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Sliding Modes, Differential Flatness and Servosystem Control for a Mobile Platform with 8 Degrees of Fredom

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Abstract: The first step in the mobile manipulator control is to know in detail its configuration it means to analyze the main elements distribution that represent the mechanism such as: joints, wheels, motors, sensors, among others. When defining the configurations, the degrees of freedom that must be considered for the design of the controllers are determined. Therefore, in the present research the algorithms of control of a mobile manipulator of 8 degrees of freedom constituted by a manipulator of 5 degrees of freedom are designed of which four are controllable, besides a mobile platform of 6 wheels of which four are addressable.

Key words: Arm manipulator, articulated pair, degrees of freedom, dynamic, flatness differential controller, generalized coordinates, servosystem controller, sliding modes controller

INTRODUCTION

Mobile manipulators are defined by the joining of a manipulator arm and a mobile platform. The manipulator arm is characterized by a joint of a set of rigid bodies connected to each other using articulations. This union results in a relative movement of the different components producing a consequence of displacements of the final link of the kinematic chain (Lopez, 2009). On the other hand, a mobile platform is a mechanical structure equipped with a locomotion system capable of navigation through a certain working environment, endowed with autonomy for its displacement with loads. Their applications can be varied and are always related to risk tasks harmful consequences for human health in areas such as, agriculture, transporting of dangerous cargoes or exploration tasks with unmanned aerial vehicles.

For the control of this mechanism it is required the kinematic and dynamic models that define the movements of the system. The kinematic model relates the space of articular trajectories to the cartesian space located on the final link. Besides, it is defined as the basis of the dynamic studies for the force calculation required to produce movement in the system (Yague, 2013; Batz, 2005).

In mobile robots, it is necessary to understand the kinematic behavior to design mechanism to inspection tasks, besides the possibility to make control laws that allows improve its operation (Craige, 1989). Furthermore, the dynamic of a mechanism is used to analyze the forces required to cause movement in other words, the torque to manipulate the position and velocity variables in the hybrid platform.

MATERIALS AND METHODS

Mobile manipulator scheme: To develop this research, it is used the scheme in Fig. 1 that shows the structure of the mobile platform. It is necessary to consider for the design of the controller the mobile platform and the manipulator mechanism is decoupled where by the frame of reference and manipulator movement is considered at the beginning of the kinematic chain but no as the set of global coordinates of the hybrid platform. Besides, the second derivatives of signals generated in the algorithm must be saturates due to the jerk or over acceleration effects.

Manipulator control: In Fig. 2, the manipulator structure is defined where it is specified the joint configuration for a manipulator with four DOF. It should be noted that in this system the masses are locates on the center of each link.

Inverse dynamic of manipulators: When the manipulator configuration is specified it is used the lagrange formulation of the Eq. 1 which determine the system dynamics with the aim to calculate and controller the required pair in each joint in terms of their masses, lengths, inertial and viscous frictions (Ogata, 2003):

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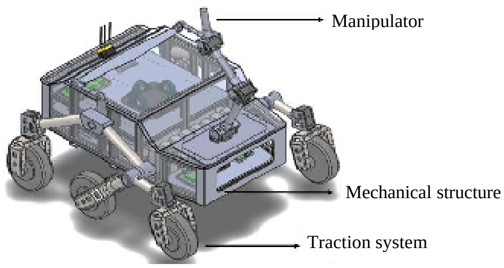

Fig. 1: Hybrid platform scheme

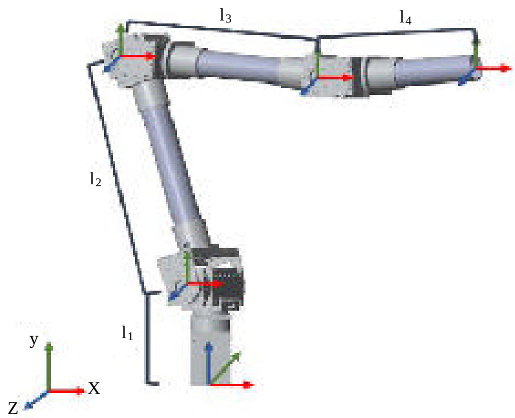

Fig. 2: Five DOF manipulator arm

$$\tau = M(q)\ddot{q}+V(q,\dot{q})+G(q)+B(\dot{q}) \quad (1)$$

For easy in the controller calculus the system is expressed as shown in Eq. 2 that determines the operation of the system in terms of the pair of the joints:

$$\begin{bmatrix}\tau_1\\ \tau_2\\ \tau_3\\ \tau_4\end{bmatrix}=\begin{bmatrix}A & B & C & D\\ E & F & G & H\\ I & J & K & L\\ M & N & O & P\end{bmatrix}\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\\ \ddot{\theta}_3\\ \ddot{\theta}_4\end{bmatrix}+\begin{bmatrix}Q\\ R\\ S\\ T\end{bmatrix}+\begin{bmatrix}U\\ V\\ W\\ X\end{bmatrix}+\begin{bmatrix}v_1 & 0 & 0 & 0\\ 0 & v_2 & 0 & 0\\ 0 & 0 & v_3 & 0\\ 0 & 0 & 0 & v_4\end{bmatrix}+\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\\ \dot{\theta}_4\end{bmatrix} \quad (2)$$

Direct dynamic: The direct dynamic model shows the temporal evolutions of articular coordinates as well as their derivatives as a function of the forces and pairs involved. To obtain it, it is used the inverse model of Eq. 2 to which cramer's rule is applied and the substitutions of null cofactors. For the management of the matrices in the calculation of the determinants it is used Eq. 3. Therefore, in solving cramer's rule it is obtained Eq. 4-7:

$$\begin{aligned}Z_1 &= A\ddot{\theta}_1\\ Z_2 &= F\ddot{\theta}_2+G\ddot{\theta}_3+H\ddot{\theta}_4\\ Z_3 &= J\ddot{\theta}_2+K\ddot{\theta}_3+L\ddot{\theta}_4\\ Z_4 &= N\ddot{\theta}_2+O\ddot{\theta}_3+P\ddot{\theta}_4\end{aligned} \quad (3)$$

$$\ddot{\theta}_1=\frac{\begin{bmatrix}Z_1 & 0 & 0 & 0\\ Z_2 & F & G & H\\ Z_3 & J & K & L\\ Z_4 & N & O & P\end{bmatrix}}{\begin{bmatrix}A & 0 & 0 & 0\\ 0 & F & G & H\\ 0 & J & K & L\\ 0 & N & O & P\end{bmatrix}}=\frac{Z_1}{A} \quad (4)$$

$$\ddot{\theta}_2=\frac{\begin{bmatrix}A & Z_1 & 0 & 0\\ 0 & Z_2 & G & H\\ 0 & Z_3 & K & L\\ 0 & Z_4 & O & P\end{bmatrix}}{\begin{bmatrix}A & 0 & 0 & 0\\ 0 & F & G & H\\ 0 & J & K & L\\ 0 & N & O & P\end{bmatrix}}=\frac{(GLZ_4\text{-}HKZ_4\text{-}HOZ_3+HOZ_3+KPZ_2\text{-}LOZ_2)}{(FKP\text{-}GJP+GLN\text{-}HKN\text{-}FLO+HJO)} \quad (5)$$

$$\ddot{\theta}_3=\frac{\begin{bmatrix}A & 0 & Z_1 & 0\\ 0 & F & Z_2 & H\\ 0 & J & Z_3 & L\\ 0 & N & Z_4 & P\end{bmatrix}}{\begin{bmatrix}A & 0 & 0 & 0\\ 0 & F & G & H\\ 0 & J & K & L\\ 0 & N & O & P\end{bmatrix}}=\frac{(FLZ_4\text{-}HJZ_4\text{-}FPZ_3+HNZ_3+JPZ_2\text{-}LNZ_2)}{(FKP\text{-}GJP+GLN\text{-}HKN\text{-}FLO+HJO)} \quad (6)$$

$$\ddot{\theta}_4=\frac{\begin{bmatrix}A & 0 & 0 & Z_1\\ 0 & F & G & Z_2\\ 0 & J & K & Z_3\\ 0 & N & O & Z_4\end{bmatrix}}{\begin{bmatrix}A & 0 & 0 & 0\\ 0 & F & G & H\\ 0 & J & K & L\\ 0 & N & O & P\end{bmatrix}}=\frac{(FKZ_4\text{-}GJZ_4+GNZ_3\text{-}FOZ_3\text{-}KNZ_2+JOZ_2)}{(FKP\text{-}GJP+GLN\text{-}HKN\text{-}FLO+HJO)} \quad (7)$$

This model allows to obtain the current position output of the mechanism used to be compared with the error signal data of the system. This signal is taken as the

