Journal of Engineering and Applied Sciences 14 (3): 675-683, 2019
ISSN: 1816-949X

A Robust Neural Network Approach for the Portfolio Selection Problem Basing on New Rational Models

[1]Kaoutar Senhaji, [2]Karim El Moutaouakil and [2]Mohamed Ettaouil
[1]Laboratory of Modeling and Scientific Calculating, Faculty of Sciences and Technics, University of Sidi Mohamed Ben Abdellah, Fes, Morocco
[2]ENSAH Ecole Nationale des Sciences Appliquees Al Hoceima, Hoceima, Morocco

Abstract: The portfolio management is a very important problem in the econometric field. In this research, we propose a new model by adding a new constraint to the Markowitz's Models to avoid the investigation into the assets of a negative return. Because of its effectiveness, continuous hopfield network is used to solve the proposed models. In this regard, we construct an original energy function that makes a compromise between the risk, profit and cardinality constraints. To ensure the equilibrium point feasibility, the parameters of the energy function are chosen based on a consistence mathematical results; In addition, the slop of the activation functions is chosen such that the behavior of each neuron is almost leaner. We compare our method to several other ones, basing on real financial data. The proposed method produces the best solutions.

Key words: Portfolio problem, mean-variance, semi-variance, efficient frontier, hopfield neural networks, energy function, Genetic algorithm

INTRODUCTION

The portfolio management is a very important problem in econometric science he gives the answer to the investor for his must difficult question, how allocate his capital between many actives. The first model was the Markowitz Model or mean-variance model by Markowitz (1952). He was the pioneer of the first rigorous treatment of the investor's dilemma and his objective was to reach a lower level risk with an expected return. Jarrow (1988) introduces a general equilibrium mode between the level risk and the portfolio return by the mean-variance efficient frontier. The Markowitz Model uses the variance formula of each assets return as a risk measure and the mean of return tocalculate the profitt, hence, his name. Indeed, the problem was solved with many methods, deterministic (Thi *et al*., 2009a, b) and approached one (Sefiane and Benbouziane, 2012; Zarranezhad *et al*., 2015). In this research, we propose a new model by adding a new constraint to the mean-variance model to avoid the investing in the assets of a negative return given its effectiveness. Continuous Hopfield network is used in this research to solve the proposed model. In this regard, we construct an original energy function that makes a compromise between the risk, profit and cardinality constraints. To ensure the equilibrium point feasibility, the parameters of the energy function are chosen based on a mathematical consistency result; In addition, the slop of the activation functions is chosen such that the behavior of each neuron is almost leaner.

The Continuous Hopfield Neural network (CHN) was proposed by Hopfield and Tank (1985) to solve combinatorial problems; Some researchers have treated the Quadratic Knapsack Problem (QKP) (Yao, 1988; Tatsumi *et al*., 2002). Within these studies, the feasibility of the equilibrium points of the CHN cannot for the general case, be assured; Moreover, the solutions obtained are often not good enough. To avoid this problem, a general methodology was proposed to solve the Generalized Quadratic Knapsack Problem (GQKP) (Talavan and Yanez, 2006). Since, the differential equation which characterizes the dynamics of the CHN is analytically hard to solve, many researchers used the famous Euler method. Recently, the CHN was used to solve the travelling salesmen problem (Takeda and Goodman, 1986; Talavan and Yanez, 2002), the constraint satisfaction problem (Ettaouil and Loqman, 2008) and the placement of the electronic circuits problem (Ettaouil *et al*., 2009).

MATERIALS AND METHODS

Continuous Hopfield Network (CHN): Hopfield and Tank presented the Continuous Hopfield Networks (CHN) to solve several optimization problems including the

Corresponding Author: Karim El Moutaouakil, ENSAH Ecole Nationale des Sciences Appliquees Al Hoceima, Hoceima, Morocco

Traveling Salesman Problem (TSP), analog to digital conversion, signal processing problems and linear programming problems (Hopfield and Tank, 1985). This approach was applied, later to different problems, especially, object recognition, graph recognition, graph coloring problems, constraint satisfaction problems, economic dispatch problems, portfolio problems and image restoration (Ettaouil and Loqman, 2008; Ettaouil *et al*., 2009; Fernandez and Gomez, 2007; Joudar *et al*., 2015).

The Continuous Hopfield Networks (CHN) consist of interconnected neurons with a smooth sigmoid activation function usually a hyperbolic tangent. The differential equation which governs the dynamics of the (CHN) is:

$$\frac{du}{dt} = \frac{u}{\tau} + Tv + i^b \quad (1)$$

where, T, i, u and v are respectively, the connections weight matrix, the biais vector, the neurons state vector and the outputs vector calculated from the state using the hyperbolic tangent.

