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Numerical Evaluation of 2-EGAOR Iterative Method on Image Blurring using Non-Linear Diffusion Equation

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Abstract: The application of non-linear diffusion equation for image blurring has become a significant study in image processing field. The process is carried out by smoothing the image while preserving the crucial part of the image, i.e., edge, shape and important features using non-linear diffusion equation. However, the classical iterative method to solve the equation requires high number of computations and make the filtering process slower. This study examined 2-EGAOR iterative method as an efficient solver to the image blurring as this method involves two point in one group to solve the linear system with two weighted parameter. For the performance comparison, the results of Successive Over-Relaxation (SOR), Accelerated Over-Relaxation (AOR) and 2-EGAOR iterative methods to develop the equivalent image producing by classical Jacobi method is recorded in this study. The number of iterations and computational time in solving the linear system are used as the evaluation criteria of these iterative methods. Based on the numerical experiment, the findings has shown the 2-EGAOR method able to blurred the image slightly faster as it gives the fewer number of iterations and computational time compared to the other mention methods.

Key words: Non-linear diffusion equation, image blurring, two point Explicit Group Accelerated Over-Relaxation (2-EGAOR), iteration, evaluation criteria, filtering process

INTRODUCTION

The wide application of Partial Difference Equations (PDEs) have become a crucial study to generate better algorithms and tools for image processing applications. The PDEs based image processing methods have been applied in various problems by past researchers, i.e., image editing (Yu and Deng, 2016; Hong *et al*., 2017), edge detection (Dollar and Zitnick, 2015), image segmentation (Dhanachandra *et al*., 2015), image enhancement (Hama and Al-Ani, 2013) , image smoothing (Li *et al*., 2018) and image denoising (Sharma *et al*., 2017). Apparently, the heat diffusion PDE is used in image blurring. However, the rate of diffusion in linear heat equation is constant across the whole image domain which causes the smoothing of the entire image. A solution to this problem is by utilizing the diffusion coefficient based on image gradients that can preserve edges information that are the solutions of nonlinear diffusion equations (Weickert *et al*., 1998; Dollar and Zitnick, 2015).

The idea of utilizing nonlinear diffusion equation in image processing was first initiated by Perona and Malik (1990) with controlling the rate of diffusion using local gradient magnitude function. The successful of the algorithm to solve many image processing problems has motivated Catte *et al*. (1992) on enhancing the edge preserving behaviour of Perona Malik model. An improved nonlinear diffusion algorithm was proposed by Wu and Zhong (2010) applied on image denoising problem. Noise is refers to the random signal that appears as random speckles which significantly corrupting the image quality. Therefore, this new method has been verified as an efficient method to properly denoise the images compared to the other existing methods as it able reduce image noise while maintain important details better by using wavelet coefficient.

Recently, Ma and Nie (2016) have proposed more advanced anisotropic diffusion filtering model for image denoising. The model applied characteristics and gradient variance parameter to classify the important image information such as edges, corners, smooth regions and

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isolated noises. The differ between nonlinear diffusion and anisotropic diffusion are the diffusivity possess in anisotropic diffusion model is a tensor instead of scalar (Weickert, 1998). Then, the eigenvalues of diffusion tensor used by Ma and Nie (2016) are designed for adaptive diffusion from the different image information. Finally, an edge combination scheme is presented to maintain the edges after denoising by fusing the distinctive denoising and edge detection techniques.

Several other tools are also been proposed to solve image processing problems by using diffusion equation such discussed earlier in this study. However, the primary motivation of this study to evaluate the performance of iterative methods in solving nonlinear diffusion equation numerically. The focus of research is same as conducted by recent researcher on Poisson image blending problems (Hong *et al*., 2017; Eng *et al*., 2017a, b, 2018). Hence, this study utilized the potential of 2-EGAOR iterative method to solve the nonlinear diffusion equation for image blurring. Then, the findings of SOR and AOR iterative methods are determined to conduct comparison performances with 2-EGAOR iterative method. The performance of those iterative methods are examined by the number of iterations and the computational time taken.

The remains of the study is sorted out as follows. Section 2 introduces the nonlinear diffusion equation based on model developed by Perona and Malik (1990) and provide the formulation of iterative method using two-point Explicit Group Accelerated Over-Relaxation by Yousif and Evans (1986) and Martins *et al*. (2002) to solve the linear system.

