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The Z-Transform with Reduction in the Fluctuations of the Calculation of Reactivity

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Abstract: In this study, we show a study to solve the point kinetic equations using the Z-transform. It is possible to decrease the fluctuations during the calculation of the reactivity in nuclear reactors starting from the inverse point kinetic equation and simulating noise with a Gaussian distribution around the average value of the neutron population density. For such cases, we use two filters simultaneously. The Finite Impulse Response (FIR) filter and the first-order delayed low-pass filter. We show that it is possible to reduce the fluctuations during the calculation of reactivity in order to achieve a more precise value compared to other methods that exist in the literature.

Key words: Inverse point-kinetic equation, FIR filter, first-order delayed low-pass filter, reactivity, fluctuations, simultaneously

INTRODUCTION

Inside a nuclear reactor the precise calculation of the reactivity is very important, since, it represents a control and safety parameter. This is because the operation of the nuclear reactor depends on it. The measurement of reactivity is essential to monitor the control systems of the reactor, since, this information is a good source to take the right decision when moving the control rods. The calculation of reactivity is done by the inverse point-kinetic method which consists in modeling the temporal behavior of the neutron flux. This method has an integral-differential equation in which the term related to the integral part conserves the history of the neutron population density and the differential part is directly related with the period of the reactor.

There are several researchers that have calculated reactivity (Shimazu *et al*., 1987; Hoogenboom and Sluijs, 1988; Binney and Bakir, 1989; Ansari, 1991; Tamura, 2003; Malmir and Vosoughi, 2013). These diverse works are mainly based in the discretization of the term related with the integral in the inverse method equation, known as the history of the neutron population density.

There are works in the literature that deal with the problem of the fluctuation of the neutron population density with the use of the first-order delayed low-pass filter (Shimazu *et al*., 1987). On a later paper Diaz *et al*. (2008), proposed the FIR filter that allows to calculate the reactivity in real time with a time step of $\Delta t = 0.01$ s but with a noise level of up to $\sigma = 0.001$. From there Diaz and Martinez (2009) improved the calculation using a double FIR filter and the method of adjustments using the least squares method to reduce the fluctuations in the reactivity calculation. Later, the low-pass filter and the finite differences method was presented (Diaz and Martinez, 2010). Other works do not consider including the generation of random noise which is present in the real situations.

Since, during the operation of a nuclear reactor fluctuations are present, we simulate the random Gaussian noise which we suppose is distributed around the mean value of the neutron population density and we reduce the fluctuations on the reactivity by means of a combination of two digital filters. These filters are: the FIR filter and the first-order delayed low-pass filter.

MATERIALS AND METHODS

The inverse point kinetic equation: As a starting point for this study, we use the point kinetic equations and represent them mathematically in the form (Duderstadt and Halmiton, 1976):

$$\frac{dP(t)}{dt} = \left[\frac{\rho(t)-\beta}{\Lambda}\right]P(t)+\sum_{i=1}^{6}\lambda_i C_i(t) \quad (1)$$

$$\frac{dC_i(t)}{dt}+\lambda_i C_i = \frac{\beta_i}{\Lambda}P(t) \quad (2)$$

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Equation 1 and 2 are constrained by the conditions:

$$P(t=0)=P_0 \quad (3)$$

$$C_i(t=0)=\frac{\beta_i}{\Lambda\lambda_i}P_0 \quad (4)$$

Where:

P(t) = The neutron population density
$C_i(t)$ = The Concentration of the i-th group of the delayed neutron precursors
$\rho(t)$ = The nuclear reactivity
Λ = The prompt neutron generation time
β_i = The effective fraction of the i-th group of delayed neutron precursor
β = The total effective fraction of the delayed neutron precursor
λ_i = The decay constant of the i-th group of delayed neutron precursor

The point kinetic equations can be derived from the transport equation or from the neutron diffusion equation. These equations are a set seven coupled non-linear differential equations. Physically these equations describe the time evolution of the neutron distribution and also the delayed neutron precursor concentrations in the core of the nuclear reactor.