A point u^e is called an equilibrium point of the system (Eq. 1) if for an input vector u^o, u^e satisfies $u(t) = u^e$ for some $t_e \geq 0$. Hopfield proved that the symmetry of matrix T with zero diagonal are sufficient conditions for existence of equilibrium point (Hopfield, 1984). The Continuous Hopfield Networks (CHN) will solve combinatorial problems that have an energy function taking the following form:

$$E(v) = -\frac{1}{2} v^t T v - \left(i^b\right)^t v \quad (2)$$

For a given combinatorial optimization problem with S variables and m linear constraints, the energy function can be assumed as:

$$E(v) = E^e(v) + E^p(v) \forall v \in H \quad (3)$$

Equation 3 is the Hamming hypercube such that:

- $E^e(v)$ is directly proportional to the objective function
- $E^p(v)$ is a quadratic function that ensure the feasibility of the solution obtained by the CHN

From now on, we denote by the Hamming hypercube, the Hamming hypercube corners set and $H_F\{v \in H_c / Rv = b\}$ the feasible solutions set where Rv = b the constraint system of the problem under study.

The portfolio modeling: Let us say that, an investor with a capital desire to invest in a number of financial assets. Then the investor is faced with a decision to make, how to allocate capital among active. Portfolio management allows us to provide an answer to this question from the various corner (or objective); it may be to choose between many actives to reach the lower risk possible for a fixed return or to achieve the higher return for a fixed risk level. Therefore, the two fundamental dimensions of a financial investment are the return and risk.

Profitability

Return of financial assets: Let p_t be the price of a financial asset at time t, n be the number of active and m the number de period. Several formula were used measure profitability appreciation (or depreciation) on the value of a financial asset or a portfolio of financial assets between two successive moments (Aftalion, 2004).

Simple or arithmetic profitability:

$$r_{tj} = \frac{P_{t+1} - P_t}{P_t} \quad (4)$$

If financial flows D_{t+1} such as a dividend is received between t and t+1 this formula becomes:

$$r_{tj} = \frac{P_{t+1} - P_t + D_{t+1}}{P_t} \quad (5)$$

Logarithmic profitability: We called logarithmic profitability observed in [t, t+1] the value defined by:

$$r_{tj} = \ln\left(\frac{P_{t+1}}{P_t}\right) \quad (6)$$

If financial flows D_{t+1} such as a dividend is received between t and t+1, this formula becomes:

$$r_{tj} = \ln\left(\frac{P_{t+1} + D_{t+1}}{P_t}\right) \quad (7)$$

Portfolio return: Basing on these formulas, we define the anticipate profitability as the weighted mean of the possible return. It is calculated by:

$$R = \sum_{i=1}^{n} r_i x_i \quad (8)$$

Where:

x_i, i = 1, ... , n = Ponderation of active i
r_i = The mean return of the active i

Calculate by:

$$r_i = \frac{1}{m} \sum_{t=1}^{m} r_{ti} \quad (9)$$

Generally, the profitability of the portfolio is calculated by using the anticipate profitability formula. View its simplicity, the simple profitability is used to evaluate each asset.

Risk: The risk is taking into account the uncertainty. The reel return can by different to the anticipated profitability, this is the difference apprehended by the risk.

Variance: The variance is a classic measure of risk Markowitz is the first one to use the variance like a measure of risk by Markowitz (1952) and the first measure of risk. The variance formula is given by:

$$\sigma_i^2 = E\left[\left(r_i - \mu_i\right)^2\right] = \frac{1}{T}\sum_{t=1}^{T}\left(r_{it} - \mu_i\right)^2 \quad (10)$$

Where:
μ_i = The mean return of asset i
T = The number of observations

In addition, the covariance between two assets i and j is given by:

$$\sigma_{ij} = E\left[\left(r_i - \mu_i\right)\left(r_j - \mu_j\right)\right] = \frac{1}{T}\sum_{t=1}^{T}\left(r_{it} - \mu_i\right)\left(r_{jt} - \mu_j\right) \quad (11)$$

Using the variance like a measure of risk have so much advantage, the principal advantage is his simplicity, using the variance-covariance matrix lit as construct a quadratic program model who is convex to calculate the efficient portfolio.

This model has inconvenient to the first one, she supposes that the return has a normal distribution what is unusually realizable. The next one is the insensitivity of the model as the difference between gains and losses; The model penalizes the same way the wastes and gains. Every gain that is too far from the average is removed. Markowitz proposed the semi-variance to overcome the defect of the MV Model.

Semi-variance: The semi-variance is a downside risk measure, proposed by Markowitz (1995), concentrate essentially about the wastes, it takes into account only the gaps that are below a target for this specific reason, we recommend that the returns follow a non-Gaussian distribution. The semi-variance of asset I's returns with respect to Benchmark B is given by:

$$\sum_{iB}^2 = E\left[\text{Min}\left(r_i - B, 0\right)^2\right] = \frac{1}{T}\sum_{t=1}^{T}\left[\text{Min}\left(r_{it} - B, 0\right)\right]^2 \quad (12)$$

Where:
r_i = The return of asset i
r_{it} = The return of asset i in t time
B = Any Benchmark return chosen by the investor

The semi-covariance is more difficult the define, (Hogan and Warren, 1974) define it as:

$$\sum_{ij}^{HW} = E\left[\left(r_i - r_f\right)\min\left(r_i - r_f, 0\right)\right] \quad (13)$$

where, the superscript HW indicates that this is a definition proposed by Hogan and Warren. However, this definition has two drawbacks: the benchmark return is limited to the risk-free rate and cannot be tailored to any desired benchmark.