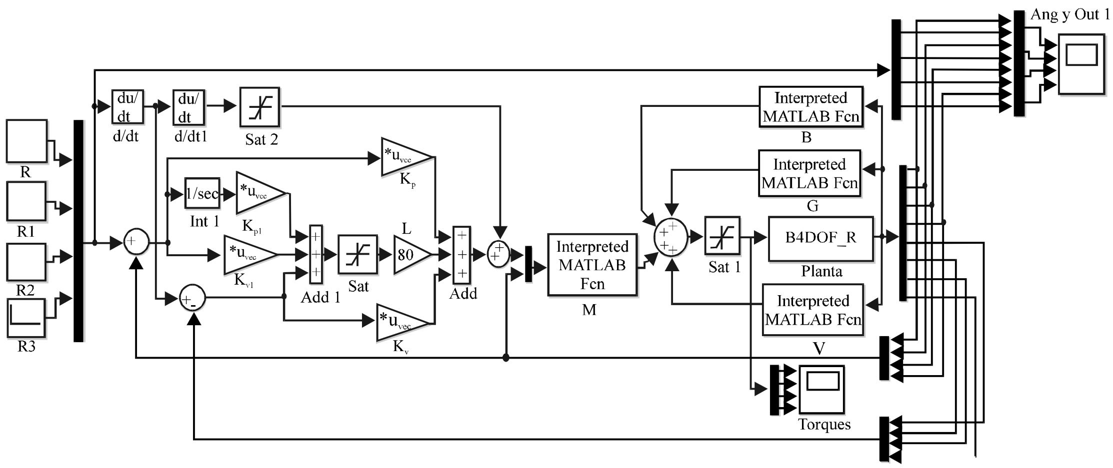

Fig. 3: Sliding mode controller scheme

input parameter of the controller to calculate the control signal to be considered for each joint of the manipulator.

Sliding mode controller: The physical principle for the sliding mode control consists in taking the trajectories of the system in a sliding surface and forcing them to evolve over this surface, so, the dynamic controller is determinate by the equations that define the previous surface in the state space in Eq. 8 and 9 (Evangelista, 2012). It is necessary to consider that $\dot{e} = \dot{e}(t) = de(t)/dt$ the same meaning for the generalize coordinates:

$$\sigma = \dot{e}+k_v e+k_p \int e dt \quad (8)$$

$$\dot{\sigma} = \ddot{e}+k_v \dot{e}+k_p e \quad (9)$$

When de sliding surface is defined, it is important to ensure the existence of the sliding mode which exist if near the surface there are vectors of tangent or velocity that point in the direction of the sliding surface. When certain conditions are satisfied, the state "slides" on said surface, remaining insensitive to variations in the system parameters and external disturbances which constitutes the fundamental characteristic for its application in the control systems as shown in Fig. 3 (Naranjo, 1991).

To develop the controller, it starts with Eq. 1 which describes the dynamics of the manipulator which the joint variable $\ddot{q}$ is shown in Eq. 10:

$$\ddot{q} = \frac{\tau}{M(q)} - \frac{V(q,\dot{q})}{M(q)}\dot{q} - \frac{G(q)}{M(q)} - \frac{B(\dot{q})}{M(q)} \quad (10)$$

Then, Eq. 11-23 shows the solution of equations necessary to obtain the control signal of Eq. 24 where, $\dot{\sigma}$ of Eq. 12 is equal to 0:

$$\ddot{e} = \ddot{q}_d - \ddot{q} \quad (11)$$

$$\dot{\sigma} = \ddot{q}_d - \frac{\tau}{M(q)} + \frac{V(q,\dot{q})}{M(q)}\dot{q} + \frac{G(q)}{M(q)} + \frac{B(\dot{q})}{M(q)} + K_v \dot{e} + k_p e \quad (12)$$

$$\tau_{eq} = V(q,\dot{q})\dot{q} + G(q) + B(\dot{q}) + M(q)\left[\ddot{q}_d + K_v \dot{e} + k_p e\right] \quad (13)$$

$$\dot{\sigma} = \ddot{q}_d - \frac{\tau_{eq}+\tau_N}{M(q)} + \frac{V(q,\dot{q})}{M(q)}\dot{q} + \frac{G(q)}{M(q)} + \frac{B(\dot{q})}{M(q)} + K_v \dot{e} + k_p e \quad (14)$$

$$\tau_N = \begin{cases} -L\operatorname{sgn}(\sigma), & \sigma(x) \neq 0 \\ 0, & \sigma(x) = 0 \end{cases} \quad (15)$$

