frac{-s^2}{2\alpha^2}\right]} \quad \alpha>0 \text{ where } s^2 = \sum_{i=1}^{n} x_i^2$$

$$\pi_2(\beta|y) = \frac{2\left(\frac{u^2}{2}\right)n+c-\frac{1}{2}}{\Gamma\left(m+c-\frac{1}{2}\right)\left(\beta^2\right)^{m+c}} e^{\left[\frac{u^2}{2\beta^2}\right]} \quad \beta$$

$$>0, \text{ where, } u^2 = \sum_{j=1}^{m} y_i^2, \text{ c: constant}$$

By using squared error loss function given by:

$$L_1(\hat{\alpha}_b, \alpha) = (\hat{\alpha}_b, \alpha)^2$$
$$L_2(\hat{\beta}_b, \beta) = (\hat{\beta}_b, \beta)^2$$

The risk function is:

$$R_s(\hat{\alpha}_b) = \int_0^\infty L_1(\hat{\alpha}_b, \alpha).\pi_1(\alpha|x)d\alpha =$$

$$\hat{\alpha}_b^2 - 2\hat{\alpha}_b \frac{\Gamma(n+c-1)}{\Gamma\left(n+c-\frac{1}{2}\right)}\sqrt{\frac{s^2}{2}} + \frac{\Gamma\left(n+c-\frac{3}{2}\right)s^2}{\Gamma\left(n+c-\frac{1}{2}\right)^2}$$

And:

$$R_s(\hat{\beta}_b) = \int_0^\infty L_1(\hat{\beta}_b, \beta).\pi_2(\beta|y)d\beta =$$

$$\hat{\beta}_b^2 - 2\hat{\beta}_b \frac{\Gamma(m+c-1)}{\Gamma\left(m+c-\frac{1}{2}\right)}\sqrt{\frac{u^2}{2}} + \frac{\Gamma\left(m+c-\frac{3}{2}\right)u^2}{\Gamma\left(m+c-\frac{1}{2}\right)^2}$$

The Baye's estimator of $\hat{\alpha}_b$ and $\hat{\beta}_b$ is a solution of the equation:

$$\frac{\partial R_s(\hat{\alpha}_b)}{\partial \hat{\alpha}_b} \text{ and } \frac{\partial R_s(\hat{\beta}_b)}{\partial \hat{\beta}_b}$$

Which implies:

$$\hat{\alpha}_b = \frac{\Gamma(n+c-1)}{\Gamma\left(n+c-\frac{1}{2}\right)}\sqrt{\frac{s^2}{2}} \quad (2)$$

$$\hat{\beta}_b = \frac{\Gamma(m+c-1)}{\Gamma\left(m+c-\frac{1}{2}\right)}\sqrt{\frac{u^2}{2}} \quad (3)$$

MATERIALS AND METHODS

Robustfit method: As a result of the rapid development of modern software, including the application of MATLAB, statistical functions have emerged in these programs used as a method to solve the problems experienced by the statistical models Robustfit function. This method is used to find the solid estimates of the regression model using weight functions and expresses this function as follows (Al-Aabdi and Karidi, 2018; Martinez and Martinez, 2007):

$$[b, \text{stats}] = \text{robustfit}(X, Y, \text{'wfun'}, \text{'tune'}, \text{const}) \quad (4)$$

Where, b; Vector parameter estimation of distribution, stats; A vector includes a set of statistics that relate to distribution estimation, X, Y; Vector variables in the regression model, Wfun; Is a weighted function proposed by statisticians and researchers in the method of conservative estimates to give a non-sensitive estimate and not affected by outliers tune, const; It represents the cut constant and these functions and constants can be illustrated as in the following Table 1.