It is usually the case that $\sum_{ij}^{HW} \neq \sum_{ji}^{HW}$ .This second characteristic is particularly limiting, since that the semi-covariance matrix is usually asymmetric. Moreover, it is not clear how to interpret the contribution of assets i and j the risk of portfolio. Estrada (2000, 2006) have defined the semicovariance between assets I and j with respect to Benchmark B as:

$$\sum_{ijB} = E\left[\text{Min}\left(r_j - B, 0\right).\ \text{Min}\left(r_j - B, 0\right)\right] = \frac{1}{T}\sum_{i=1}^{T}\left[\text{Min}\left(r_{it} - B, 0\right).\ \text{Min}\left(r_{jt} - B, 0\right)\right] \quad (14)$$

According to Estrada, this definition can be tailored to any desired B and generates a symmetric and exogenous semicovariance matrix. Usually, the risk of the portfolio using the semivariance is defined by:

$$\sum_{pB}^2 = \frac{1}{T}\sum_{t=1}^{T}\left[\min\left(R_{pt} - B, 0\right)\right]^2 \quad (15)$$

Where:
R_{pt} = The returns of portfolio
$\sum_{pB}^2$ = Semivariance

The problem in this formula is that the semicovariance matrix is endogenous that is a change in weights affects the periods in which the portfolio underperforms the benchmark which in turn affects the elements of the semicovariance matrix.

However, Estrada (2015) proposed to a heuristic makes it possible to solve all mean-semivariance problems with the same well-known closed-form solutions widely used to solve mean-variance problems. More precisely, the semivariance of a portfolio with respect to a Benchmark B can be approximated with the expression:

$$\sum_{pB}^2 \approx \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sum_{ijB} \quad (16)$$

This expression yields a symmetric and exogenous semivariance matrix, according to Estrada can be used in the same way the covariance matrix used in the mean-variance problems.

Quadratic programming for the portfolio problem: The mean-variance model is the first model in the modern theory of portfolio by Markowitz (1952). He reflects the choice of the investor, the Markowitz Model tries to share the richness among assets with reducing the level risk of the portfolio. The following model is the mean-variance efficient frontier (Jarrow, 1988) an equilibrium mode of the Markowitz Model.

$$(P)\begin{cases} \text{Min } \lambda\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} - (1-\lambda)\sum_{i=1}^{n} r_i x_i \\ \sum_{i=1}^{n} x_i = 1 \\ x_i \geq 0 \quad i = 1, ..., n \end{cases} \tag{17}$$

where, the first term of the Eq. 17 represents the risk of the portfolio using the variance measure. The second term is the profitability of the portfolio using the anticipate one. The λ is the aversion parameter to risk like $0\leq\lambda\leq 1$. When $\lambda = 1$ correspond to the investor a purely risk-averse (minimizing the total variance associated to the portfolio regardless of the mean returns and the optimal solution will typically consist of several assets). The $\lambda = 0$ correspond to the one who has no fear of taking the risk (maximising the portfolio return without considering the risk and the optimal solution will be formed only by the asset with the greatest return). Or Any value of λ inside the interval [0, 1] represents a trade-off between the return and the risk of the portfolio.

The constraint Eq. 18 is the affection constraint, to ashore that the investment doesn't exceed 100% of invested capital. The constraint Eq. 19 is expressing the positivity of the variables. Or x_i, i = 1, ..., n is a ponderation of active i at risk forming the portfolio.

To avoid the investing in the assets of a negative return, we add a new constraint to the Markowitz Model. To this end, we define the set i as follow:

$$I = \{\text{the asset } i \backslash r_i \leq 0\} \tag{18}$$

In this sense, the proposed constraint is given by:

$$\sum_{i\in I} x_i = 0 \tag{19}$$

The new model that minimizing the risk and maximizing the return and avoid the investing in the assets of a negative return the mean-variance rational model is as follows:

$$(PM)\begin{cases} \text{Min } \lambda\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} - (1-\lambda)\sum_{i=1}^{n} r_i x_i \\ \text{S. C} \\ \sum_{i=1}^{n} x_i = 1 \\ \sum_{i\in I} x_i = 0 \end{cases} \tag{20}$$

$$x_i \geq 0 \quad i = 1, ..., n \tag{21}$$

What for the modsel selection using the semivariance as a measure of risk is the same as the Model (PM), just replace the variance formula by semi-variance formula to get the mean-semivariance rational model. To resume the portfolio problem selection under the downside risk is solved by the same way thin the mean-variance rational model:

$$(PM')\begin{cases} \text{Min } \lambda\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sum_{ijB} - (1-\lambda)\sum_{i=1}^{n} r_i x_i \\ \text{S. C} \\ \sum_{i=1}^{n} x_i = 1 \\ \sum_{i\in I} x_i = 0 \end{cases} \tag{22}$$

$$x_i \geq 0 \quad i = 1, ..., n \tag{23}$$

To solve this model via. enumerative methods, we can use different types of relaxations as Lagrangian relaxation, semidefinite relaxation or convex quadratic relaxation (Billionnet and Soutif, 2004; Goemans, 1997). Nevertheless, these methods are very slow. As it's will knowing, the continuous Hopfield networks is a fast neural network method that is why we use it, in the coming sections to solve the proposed model.

Continuous Hopfield network for the portfolio problem: In this part, we propose a new continuous Hopfield network to solve the Markowitz Model for portfolio problem. This approach must make a compromise between the objective function and the constraint.

Energy function: To solve the (PM) via. the CHN, we propose the following energy function:

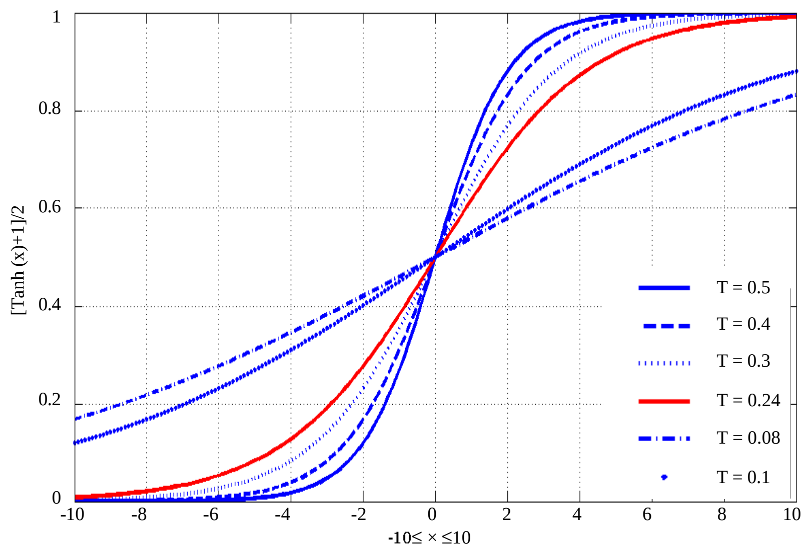

Fig. 1: The slope activation function for slope values (0.5, 0.4, 0.3, 0.24, 0.1 and 0.08)

$$E(x) = \alpha\frac{\lambda}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}x_i x_j \sigma_{ij} - \frac{\alpha}{2}(1-\lambda)\sum_{i=1}^{n} r_i x_i + \beta_1\left(\mathbb{1}^t x\right) + \beta_2\left(\mathbb{1}^t x\right)^2 + \phi_1 \mathbb{1}_I^t x + \phi_2\left(\mathbb{1}_I^t x\right)^2 \quad (24)$$

Where:

$$\mathbb{1}^t = (1, ..., 1)^t \in \mathbb{R}^n \quad (25)$$

$$\mathbb{1}_I^t = (\delta_1, ..., \delta_n)^t \in \mathbb{R}^n \quad (26)$$

Such that:

$$\delta_i \begin{cases} 1 & \text{if } i \in 1 \\ 0 & \text{else} \end{cases} \quad (27)$$

$$1 = \{i \backslash i \text{ is an asset such that } r_i \leq 0\} \quad (28)$$

α, β_1, β_2, ϕ_1, ϕ_2 are the penalty parameters. The gradient function of the function E is:

$$\nabla E(x) = \alpha\lambda Q x - (1-\lambda)\alpha r + \beta_1 \mathbb{1} + \beta_2 \mathbb{1}(\mathbb{1}^t x) + \phi_1 \mathbb{1}_I + \phi_2 \mathbb{1}_I(\mathbb{1}_I^t x) \quad (29)$$

The bias are given by the vector:

$$I = -\nabla E(0) = \alpha(1-\lambda)r - \beta_1 \mathbb{1} - \phi_1 \mathbb{1}_I \quad (30)$$

On the other hand, we have:

$$\nabla^2 E(x) = \alpha\lambda Q + \beta_2 \mathbb{1}\mathbb{1}^t + \phi_2 \mathbb{1}_I \mathbb{1}_I^t \quad (31)$$

Then, the weight matrix is:

$$T = -\alpha\lambda Q - \beta_2 \mathbb{1}\mathbb{1}^t - \phi_2 \mathbb{1}_I \mathbb{1}_I^t \quad (32)$$

The weight matrix and the bias vector depend on the penalty parameters to ensure the feasibility of the equilibrium point, we use a mathematical method to select an adequate weight and bias.