$$\dot{\sigma} = \ddot{q}_d - \frac{V(q,\dot{q})\dot{q} + G(q) + B(\dot{q}) + M(q)\left[\ddot{q}_d + K_v \dot{e} + k_p e\right] + \tau_N}{M(q)} + \frac{V(q,\dot{q})}{M(q)}\dot{q} + \frac{G(q)}{M(q)} + \frac{B(\dot{q})}{M(q)} + K_v \dot{e} + k_p e \quad (16)$$

$$\dot{\sigma} = \ddot{q}_d - \left[\ddot{q}_d + K_v \dot{e} + k_p e\right] - \frac{\tau_N}{M(q)} + K_v \dot{e} + k_p e \quad (17)$$

$$\dot{\sigma} = -\frac{\tau_N}{M(q)} \quad (18)$$

$$L.\operatorname{sgn}(\sigma) = \frac{\tau_N}{M(q)} \quad (19)$$

$$M(q).L\operatorname{sgn}(\sigma) = \tau_N \quad (20)$$

$$\tau = \tau_{eq} + \tau_N \quad (21)$$

$$\tau = V(q,\ \dot{q})+G(q)+B(\dot{q})+M(q)\left[\ddot{q}_d+K_v\dot{e}+k_p e\right]+ M(q)L\,\mathrm{sgn}(\sigma) \quad (22)$$

$$\tau = V(q,\ \dot{q})+G(q)+B(\dot{q})+M(q)\left[\ddot{q}_d+K_v\dot{e}+k_p e+L\,\mathrm{sgn}(\sigma)\right] \quad (23)$$

$$\tau = V(q,\ \dot{q})+G(q)+B(\dot{q})+M(q)\left[\ddot{q}_d+K_v\dot{e}+k_p e+L\,\mathrm{sgn}(\dot{e}+k_v e+k_p\int e\,dt)\right] \quad (24)$$

Differential flatness controller: When performing control over nonlinear dynamical systems such as the manipulator arm, there are drawbacks such as uncertainties in the estimation of faults in the behavior of the mechanism, so, in some cases for the development of controllers it is used the algebraic-differential approach (Chumacero, 2010).

This methodology is based on concepts of differential algebra which allows the management of differential equations in a systematic way, proposing the implementation of the property of differential flatness. The notions of the controllers based on this property correspond to prepare a flight plan that consists of the offline generation of a trajectory and the associated control actions for that trajectory based on the knowledge of the system model (Antritter, 2008). For the development of thecontroller the first thing to do will be the definition of the plane outputs that constitute the system, described in Eq. 25:

$$F = q \qquad \dot{F} = \dot{q} \qquad \ddot{F} = \ddot{q} \quad (25)$$

Then the dynamic model of the manipulator of Eq. 1 is taken up and a new system is proposed in terms of the previously defined flat outputs as observed in Eq. 26:

$$\ddot{F} = \frac{\tau}{M(q)}-\frac{V(q,\dot{q})}{M(q)}\dot{F}-\frac{G(q)}{M(q)}-\frac{B(\dot{q})}{M(q)} \quad (26)$$

The controller based on flatness difference must be complemented with the implementation of a sliding mode controller, so, the sliding surface is defined in terms of the flat outputs of the dynamic model as shown in Eq. 27:

$$\ddot{F} = \vartheta_{aux} = \ddot{F}^*+k_v\left(\dot{F}^*-\dot{F}\right)+k_v\left(\dot{F}^*-F\right)+k_i\int\left(\dot{F}^*-\dot{F}\right)dt \quad (27)$$

With the auxiliary control ϑ_{aux} the integral compensator is combined with the attractive control and the sliding surface of Eq. 8 but in this case the error is represented as $\dot{E} = \dot{F}^*-\dot{F}$

$$\ddot{F} = \ddot{F}^*+kv\left(\dot{F}^*-\dot{F}\right)+k_v\left(\dot{F}^*-\dot{F}\right)+k_i\int\left(\dot{F}^*-\dot{F}\right)dt+L\,\mathrm{sgn}(\sigma) \quad (28)$$