It is possible to write an expression to calculate reactivity. Equation 2 can be expressed as a set of subtracting equations such that:

$$C_i[n+1]=\frac{\beta_i}{\Lambda}P[n]-\lambda_i C_i[n],\ i=1,2,...,6 \quad (5)$$

where, n is the discretized time.

The properties of the Z-transform for a delayed signal are such that:

$$X[n+m]\Leftrightarrow Z^m X[Z]-Z^m\sum_{n=0}^{m-1}X[n]Z^{-n} \quad (6)$$

Using Eq. 6 on Eq. 5, we get:

$$ZC_i-C_i[0]=\frac{\beta_i}{\Lambda}P[Z]-\lambda_i C_i[Z] \quad (7)$$

Grouping and factoring terms on Eq. 7, we obtain:

$$C_i[Z]=\frac{C_i[0]}{Z+\lambda_i}+\frac{\beta_i}{\Lambda}\frac{P[Z]}{Z+\lambda_i} \quad (8)$$

If we apply the inverse Z-transform on Eq. 8, we get:

$$C_i[n]=C_i[0]e^{-\lambda_i n}+\frac{\beta_i}{\Lambda}Z^{-1}\left\{Z\left[P[n]\right]Z\left[e^{-\lambda_i n}\right]\right\} \quad (9)$$

Replacing the condition given by Eq. 4 on Eq. 9:

$$C_i[n]=\frac{\beta_i\langle P_0\rangle}{\lambda_i\Lambda}e^{-\lambda_i n}+\frac{\beta_i}{\Lambda}Z^{-1}\left\{Z\left[P[n]\right]Z\left[e^{-\lambda_i n}\right]\right\} \quad (10)$$

Solving for the reactivity on Eq. 1, we can write in the discrete form:

$$\rho[n]=\beta+\frac{\Lambda}{P[n]}P^{(1)}[n]-\frac{\Lambda}{P[n]}\sum_{i=1}^{6}\lambda_i C_i[n] \quad (11)$$

Replacing Eq. 10 on Eq. 11, we obtain:

$$\rho[n]=\beta+\frac{\Lambda}{P[n]}P^{(1)}[n]-\frac{\langle P_0\rangle}{P[n]}\sum_{i=1}^{6}\beta_i e^{-\lambda_i n}-\frac{1}{P[n]}Z^{-1}\left\{Z\left[P[n]\right]Z\left[\sum_{i=1}^{6}\lambda_i\beta_i e^{-\lambda_i n}\right]\right\} \quad (12)$$

Equation 12 can be compared with the inverse point kinetic equation in its continuous form (Duderstadt and Hamilton, 1976):

$$\rho(t)=\beta+\frac{\Lambda}{P(t)}\frac{dP(t)}{dt}-\frac{\langle P_0\rangle}{P(t)}\sum_{i=1}^{6}\beta_i e^{-\lambda_i t}-\frac{1}{P(t)}\int_0^t\sum_{i=1}^{6}\lambda_i\beta_i e^{-\lambda_i(t-t')}P(t')dt' \quad (13)$$

Equation 13 is an integral-differential equation frequently used to program the movement of the control rods to obtain the desired variation of the neutron population density and during physical tests for nuclear power plants. This equation is used for the construction of a digital reactivity meter. The integral part is known as the history of the neutron population density because it uses all the values of the existing neutron density and the differential part represents the period of a reactor. Equation 12 is the base for the method, we present in the following section.