Non-linear diffusion equation: The process of blurring an image using non-linear diffusion equation is one of the Partial Differential Equation (PDEs) based image filtering technique. This equation also can has widely been used in denoising (Chen *et al*., 2015), smoothing (Liu *et al*., 2017) and edge detection (Nomura *et al*., 2016) an image. According to Kamalaveni *et al*. (2015), this method improve the problem of linear diffusion filtering by avoiding the blurring the important features and localization. Equation 1 can be represent as follows (Perona and Malik, 1990):

$$I_t = \operatorname{div}\left(g(x,y,t)\nabla I(x,y)\right) \tag{1}$$

Equation 1 apply the diffusion coefficient $g(x,y,t) = c(\|\nabla I(x,y,t)\|)$ that reduces the diffusivity at any point probable to be the edges. Based on the above equation, the symbol of div is known as the divergence operator. Meanwhile, ∇I represent the gradient magnitude operator respect to the spatial of x and y. The diffusivities used from Eq. 1 is based on the diffusion coefficient c(.) given as Eq. 1:

$$c(\nabla I) = \frac{1}{1+\left(\frac{\|\nabla I\|}{K}\right)^2} \tag{2}$$

Equation 2 is known as edge stopping function that control the diffusivities suggested by Perona and Malik (1990). The diffusivity is enhancing in the interior of homogenous region while zero at the boundaries. Its means the blurring impact will be lessen at any location with high potential be the boundaries monitored by the local gradient magnitude function, $|\nabla I|$ where, g (x, y, t) = $c(|\nabla I|)$. Therefore, implicit finite difference discretization approach is applied in this study to form the approximate equation followed by linear systems and then solved by using 2-EGAOR.

MATERIALS AND METHODS

Implementation of proposed iterative methods

Point iterative methods: The discretization of Eq. 1 through finite difference method by using implicit scheme has been developed to form the approximation equation of standard five-point as in Eq. 3:

$$\begin{aligned}&\left(1+\lambda c_N+\lambda c_S+\lambda c_E+\lambda c_W\right)U_{i,\,j,\,k+1}-\lambda c_N U_{i,\,j+1,\,k+1}-\\&\lambda c_S U_{i,\,j-1,\,k+1}-\lambda c_E U_{i+1,\,j,\,k+1}-\lambda c_W U_{i-1,\,j,\,k+1}\cong U_{i,\,j,\,k}\end{aligned} \tag{3}$$

where, $\lambda = \Delta t/h^2$ as $h = \Delta x = \Delta y$. To simplify Eq. 3, let $\beta = 1+\lambda c_N+\lambda c_S+\lambda c_E+\lambda c_W$, $\delta_N = \lambda c_N$, $\delta_S = \lambda c_S$, $\delta_E = \lambda c_E$ and $\delta_W = \lambda c_W$ then, developed Eq. 4:

$$\begin{aligned}&\beta U_{i,j,k+1}-\delta_N U_{i,j+1,k+1}-\delta_S U_{i,j-1,k+1}-\\&\delta_E U_{i+1,j,k+1}-\delta_W U_{i-1,\,j,\,k+1}\cong U_{i,j,k}.\end{aligned} \tag{4}$$

The developed approximation (Eq. 4) consists of the sparse of linear system can be reconstruct in the matrix form composed as Eq. 5:

$$A\underline{U} = \underline{b} \tag{5}$$

where, A and b are known and U is unknown. Since, the coefficient of the main diagonal is strictly dominant, the linear system (Eq. 5) comprises a unique solution (Young, 1954). Matrix A then be decomposed into three different matrices shown as Eq. 6:

$$A = S\text{-}T\text{-}V \tag{6}$$

which the matrix of S refers to the diagonal part of matrix A while matrices of -T and -V are the strict lower triangular and strict upper triangular parts, respectively. Thus, the corresponding point SOR (Young, 1954) and AOR (Hadjidimos, 1978) iterative schemes in the matrix form as in Eq. 7:

$$\begin{aligned} U^{(n)} &= (S\text{-}\omega T)^{-1}\left[\omega V+(1\text{-}\omega)S\right]U^{(n-1)}+\omega(S\text{-}\omega T)^{-1}b \\ U^{(n)} &= (S\text{-}rT)^{-1}\left[(1\text{-}\omega)S+(\omega\text{-}r)T+\omega V\right]U^{(n-1)}+\omega(S\text{-}rT)^{-1}b \end{aligned} \tag{7}$$