Activation function: As the portfolio problem looks for continuous values, the activation function must be chosen such that every value in $[0, 1]^n$ has the same probability to be taken. Figure 1 gives the plot of the function f(x) = Arctanh (Tx) for several values of the slope T under [a, b]. The best slop is the one over spending to:

- Almost leaner plot
- Almost surjective f under [a, b]

In our case, we must, first, determine the set $[a, b]^n$ that contains every possible values of the neuron states noted U_i. As it known, the state U_i is given by $U_i = T_i x + I_i$, where, T_i is the vector line i of matrix T. in this sense, U_i verify the follow equality:

$$\sum_{j=1}^{n} \text{Min}\left(0, T_{ij}\right) + I_i \leq U_i \leq \sum_{j=1}^{n} \max\left(0, T_{ij}\right) + I_i \quad (33)$$

Let $\overline{U}$ and $\underline{U}$ be two instants defend by:

$$\underline{U} = \text{Min}\left\{\sum_{j=1}^{n} \text{Min}\left(0, T_{ij}\right) + I_i\right\} \quad (34)$$

And:

$$\overline{U} = \text{Max}\left\{\sum_{j=1}^{n} \text{Max}\left(0, T_{ij}\right)+I_i\right\} \quad (35)$$

In this sense, we have:

$$\forall_i \in \{1, ..., n\} \underline{U} \le U_i \le \overline{U} \quad (36)$$

As f is an increasing function, we have:

$$\forall_i \in \{1, ..., n\} f\left(\underline{U}\right) \le f\left(U_i\right) \le f\left(\overline{U}\right) \quad (37)$$

Consequently, the parameters a and b are given by the formulas:

$$a = f\left(\underline{U}\right) \text{ and } b = f\left(\overline{U}\right) \quad (38)$$

Basing on the parameters a and b, we chose the slope of the activation function such that this latter is almost leaner and subjective from its start set to the interval [a, b]. By doing this, the activation function doesn't favourites any asset.

Parameter setting: As we want to minimize the quantity:

$$\frac{1}{2}\left(1\text{-}\lambda\right)x^t Qx\text{-}\lambda r^t x\Big) \quad (39)$$

The parameter α must be strict positive $\alpha>0$. If for some x_i have exist a neuron i in I such as $x_i>0$. Then, $E_i(x) = \delta E(x)/\delta x_i$ must decrease $E_x(x)<\varepsilon$ ($\varepsilon\epsilon R^{*+}$. Therefore, the sufficient condition is:

$$\left(1\text{-}\lambda\right)\sum_{j=1}^{n} q_{ij}\text{-}\alpha\lambda r_i+\beta_1+\phi_1\delta_i+\phi_2\delta_i \mid I \mid\le \varepsilon \quad (40)$$

Such that, $\phi^2>0$ and $\beta_1<0$. Or the sufficient free condition is:

$$\alpha\left(1\text{-}\lambda\right)\overline{\Sigma Q}\text{-}\alpha\lambda\underline{r}+\beta_1+\phi_1+\phi_2 \mid I \mid\le \varepsilon \quad (41)$$

Such that:

$$\phi_1 \succ 0;\ \overline{\Sigma Q} = \min\sum_{j=1}^{n} q_{ij};\ \underline{r} = \min r_i \quad (42)$$

Finally, any solution of the coming system:

$$(S)\begin{cases} \varepsilon>0 \\ \alpha>0 \\ \beta_{1<0} \text{ and } \phi_{1>0} \\ \phi_2>0 \\ \alpha\left(1\text{-}\lambda\right)\overline{\Sigma Q}\text{-}\alpha\lambda\underline{r}+\beta_1+\phi_1+\phi_2 \mid I \mid\le \varepsilon \end{cases} \quad (43)$$

Leads to a feasible portfolio: For the second Model (PM'), the mean-semivariance rational model, we construct a similar continuous Hopfield network by replacing the variance formula by the Estrada semivariance one. By doing this, only the weight matrix is changed.

RESULTS AND DISCUSSION

To show out the effect of the chosen risk measure, we solve the proposed model two time in the first one, we use the variance as risk measure in the second one, the semivariance is required as risk measure of the portfolios. As these measures have a symmetric form, the symmetric of the Hopfield weight matrix is symmetric too consequently, the stability of the proposed neural network is ensured (Hopfield and Tank, 1985). In addition, we consider the mean return is the mean objective.

We have useda real financial data from the database Yahoo finance. Our data includes three portfolios, each one included 20 active over 20 periods our study will cover the period from 29 September, 2014-17 February, 2015 in weekly.

Mean-variance rational model: In this section, we will solve the problem of selection of the portfolio using variance risk measure and then we draw a comparison between the CHN and Genetic algorithm to evaluate the performance of our approach.

The result of our approach for the mean-variance Model (MP) is reported in Table 1. In this sense, the CHN is turned many times staring from different initial solutions then, the best solutions are taken. It should be noted that the results are presented in terms of percentage.