Proposed method: It is possible to compare Eq. 12 and 13 using the Z-transform which is known as the discrete version of the Laplace transform. This way, we use the calculation of the Z-transform and its inverse where the following conclusions are obtained:

$$y[n]=Z^{-1}\left\{Z\left[P[n]\right]Z\left[h_i[n]\right]\right\}T \quad (14)$$

Where T represents the time evolution in the calculation. This term appears when we approximate the continuous version of Eq. 13 in a discrete form given by Eq. 12. On Eq. 14, we have considered $h_{\beta i}[n]=\sum_{i=1}^{6}\lambda_i\beta_i e^{-\lambda_i nT}$ interpreting this function as the response on a unitary pulse. When calculating the inverse Z-transform, we get the expression:

$$y[n]=T\sum_{m=0}^{N}h_i[n-m]x[m]=Th_i[n]*x[n] \quad (15)$$

However, Eq. 12 realizes too many operations, since, $h_i[k]=0$ for values outside the interval $0\leq k\leq N$, therefore, we write n instead of N, denoting the following expression:

$$y(t)\approx y(nT)=y[n]=T\sum_{m=0}^{n}h_i[n-m]X[n]=Th_i[n]*x[n] \quad (16)$$

Equation 16 was obtained by (Diaz and Martinez, 2009) using the Laplace transform. Here, it was deduced using the Z-transform which is more efficient for the calculation of reactivity. On Eq. 15 and 16 the symbol (*) represents the product of the convolution $h_i[n]$ is the response of the system when the input is an unitary impulse function.

Equation 16 is called the Finite Impulse Response (FIR) filter, the reactivity can be calculated of the following form:

$$\rho[t]=\beta+\frac{\Lambda}{P(t)}\frac{dP(t)}{dt}-\frac{\langle P_0\rangle}{P(t)}\sum_{i=1}^{6}\beta_i e^{-\lambda_i t}-\frac{T}{P(t)}\left(\sum_{i=1}^{6}\lambda_i\beta_i h_i(t)*P(t)\right) \quad (17)$$

To fulfill the conditions on reactors of the critical type for n = 0, we take:

$$\tilde{y}[n]=y[n]-\frac{1}{2}\left[h_i[n]x[0]x[n]\right] \quad (18)$$

The condition given by Eq. 18 gives a better approximation to the FIR filter, known as the trapezoidal rule.

To generate Gaussian noise around the mean of the neutron population density, we employ Eq. 19 with a standard deviation, noted by σ according to (Kitano *et al*., 2000):

$$\bar{P}_i=\frac{1}{N}\sum_{j=1}^{N}Pj \quad (19)$$

The proposed method in this study consists in effectively combining two digital filters in a simultaneous manner. This allows the reduction of fluctuation on the reactivity calculation. For this task, we take as an indicator, the mean absolute error, obtained from the data sample when filtering, instead of other established indicators (Diaz *et al*., 2008; Diaz and Martinez, 2010).

We express the implemented combinations as cases on the simulation, among which we find Case I which consists in the FIR method and allows the discrete calculation of the reactivity by means of the convolution of the neutron population density and the transfer function as noted on Eq. 17.

The proposed Case II consist on implementing a first-order delayed low-pass filter with a filtering constant represented by a value of τ = 0.5 s and with a time step equal to T. The mathematical expression for the filter presented in the work (Shimazu *et al*., 1987) is:

$$P_i=P_{i-1}+\frac{T}{T+\tau}\left(\bar{P}_i-P_{i-1}\right) \quad (20)$$

To simulate a real situation in the reactivity control of a nuclear power plant, we propose that the neutron population density, obtained by different sensors can be calculated using:

$$P_{m,noise}=P_m+\sigma\bar{P}_i\eta N(0,1) \quad (21)$$

where, σ is the standard deviation, $\eta_N(0,1)$ N data with a Gaussian distribution with a statistical median of zero and a variance equal to one. $\bar{P}_i$ is the average of the N values of neutron population density as given by Eq. 19 $P_{m,\ noise}$ and P_m denote the neutron population density with and without noise at step m, respectively.