for n = 1, 2, 3. The approximate (Eq. 4) also brings about the large linear system with sparse coefficient matrix which can be stated in algebraic Eq. 8 as:

$$\sum_{j=1}^{m} a_{i,j}U_j = b_i; \qquad i = 1,2,3,\cdots, m \tag{8}$$

By applying the iterative methods of SOR and AOR (Basran *et al*., 2018) for solving Eq. 4 in the form of algebraic Eq. 8, the corresponding iteration scheme can be shown in Eq. 9:

$$\begin{aligned} U_{i,j}^{k+1} &\cong \frac{\omega}{\beta}\left(U_{i,j}^{k}+\delta_N U_{i,j+1}^{k}+\delta_S U_{i,j-1}^{k+1}+\delta_E U_{i+1,j}^{k}+\delta_W U_{i-1,j}^{k+1}\right)+ \\ &\quad (1\text{-}\omega)U_{i,j}^{k} \\ U_{i,j}^{k+1} &\cong \frac{r}{\beta}\left(\delta_S U_{i,j-1}^{k+1}\text{-}\delta_S U_{i,j-1}^{k}+\delta_W U_{i-1,j}^{k+1}\text{-}\delta_W U_{i-1,j}^{k}\right)+ \\ &\frac{\omega}{\beta}\left(U_{i,j}^{k}+\delta_N U_{i,j+1}^{k}+\delta_S U_{i,j-1}^{k}+\delta_E U_{i+1,j}^{k}+\delta_W U_{i-1,j}^{k}\right)+(1\text{-}\omega)U_{i,j}^{k} \end{aligned} \tag{9}$$

for k = 1,2,3, ..., n. The SOR iterative method can be reconstruct from AOR method by placing the value of equivalent to ω. The iteration process of Eq. 9 continues until the convergence criterion is satisfied which in this study we imply the convergence error tolerance when the overall pixel difference between the images produced by using Jacobi iterative methods which is <5.0%. The weighted parameter, ω is set from the range of 1<ω<2. The optimal value of the weighted parameter, ω can be obtained through several test that gives the least number of iterations, k. Then, choosing the consecutive values of r with precision 0.01 within the interval 0.1 for AOR method.

Study conducted by Dhanachandra *et al*. (2015) has used the Gauss-Seidel iterative method as the faster solver than the Jacobi iterative method. Besides, recent studied in image processing also has employed block iterative method for solving Poisson image blending (Eng *et al*., 2017a, b, 2018). Other than that block iterative methods also had widely been studied in robot path planning problems (Saudi *et al*., 2017; Dahalan *et al*., 2017) and other numerical problems (Akhir and Sulaiman, 2015; Chew and Sulaiman, 2017).

Two point explicit group accelerated over-relaxation iterative method: Two point explicit group iterative has been constructed to solve the linear system of Eq. 4 where two node points as shown in Eq. 10 are applied form two-point block such:

$$\begin{aligned} U_{i,j}^{k+1} &\cong \frac{1}{\beta}\left(U_{i,j}^{k} + \delta_N U_{i,j+1}^{k} + \delta_S U_{i,j-1}^{k+1} + \delta_E U_{i+1,j}^{k} + \delta_W U_{i-1,j}^{k+1}\right) \\ U_{i+1,j}^{k+1} &\cong \frac{1}{\beta_1}\left(U_{i+1,j}^{k}+\delta_{N1} U_{i+1,j+1}^{k}+\delta_{S1} U_{i+1,j-1}^{k+1}+\delta_{E1} U_{i+2,j}^{k}+\delta_{W1} U_{i,j}^{k+1}\right) \end{aligned} \tag{10}$$

Equation 10 above can be formulated into matrix form of Eq. 5 which constructed as:

$$\begin{bmatrix} \beta & -\delta_E \\ -\delta_{W1} & \beta_1 \end{bmatrix}\begin{bmatrix} U_{i,j}^{k+1} \\ U_{i+1,j}^{k+1} \end{bmatrix} = \begin{bmatrix} U_{i,j}^{k}+\delta_N U_{i,j+1}^{k}+\delta_S U_{i,j-1}^{k+1}+\delta_W U_{i-1,j}^{k+1} \\ U_{i+1,j}^{k}+\delta_{N1} U_{i+1,j+1}^{k}+\delta_{S1} U_{i+1,j-1}^{k+1}+\delta_{E1} U_{i+2,j}^{k} \end{bmatrix} \tag{11}$$