In Table 1, the sign P_i(i = 1, ..., 6) refers to the obtained portfolio, R: his risk (portfolio P_i risk), MR: his Mean Return (portfolio P_i mean return or profitability) and the vector (ACA.P, ..., TEF.MC) is the assets abbreviated names as in Yahoo finance site web.

As we can remark at T difference between level risks of each presented portfolio, nevertheless, this difference is representative at a big budget (billion).

In the portfolio problem, solutions that form the efficient frontier are mathematically incomparable, including the solutions shown in Table 1 in such case, it is essential to know what investor face us. Indeed, it is important to remember that several investor typesexists, well the must knowing is as follow:

- Only considering the level risk
- Only considering the return result
- Be satisfied with a compromise between the risk level and the return

Table 1: Optimals portfolios

Percentage	AC A.PA	BN.P A.	OR. PA	CA. PA	RNO. PA	VIE. PA	EN. PA	KER. PA	GLE. PA	DIE. BR	UCB. BR	YOOX. MI	BPE. MI	ADS. DE	IFX. DE	MUV2. DE	KBC. BR	ENI. MI	AURS. PA	TEF. MC	R	MR
P_1	8.9	23.55	31.23	0	0	0	0.14	0	0	0	0	0	0	0	0	0	0	0	36.99	0	0.0297	0.99
P_2	6.41	20.5	34	0	0	0	5.09	0	0	0	0	0	0	0	0	0	0	0	34.00	0	0.0288	1.04
P_3	0	16.7	28	0	0	0	14.65	0	0	0	0	0	0	0	0	12.65	0	0	28.00	0	0.0276	1.11
P_4	0	5.85	28	0	0	0	28.00	0	0	0	0	0	0	10.15	0	0	0	0	28.00	0	0.0285	1.22
P_5	0	1.95	30	0	0	0	30.00	0	0	0	0	0	0	0	8.05	0	0	0	30.00	0	0.0282	1.27
P_6	0	7.93	9.96	0	0	0	36.11	0	0	0	0	0	0	0	0	0	0	0	46.00	0	0.0312	1.32

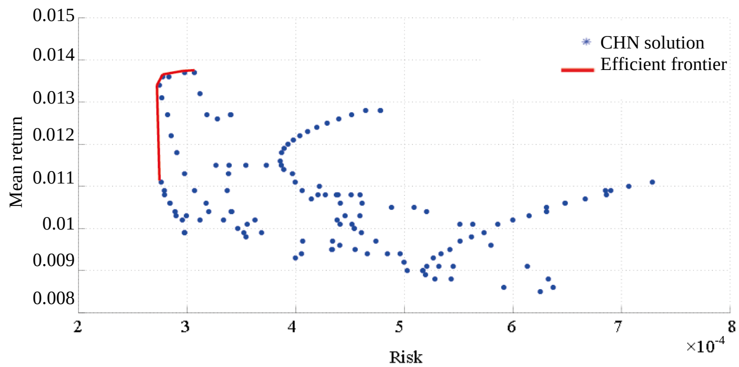

Fig. 2: Mean-variance efficient frontier

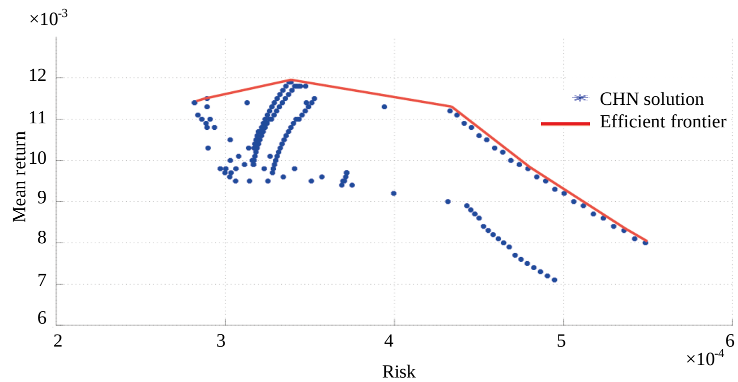

Fig. 3: Mean-semivariance efficient frontier

As explained in the previous paragraph, we can draw the efficient frontiers basing on the obtained results using the proposed continuous Hopfield network. In this regard, we have generated many solutions of each risk averse parameter values from the interval [0, 1], the Fig. 2 represent the efficient frontier.

The red line in Fig. 3 is the constructed shape of the mean-variance rational model efficient front or the blue points are the obtained solutions for each time simulation.

From the Fig. 2 for low-risk levels, corresponding to high values of the risk aversion parameter λ, we obtain an adequate representation. Whereas for low values of λ when the objective function converges to the mean return, regardless of the risk, the obtained solutions are not enough good to trace a complete efficient frontier.