RESULTS AND DISCUSSION

Ahead, we show some results of the numerical experiments realized for different values of ω calculating the resulting positive square root from the Inhour equation. The simulated tests were made considering parameters such as: λ_i the decay constant for delayed neutrons and β_i the delayed neutron fraction. The typical precursor coefficients are presented on Table 1. The generation time is $\Lambda=2\times10^{-5}$ s but the derivative term that

Table 1: Typical precursor coefficients for ^{235}U

Group	1	2	3	4	5	6
$T_{1/2}$ (s)	54.5785	21.8658	6.0274	2.2288	0.4951	0.1791
λ_i	0.0127	0.0317	0.115	0.311	1.4	3.87
β_i/β	0.0380	0.2130	0.1880	0.4070	0.1280	0.0260
β_i	0.000266	0.001491	0.001316	0.002849	0.000896	0.000182

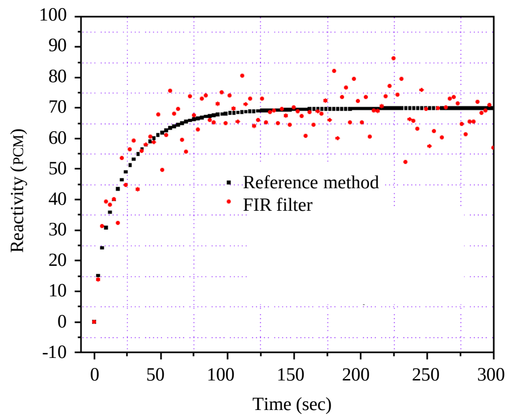

Fig. 1: Variation on the reactivity over time for a value of ω = 0.01046 with a time step of T = 0.01 s using the FIR filter (P(t) = exp(ωt), σ = 0.01, τ = 0.5, sample number = 100, Max. Diff = 20.50 PCM in t = 107 s)

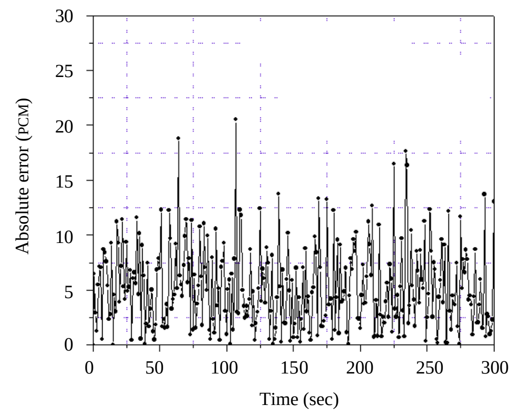

Fig. 2: Variation on the absolute error over time for a value of ω = 0.01046 with a time step of T = 0.01 s using the FIR filter (P(t) = exp(ωt), σ = 0.01, mean absolute error = 0.05 PCM)

accompanies this constant will not be considered on this work. The results presented on this paper are for a real time of t_R = 1s using 100 samples for each calculation. Additionally, the exact value of the reactivity in the experiments can be calculated using Eq. 13 which is called as the reference method. It is worth noting that during the simulation tests, we employed a quasi-random number generator with a normal distribution and a seed that is reestablished in the 2^{31}-1, to allow the reader to reproduce the results, we obtained for this study.

We also stablished two simulation cases and the obtained results are shown in the presentation order that are shown in the next section. The form of the neutron population density is P(t) = exp(ωt) with different values for ω in the first case and others for the second. The relative standard deviation is σ = 0.01 for a Time step on the calculation of T = 0.01s which is applied in the simulation experiments.

Case I; FIR filter: Case I applies Eq. 17, called FIR filter, plus the trapezoidal approximation given by Eq. 18. Using these considerations, we calculate discretely the reactivity. The form of the neutron population density P(t) = exp(ωt) with ω = 0.01046, for this case, we obtain a maximum difference of 20.50 PCM at a time of t = 107 s as shown on Fig. 1. However, as shown on Fig. 2, the mean absolute error is 5.05 PCM, meaning that there are high fluctuations on the reactivity. We conclude that the FIR filter does not reduce the mean absolute error in a significant manner in comparison with the results shown by Diaz *et al.* (2008) because the precision of the FIR filter with the correlation given by Eq. 18 is of order h^2. Therefore, it is convenient to use another filter to reduce the fluctuations.