From the above it is derived from auxiliary sliding surface to continue with the controller design as shown in Eq. 29, considering that ϑ = 0:

$$\vartheta_{aux} = \ddot{E}+k_v\dot{E}+k_v E+k_i\int E\,dt+L\,\mathrm{sgn}(\sigma) \quad (29)$$

$$\vartheta_{aux} = \ddot{F}^*-\frac{\tau}{M(q)}-\frac{V(q,\dot{q})}{M(q)}\dot{F}-\frac{G(q)}{M(q)}-\frac{B(\dot{q})}{M(q)}+k_v\dot{E}+ k_v E+k_i\int E\,dt+L\,\mathrm{sgn}(\sigma) \quad (30)$$

$$\frac{\tau}{M(q)} = -\frac{V(q,\dot{q})}{M(q)}\dot{F}-\frac{G(q)}{M(q)}-\frac{B(\dot{q})}{M(q)}+\ddot{F}^*+k_v\dot{E}+k_v E+ k_i\int E\,dt+L\,\mathrm{sgn}(\sigma) \quad (31)$$

$$\tau = V(q,\dot{q})\dot{F}+G(q)+B(\dot{q})+M(q)\left[\ddot{F}^*+k_v\dot{E}+k_v E+k_i\int E\,dt+L\,\mathrm{sgn}(\sigma)\right] \quad (32)$$

After obtaining the control Eq. 32, it proceeds with the calculation of the error dynamics as shown in Eq. 33 and 34:

$$M(q)\ddot{F}+V(q,\ \dot{q})\dot{F}+G(q)+B(\dot{q}) = V(q,\dot{q})\dot{F}+G(q)+B(\dot{q}) + M(q)\left[\ddot{F}^*+K_v\dot{E}+k_p E+k_i\int E\,dt+L\,\mathrm{sgn}\left(\dot{E}+k_p E+k_i\int E\,dt\right)\right] \quad (33)$$

$$0 = \ddot{q}_d-\ddot{q}+K_v\dot{e}+k_p e+L\,\mathrm{sgn}\left(\dot{e}+k_v e+k_p\int e\,dt\right) \quad (34)$$

To develop the manipulator controller of four DOF are proposed as control constants k_p in Eq. 35 k_v in Eq. 36 and L = 30:

$$k_p = \begin{bmatrix} 1200 & 0 & 0 & 0 \\ 0 & 900 & 0 & 0 \\ 0 & 0 & 1500 & 0 \\ 0 & 0 & 0 & 10000 \end{bmatrix} \quad (35)$$

$$k_v = \begin{bmatrix} 90 & 0 & 0 & 0 \\ 0 & 70 & 0 & 0 \\ 0 & 0 & 120 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix} \quad (36)$$

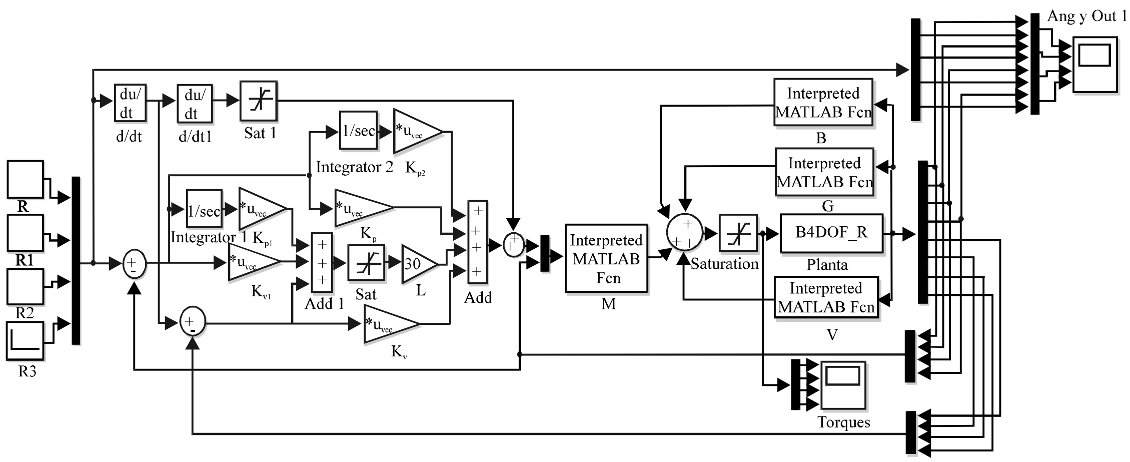

Fig. 4: Diffrential flatness controller scheme

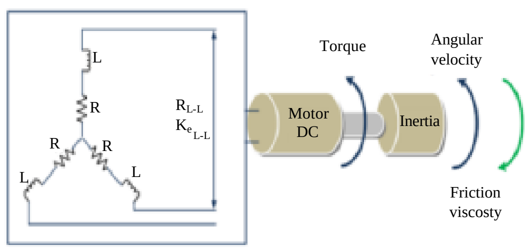

Fig. 5: Brushless motor scheme

For the simulation of the system is proposed the schematic of Fig. 4 which presents the layout of the controller as well as the model of the manipulator described in B4DOF_R. In addition as noted the control signal must be saturated at the output due to the torque limit stipulated by the actuators in the mechanism joints.