This phenomenon can be explained basing of the behavior of the risk aversion parameter λ, Indeed when λ converge to zero, the energy function formula becomes a leaner equation, consequently, the proposed function for the portfolio problem via continuous Hopfield network is no morean adequate energy function.

This fact will be reinforced in the comingsection when we compare the Genetic algorithm and CHN results. Table 2 represents a comparison between the CHN and GA solutions for the model PM in which mean return level is fixed. From the Table 2, we remark that the proposed continuous Hopfield network gives the lowest risk values. So, it is recommended to use our method to solve the problem under study for $\lambda \geq 0.5$.

In otherwise, we fix the risk value and we compare the obtained results by CHN and GA for the mean return. Table 3 represents a comparison of different solutions obtained for a low-risk aversion parameter.

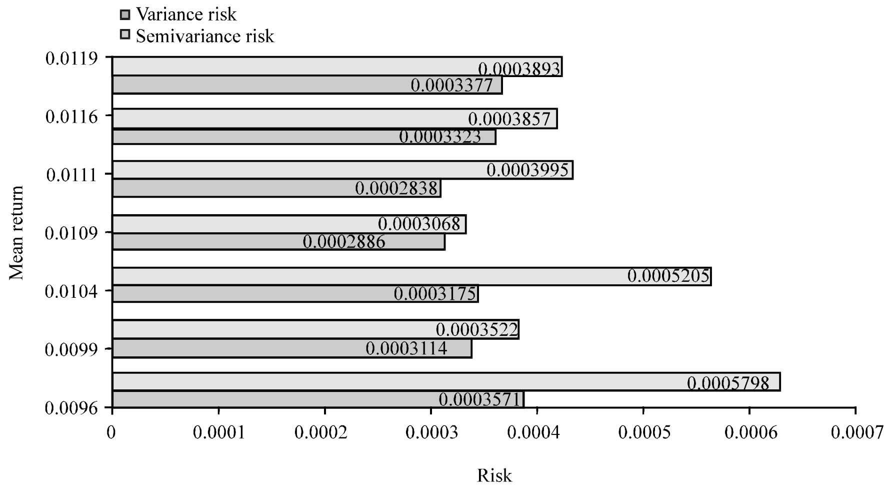

Fig. 4: Graph compsaring the semivariance and variance

Table 2: Comparison mean return result between CHN and GA for a fixed level risk

Risk		Mean return	
CHN	AG	CHN	AG
0.000297	0.000351	0.0099	
0.000288	0.000300	0.0104	
0.000276	0.000399	0.0111	
0.000285	0.000328	0.0122	
0.000282	0.000323	0.0127	
0.000312	0.000534	0.0132	

Table 3: Comparison risk result between CHN and GA for a fixed return

Mean return		Risk	
CHN	AG	CHN	AG
0.0101	0.0162	0.000586	
0.0105	0.0138	0.000508	
0.0107	0.0145	0.000663	
0.0085	0.0134	0.000625	
0.0110	0.0152	0.000707	
0.0093	0.0140	0.000527	

We can see that for a fixed risk, a lewder risk aversion parameter λ, the GA perform better that CHN. As conscience, we can compare, experimentally, the genetic algorithm and the continuous Hopfield network on the portfolio problem but it well be important to use both CHN and GA to solve this problem:

- Genetic algorithm for λ∈[0, 0.5]
- Continuous Hopfield network for λ∈[0, 5, 1]

Mean-semivariance rational model: In this study, we solve the portfolio selection problem using the Estrada semi-variance (Estrada, 2002, 2006) as risk measure via. our approach using deferent value = of λ. In this regard, we compare the systems (mean-variance, CHN) and (Estrada mean-semivariance, CHN). The obtained solutions are presented in terms of efficient frontier Fig. 3.

The red line in Fig. 3 is the constructed shape of the mean-semivariance rational modelefficient front or the blue points are the obtained solutionsfor each time simulation.

From the Fig. 3, we can observe the shape of the obtained efficient frontier. We can say that the obtained shape Simulates the expected shape as known in the literature. On the other hand, Table 4 gives an example of an efficient portfolio produced by the proposed continuous Hopfield network.

Table 4: Example of efficient portfolio

Assets	x_i (%)
ACA.PA	0
BN.PA	32
OR.PA	32
CA.PA	0
RNO.PA	0
VIE.PA	0
EN.PA	2.30
KER.PA	0
GLE.PA	0
DIE.BR	0
UCB.BR	0
YOOX.MI	0
BPE.MI	0
ADS.DE	0
IFX.DE	0
MUV2.DE	1.70
KBC.BR	0
ENI.MI	0
AURS.PA	32
TEF.MC	0

In our knowledge, it the first time that the Estrada semivariance and the continuous Hopfield network are used in the literature to solve the portfolio selection problem. To point out to the effectiveness of this strategy results we compare its performance to mean-variance and CHN one (Fig. 4). To this end, we fix the return.