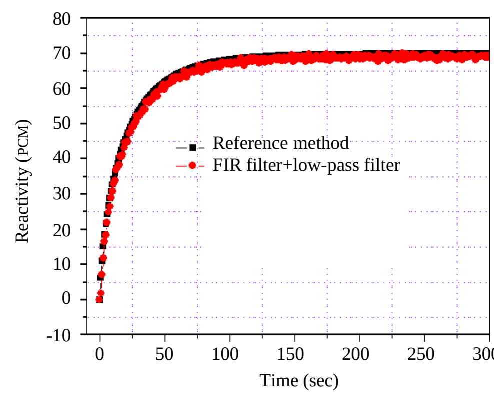

Fig. 3: Variation on the reactivity over time for a value of ω = 0.01046 with a time step of T = 0.01 s using the low-pass filter+FIR filter to diminish a Gaussian noise σ = 0.01 (P(t) = exp(ωt), σ = 0.01, τ = 0.5, sample number = 100, Max. Diff = 4.35 PCM in t = 1 s)

Case II: combination of a low pass filter and FIR filter: Case II uses the first-order delayed low-pass filter, given by Eq. 20 combined with the FIR filter, shown on Eq. 17, to filter the fluctuations on the reactivity. We use the form of the neutron population density P(t) = exp(ωt) with ω = 0.01046, σ = 0.01 a Time step of T = 0.01 s and the filter constant of τ = 0.5 s. We obtain a reduction on the fluctuations for the nuclear reactivity with a maximum difference of 4.35 PCM which appears at t = 1 s as seen on Fig. 3. However, when the stabilization time has passed, the maximum difference is of 2.26 PCM at t = 18 s. On the other hand, there is an increasing and stable tendency for this combination, given that the mean absolute error is 0.51 PCM as shown on Fig. 4.

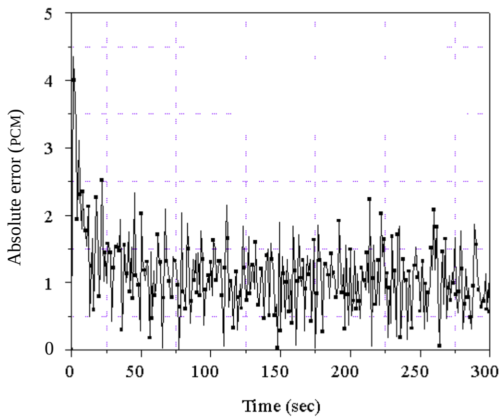

Fig. 4: Variation on the absolute error over time for a value of ω = 0.01046 with a time step of T = 0.01 s using the low-pass filter and FIR filter (P(t) = exp(ωt), σ = 0.01, mean absolute error = 0.51 PCM)

Table 2: Mean absolute error in PCM for a combination of the filtering using the neutron population density P (t) = exp(ωt) with different values for ω

P (t) = exp (ωt), τ = 0.5, σ = 0.01		Mean absolute error [PCM] Trapezoidal FIR	Trapezoidal FIR+ low-pass filter
t_f = 500	ω = 0.006881	5.21	0.43
t_f = 1000	ω = 0.00243	5.43	0.35
t_f = 800	ω = 0.01046	5.05	0.51
t_f = 600	ω = 0.02817	4.49	1.04
t_f = 300	ω = 0.12353	3.23	3.45

Table 3: Mean absolute error in PCM for a combination of the filtering using the neutron population density P (t) = a+bt^4 with different values for b