Mobile platform controller: For the calculation of the controller it is obtained the mathematical models of the traction motors of the mobile platform. For this it is purposed a brushless DC motor with permanent magnet brushes as shown in Fig. 5 (Shao, 2003; Becerra-Vargas *et al*., 2014).

To get the model, it defines the mechanical time counter of the system according to Eq. 37 and the electric time constant with Eq. 38:

$$\tau_m = \sum \frac{RJ}{K_e K_m} = \frac{J\sum R}{K_e K_m} \quad (37)$$

$$\tau_e = \sum \frac{L}{R} = \frac{L}{\sum R} \quad (38)$$

But having a symmetrical and three-phase configuration, the mechanical and electrical constants are expressed according to Eq. 39 and 40:

$$\tau_m = \frac{J(3R)}{K_e K_m} \quad (39)$$

$$\tau_e = \frac{L}{3R} \quad (40)$$

So, the phase effectis considered Eq. 41 which allows defining the final mechanical time constant to be used in the mathematical model:

$$\tau_m = \frac{J(3R)}{(K_{e_{L\text{-}L}}/\sqrt{3})K_m} \quad (41)$$

There is also the respective, relationship between this voltage constant and the mechanical constant determined by $K_e = K_m$ 0.0605. Finally, it is obtained the transfer function that models all parameters of the brushless DC motor in Eq. 42:

$$G(s) = \frac{\frac{1}{K_e \tau_m \tau_e}}{s^2 + \frac{1}{(\tau_e)}s + \frac{1}{\tau_m \tau_e}} \quad (42)$$

For this control is implemented a motor with specifications that are shown in Table 1, finally to obtain Eq. 47, characteristic of this type of motors (Delgado and Bolanos, 2013):

Table 1: Brushless golden motor specifications

Parameters	Units	Values
R_a	Ω	1.05
L_a	H	1.95×10^{3}
J	$kg.m^2$	8.5×10^{-6}
B	N.m.sec/rad	0
τ_m	sec	13.5×10^{3}
τ_e	sec	1.85×10^{3}

$$G(s)=\frac{1312800}{s^2+540,\ 5405s+40040} \tag{43}$$

For the design of a servosystem controller, the transfer function representation of Eq. 43 must be changed by the representation in state spaces as shown in Eq. 44 for this, the system state variables in Eq. 45 as well as their equations in difference in Eq. 46 are defined:

$$\begin{aligned}\dot{x}&=Ax+Bu\\ y&=Cx+Du\end{aligned} \tag{44}$$

$$\begin{aligned}x&=[x\quad \alpha]\\ \dot{w}&=\alpha\end{aligned} \tag{45}$$

$$\dot{\alpha}=-\frac{1}{\tau_e}\alpha-\frac{1}{\tau_m\tau_e}w+\frac{1}{k_e\tau_m\tau_e} \tag{46}$$

From the above it is obtained Eq. 47 for the system matrix A , Eq. 48 for the input matrix B and Eq. 49 for the output matrix C:

$$A=\begin{bmatrix}0 & 1\\ -\frac{1}{\tau_m\tau_e} & -\frac{1}{\tau_e}\end{bmatrix}=\begin{bmatrix}0 & 1\\ -40040 & -540.5405\end{bmatrix} \tag{47}$$

$$B=\begin{bmatrix}0\\ \frac{1}{k_e\tau_m\tau_e}\end{bmatrix}=\begin{bmatrix}0\\ 1312800\end{bmatrix} \tag{48}$$

$$C=[1\quad 0] \tag{49}$$

Servosystem controller: The servosystem controller is presented as a control in which all the system information is used to calculate the manipulated input. In addition, it is implemented when in real systems it is not possible to have access to all the variables but that can be estimated using an observer (Goodwin and Sang, 1989). For the calculation of a controller by servosystem is considered a linear system defined by Eq. 44 which represents the mathematical model of the mechanism in state variables.