Goodness of fit analysis: For the purpose of access to the best it has been estimated to rely on the following statistical standards as a basis for comparison between the results of the two methods (Martinez and Martinez, 2007; Afify, 2004):

Table 1: Weighted function

Weight function	Meaning	Tuning constant
'andrews'	W = (abs(r) <pi).*sin (r)./r	1.339
'bisquare'	W = (abs(r) <1).*(1.r.^2).^2	4.685
'cauchy'	W = 1./(1+r.^2)	2.385
'fair'	W = 1./(1+abs(r))	1.400
'huber'	W = 1./max (1, abs(r))	1.345
'logistic'	W = tanh (r)./r	1.205
'talwar'	W = 1* (abs (r) <1)	2.795
'welsch'	W = exp (-(r.^2))	2.985

Mean Square Error (MSE): The Mean Squares of Error (MSE) can be calculated through the following mathematical equations:

$$\text{MSE}\left(\int(x_i, y_i)\right) - \frac{\sum_{i=1}^{n}\left(\left(\hat{f}(x_i, y_i) - f(x_i, y_i)\right)^2\right.}{n} \quad (5)$$

such that:

$$f(x, y) = \frac{x}{\alpha^2} e^{\frac{-x}{e^2\alpha^2}} \; \frac{y}{\beta^2} e^{\frac{-y}{e^2\beta^2}}$$

$$\hat{f}(x, y) = \frac{x}{\hat{\alpha}^2} e^{\frac{-x}{e^2\hat{\alpha}^2}} \; \frac{y}{\hat{\beta}^2} e^{\frac{-y}{e^2\hat{\beta}^2}}$$

Correlation coefficient (R^2): The reasonable relationship between treatment and observed responses, as confirmed by R^2 statistic:

$$\text{TSS} = \text{sum}\left(\left(y - \bar{y}\right)\right)^2$$
$$\text{RSS} = \text{sum}\left(\left(y - \hat{y}\right)\right)^2 \quad (6)$$
$$R^2 = 1 - \frac{\text{TSS}}{\text{TSS}}$$

Total Deviations (TD): Were employed some methods appropriate analysis here is the sum of the deviations of each method as follows:

$$\text{TD} = \left|\frac{\hat{\alpha} - \alpha}{\alpha}\right| + \left|\frac{\hat{\beta} - \beta}{\beta}\right| \quad (7)$$

RESULTS AND DISCUSSION

In this research, we offer some results to compare the performance of the methods used for different sizes of samples. We perform this comparison taking sample sizes as n, m = 10, 50, 100 and 150 the results are based on 3000 simulation runs. Both stress and strength is created for the population ($\alpha = 1$, $\beta = 2$). You will generate random samples of different sizes by observing using the following code:

$$x = \text{raylrand}(\text{alpha}, n, 1)$$

$$y = \text{raylrand}(\text{beta}, n, 1)$$

Such that, x is stress and y is strength. To compare between estimation of parameter (α and β) and the reliability function to Rayleigh distribution. As was the use of statistical measures that have been mentioned in the Eq. 5-7 the results are given in Table 2 and 3.

Table 2: Simulation results for Bayesian estimation method

Sample size	$\hat{\alpha}$	$\hat{\beta}$	MSE	TD	R^2	Reliability
10	0.9520	1.8992	0.0432	0.2556	0.9840	0.2107
50	0.9885	1.9801	0.0343	0.1141	0.9966	0.2014
100	0.9947	1.9913	0.0327	0.0802	0.9983	0.2007
150	0.9968	1.9948	0.0334	0.0656	0.9988	0.2005

Table 3: Simulation results for Robustfit method

Sample size	$\hat{\alpha}$	$\hat{\beta}$	MSE	TD	R^2	Reliability
10	1.0235	1.9941	0.1865	0.9101	0.9850	0.2771
50	1.0080	1.9981	0.3106	0.3727	0.9966	0.2170
100	1.0023	1.9995	0.1983	0.2580	0.9983	0.2074
150	1.0050	1.9989	0.1636	0.2097	0.9988	0.2063

CONCLUSION

In this research, we examined the reliability of the stress-strength of Rayleigh distribution when both stress and strength tracked the same population. As we have estimated reliability using Bayesian and Robusfit method of estimation. Through the use of the MATLAB program and for different sizes and three statistical measures, we note that the Bayesian method gave better results than the Robustfit method and for all sizes. The estimated reliability function was better than the other method The estimated parameters (α, β) in the method are better and closer to the initial values and therefore, we recommend adopting what gave him good values.

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