P (t) = a+bt^4 t_f = 3000s, τ = 0.5, σ = 0.01		Mean absolute error [PCM] Trapezoidal FIR	Trapezoidal FIR+ low-pass filter
a = 1	b = $(0.0127)^5/9$	4.09	0.36
a = 1	b = $(0.0127)^4/40$	5.36	0.37
a = 1	b = $(0.0127)^4/4$	5.33	0.39

Table 2 shows the mean absolute error for the different cases presented before, using the neutron population density of the form P(t) = exp(ωt) with different values for ω, a time step of T = 0.01s and standard deviation of σ = 0.01. When the FIR filter and the low pass filter are used in this case there is a mean absolute error lower than before. This combination therefore reduces in a great manner the fluctuations of the reactivity.

Table 3 shows the differences of the mean absolute error for a neutron population density defined by P(t) = a+bt^4 with different values for b. To run our numerical experiments, we used a standard deviation of σ = 0.01, a filtering constant of τ = 0.5 s with a time step of T = 0.01s and a filtering sample number of 100. The results

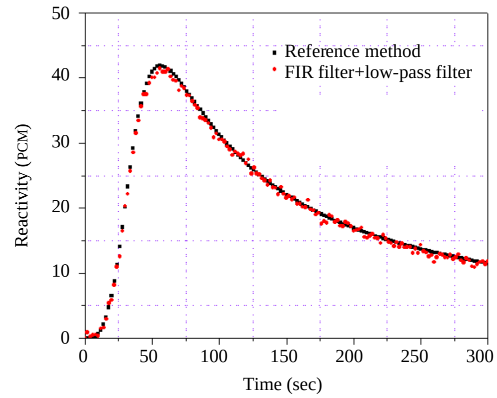

Fig. 5: Variation on the reactivity over time for a neutron population density of the form P(t) = a+bt^4 with a = 1 and b = $(0.0127)^5/9$ with a time step of T = 0.01 s using the low-pass filter and FIR filter (σ = 0.01, τ = 0.5, sample number, Max. Diff = 2.04 PCM in t = 487 s)

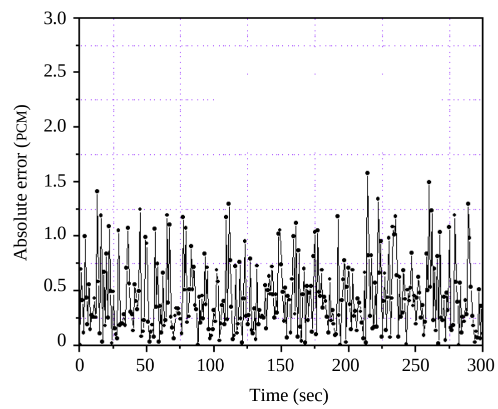

Fig. 6: Variation on the absolute error over time for a neutron population density of the form with P(t) = a+bt^4 with a = 1 and b = $(0.0127)^5/9$ with a time step T = 0.01 s of using the FIR filter and the low-pass filter (σ = 0.01, τ = 0.5, sample number, mean absolute error = 0.36 PCM)

are very accuracy with mean absolute errors no higher than 0.39 PCM. The maximum difference is of 2.04 PCM at t = 487 s and the mean absolute error is 0.36 PCM as shown on Fig. 5 and 6, respectively.

CONCLUSION

This research shows the simultaneous use of two digital filters, that allows to reduce the fluctuations on the reactivity. We show that to improve the precision in the calculation of the reactivity, the second case which is the combination between a FIR filter and a low-pass filter is adequate because it reduces the mean absolute errors.

Finally, this study stablishes results that are the fundamental base for the implementation of a reactivity meter in real time with time steps of up to T = 0.01 s. For different forms of the neutron population density when the density includes a Gaussian noise around the mean of the neutron population density with standard deviation of up to $\sigma = 0.01$. For future work, we intend to implement a digital filter that reduces greatly the fluctuations of the nuclear reactivity in comparison with the results obtained with other combinations.

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