For the design of the controller in discrete time the discretization of the matrices must be develop by means of Eq. 50 and 51 to finally obtain a representation described by Eq. 52:

$$G=e^{At} \tag{50}$$

$$H=\left(\int_0^t e^{At}\quad dt\right)B \tag{51}$$

$$\begin{aligned}\dot{x}&=Gx+Hu\\ y&=Cx+Du\end{aligned} \tag{52}$$

For the above, it is obtained the representation in state variables of the system in Eq. 53:

$$\begin{aligned}G&=\begin{bmatrix}0.399633 & -41.6851\\ 0.001041 & 0.96238\end{bmatrix}\\ H&=\begin{bmatrix}0.001041\\ 0.000000939\end{bmatrix}\\ C&=[0\quad 1312800]\end{aligned} \tag{53}$$

Then, for the calculation of the state feedback control constants, the Ackermann method is implemented but since, this controller will have an integrator inside its structure that ensures a steady state error of zero the matrices G and H must be expanded according to Eq. 54:

$$\begin{aligned}G&=\begin{bmatrix}0.2995 & -47.6743 & 0\\ 0.0019 & 0.9431 & 0\\ 1563.1092 & 1238112.2068 & 1\end{bmatrix}\\ H&=\begin{bmatrix}0.0011906\\ 0.0000014\\ 1.8653294\end{bmatrix}\end{aligned} \tag{54}$$

For the development of the Ackermann method the desired polynomial must be obtained in discrete time as observed in Eq. 55 and 56, determined by a desired settling time of 0.05 and a maximum on impulse of 0.9. To finally, match the magnitudes of each power with the characteristic polynomial of the model and thus, determine the control variables for each element:

$$|z| = e^{-T\xi w_n}$$
$$< z = Tw_d \quad (55)$$

$$z = re \pm jim \quad (56)$$

$$Pd = (z\text{-}re\text{-}jim)(z\text{-}re\text{+}jim)(z\text{-}e^{-10T\xi w_n}) \quad (57)$$

$$z = 1\text{-}1.25065z^{-1}+0.42282z^{-2}\text{-}0.00808z^{-3}+0.000039z^{-4} \quad (58)$$

Then, the values of z for the matrix G and z^0 are replaced by an identity matrix of the same order of G of which we get φ(G) as shown in Eq. 59. From this equation, the states feedback control values are obtained by Eq. 60:

$$\varphi(G) = G^n+a_{n-1}G^{n-1}+, \dots, +a_1G+a_0I \quad (59)$$

$$k = [0 \quad 0, \dots, 0 \quad 1]\text{inv}(m_c)\varphi(G) \quad (60)$$

Finally, it is said that φ(G) bases its operation on dead scillations, so, for the calculation of observer variables, Eq. 61 is implemented:

$$k_e = \begin{bmatrix} 0 \\ 0 \\ \dots \\ 0 \\ 1 \end{bmatrix} \text{inv}(m_o)\varphi(G) \quad (61)$$

Once the equations for the calculation of a servosystem controller have been completed, the control variables for the position mechanism are defined by Eq. 62, determined by a desired set-up time of 0.05 and a maximum on pulse of 0.9:

$$k = [598.96475 \quad 294515.1647 \quad 0.050977]$$
$$k_e = 1.10^{-4}\begin{bmatrix} 0.85099 \\ 0.010374 \end{bmatrix} \quad (62)$$

RESULTS AND DISCUSSION

Manipulator results: In order to verify the correct operation of the previously designed controller, the values of Fig. 6 which represent the angular paths of each joint are taken as desired positions of the system.

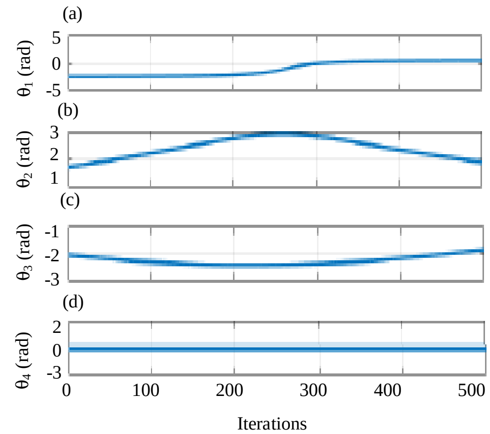

Fig. 6: Angular variation

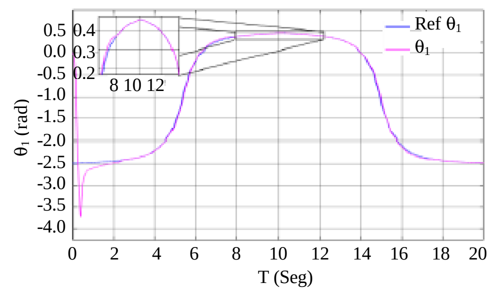

Fig. 7: Controller in joint 1

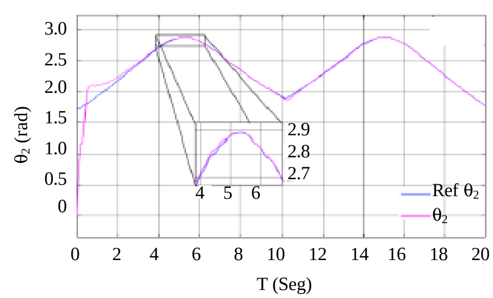

Fig. 8: Controller in joint 2

Knowing the desired trajectories of the system, proceed with the verification of the controller. Which the Fig. 7-10, it is observed the blue signal that determines the desired condition of each joint and the violet signal that describes the behavior of the manipulator under the behavior of the signal generated by the controller.