Figure 4 illustrated the said comparison the system (Estrada mean-semivariance, CHN (Estrada, 2002, 2006), produces a decision with the smallest risk in compressing with one produced by the system (mean-variance, CHN). From the Fig. 3 and 4, we can say that the mean-semivariance rational model has proved his efficiencies.

CONCLUSION

In this research, we have completed the Markowitz Model by adding a new constraint. These latter permits to select, only, the assets with a positive return. Due to the effectiveness of the neural network tools, we have used the continuous Hopfield network to solve the obtained model by proposing an original energy function with an adequate parameter that ensures the equilibrium point feasibility. The proposed system produces a portfolio with no negative return. Moreover, the representation of the return bythe risk level for several values of λ, leads to an acceptable efficient frontier, especially when the mean-semivariance is used as a risk measure.

REFERENCES

Aftalion, F., 2004. [New Finance and Portfolio Management]. 2nd Edn., Editions Economica, Paris, France, ISBN:9782717849127, Pages: 246 (In Fernch).

Billionnet, A. and E. Soutif, 2004. An exact method based on Lagrangian decomposition for the 0-1 quadratic knapsack problem. Eur. J. Oper. Res., 157: 565-575.

Estrada, J., 2002. Systematic risk in emerging markets: The D-CAPM. Emerging Markets Rev., 3: 365-379.

Estrada, J., 2006. Downside risk in practice. J. Appl. Corporate Finance, 18: 117-125.

Estrada, J., 2015. Mean-semivariance optimization: A heuristic approach. J. Appl. Finance, 18: 57-72.

Ettaouil, M. and C. Loqman, 2008. Constraint satisfaction problems solved by semidefinite relaxations. WSEAS. Trans. Comput., 7: 951-961.

Ettaouil, M., K. Elmoutaouakil and Y. Ghanou, 2009. The Continuous Hopfield Networks (CHN) for the placement of the electronic circuits problem. Wseas Trans. Comput., 8: 1865-1874.

Fernandez, A. and S. Gomez, 2007. Portfolio selection using neural networks. Comput. Oper. Res., 34: 1177-1191.

Goemans, M.X., 1997. Semidefinite programming in combinatorial optimization. Math. Program., 79: 143-162.

Hogan, W.W. and J.M. Warren, 1974. Toward the development of an equilibrium capital-market model based on semivariance. J. Financial Quant. Anal., 9: 1-11.

Hopfield, J.J. and D.W. Tank, 1985. Neural computation of decisions in optimization problems. Biol. Cybern., 52: 141-152.

Hopfield, J.J., 1984. Neurons with graded response have collective computational properties like those of two-state neurons. Proc. Natl. Acad. Sci. USA., 81: 3088-3092.

Jarrow, R.A., 1988. FInance Theory. Prentice Hall, Upper Saddle River, New Jersey, USA., ISBN:9780133148657, Pages: 298.

Joudar, N., K.E. Moutaouakil and M. Ettaouil, 2015. An original continuous hopfield network for optimal images restoration. WESEAS Trans. Comput., 14: 668-678.

Markowitz, H., 1952. Portfolio selection. J. Finance, 7: 77-91.

Markowitz, H., 1995. Portfolio Selection: Efficient Diversification of Investments. John Wiley and Sons, Hoboken, New Jersey, USA., Pages : 344.

Sefiane, S. and M. Benbouziane, 2012. Portfolio selection using genetic algorithm. J. Applied Finance Banking, 2: 143-154.

Takeda, M. and J.W. Goodman, 1986. Neural networks for computation: Number representations and programming complexity. Appl. Opt., 25: 3033-3046.

Talavan, P.M. and J. Yanez, 2002. Parameter setting of the Hopfield network applied to TSP. Neural Networks, 15: 363-373.

Talavan, P.M. and J. Yanez, 2006. The generalized quadratic knapsack problem: A neuronal network approach. Neural Networks, 19: 416-428.

Tatsumi, K., Y. Yagi and T. Tanino, 2002. Improved projection Hopfield network for the quadratic assignment problem. Proceedings of the 41st SICE Annual International Conference on SICE Vol. 4, August 5-7, 2002, IEEE, Osaka, Japan, pp: 2295-2300.

Thi, L.H.A., M. Moeini and T.P. Dinh, 2009a. DC programming approach for portfolio optimization under step increasing transaction costs. Optim., 58: 267-289.

Thi, L.H.A., M. Moeini and T.P. Dinh, 2009b. Portfolio selection under downside risk measures and cardinality constraints based on DC programming and DCA. Comput. Manage. Sci., 6: 459-475.

Yao, W., 1988. Alternative networks for solving the traveling salesman problem and the list-matching problem. Proceedings of the IEEE International Conference on Neural Networks, July 24-27, 1988, IEEE, San Diego, California, USA., pp: 333-340.

Zarranezhad, M., R.Y. Hajiabad and M . Fakherinia, 2015. Evaluation of particle swarm algorithm and genetic algorithms performance at optimal portfolio selection. Asian Econ. Financial Rev., 5: 88-101.