Mobile platform results: To verify the correct operation of the controller a random path defined by the blue signal of Fig. 11 and its respective, behavior after the control signal is applied in the red path.

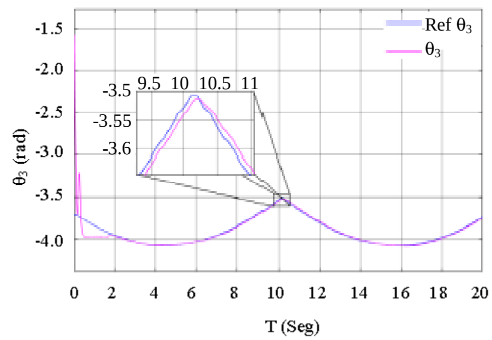

Fig. 9: Controller in joints 3

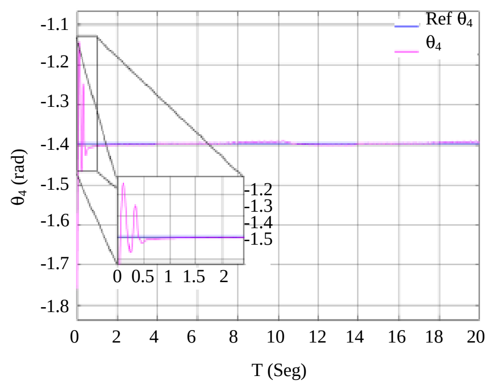

Fig. 10: Controller in joint 4

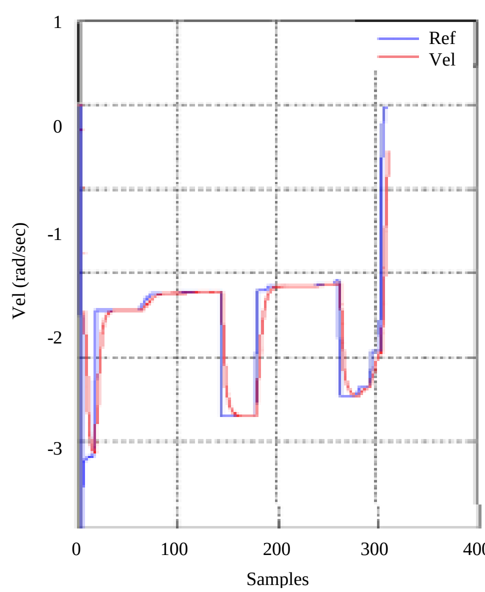

Fig. 11: Velocity trajectories

CONCLUSION

From the obtained results it is observed that, for the manipulator, the control of differential flatness with sliding modes stabilizes slowly and with oscillations in the first instants of time because the control signal directly affects the behaviors of the inertia matrix of the system but not the error signal generated by the difference between the desired signal and the system error signal. After the control signal can stabilize the two signals, it is responsible for carrying the system on the desired path without oscillations for what is left of the trajectory.

While, for the mobile platform, the control by servosystem, oversees following the reference in speed without oscillations and with rapid changes in the motors that comprise it.

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NOMENCLATURE

e	=	Derivative of error
$\dot{F}$	=	Derivative of Flat outlets
$\dot{e}$	=	Double derivative of error
$\ddot{F}$	=	Double derivative of flat outlets
F	=	Flat outlets
K_e	=	Electrical motor constant
e	=	Error
q	=	Generalize coordinates
$\ddot{q}$	=	Generalize coordinates of acceleration
$\dot{q}$	=	Generalize coordinates of velocity
$G(q)$	=	Gravitational forces vector
L	=	Inductor
$M(q)$	=	Inertial Matrices
B	=	Input matrix
K_m	=	Mechanical motor constant
C	=	Output matrix
R	=	Resistance
A	=	System matrix
$V(q, \dot{q})$	=	Vector of centrifugal and coriolis forces
$B(\dot{q})$	=	Vector friction forces

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