Then, the inverse matrix of the coefficient lattice of Eq. 11 need to be identified in order to form a general scheme of two point Explicit Group (2-EG) iterative method. Hence, the general scheme of 2-EG can be shown as below (Yousif and Evans, 1986):

$$\begin{aligned} \begin{bmatrix} U_{i,j}^{k+1} \\ U_{i+1,j}^{k+1} \end{bmatrix} &= \frac{1}{\beta\beta_1\text{-}\delta_E\delta_{W1}}\begin{bmatrix} \beta_1 & \delta_E \\ \delta_{W1} & \beta \end{bmatrix} \\ &\begin{bmatrix} U_{i,j}^{k}+\delta_N U_{i,j+1}^{k}+\delta_S U_{i,j-1}^{k+1}+\delta_W U_{i-1,j}^{k+1} \\ U_{i+1,j}^{k}+\delta_{N1} U_{i+1,j+1}^{k}+\delta_{S1} U_{i+1,j-1}^{k+1}+\delta_{E1} U_{i+2,j}^{k} \end{bmatrix} \end{aligned} \tag{12}$$

We can simplify Eq. 13 as following:

$$\begin{aligned} U_{i,j}^{k+1} &= \frac{1}{\beta\beta_1\text{-}\delta_E\delta_{W1}}\left(\beta_1 S_1+\delta_E S_2\right) \\ U_{i+1,j}^{k+1} &= \frac{1}{\beta\beta_1\text{-}\delta_E\delta_{W1}}\left(\delta_{W1} S_1+\beta S_2\right) \end{aligned} \tag{13}$$

where, S_1 and S_1 denoted as Eq. 14:

$$\begin{aligned} S_1 &= U_{i,j}^{k}+\delta_N U_{i,j+1}^{k}+\delta_S U_{i,j-1}^{k+1}+\delta_W U_{i-1,j}^{k+1} \\ S_2 &= U_{i+1,j}^{k}+\delta_{N1} U_{i+1,j+1}^{k}+\delta_{S1} U_{i+1,j-1}^{k+1}+\delta_{E1} U_{i+2,j}^{k} \end{aligned} \tag{14}$$

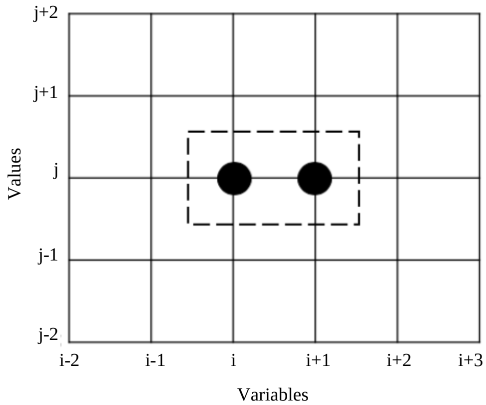

Fig. 1: The 2-EGAOR iterative method computational network in finite grid

It can be seen that, the calculations for both points $U^{k+1}_{i,j}$ and $U^{k+1}_{i+1,j}$ are totally independent. As shown in Fig. 1, the implementation of 2-EG iterative method occurs in each group of two node points meanwhile the ungroup nodes positioned next to the boundary are computed using direct method (Yousif and Evans, 1986). To form 2-EGAOR iterative method, the weighted parameter of r and ω need to be added into the Eq. 14. Then, it can be rewritten as Martins *et al.* (2002) (Eq. 15):

$$
\begin{aligned}
U^{k+1}_{i,j} &= \frac{r}{\beta\beta_1-\delta_E\delta_{W1}}\left(T_1\right)+\frac{\omega}{\beta\beta_1-\delta_E\delta_{W1}}\left(\beta_1 S_{1NEW}+\delta_E S_{2NEW}\right)+\\
&\left(1-\omega\right)U^{k}_{i,j}\\
U^{k+1}_{i+1,j} &= \frac{r}{\beta\beta_1-\delta_E\delta_{W1}}\left(T_2\right)+\frac{\omega}{\beta\beta_1-\delta_E\delta_{W1}}\left(\delta_{W1} S_{1NEW}+\beta S_{2NEW}\right)+\\
&\left(1-\omega\right)U^{k}_{i+1,j}
\end{aligned}
\tag{15}
$$

The symbols of S_{1NEW}, S_{2NEW}, T_1 and T_2 in Eq. 14 can be present as in Eq. 16 and 17:

$$
\begin{aligned}
S_{1NEW} &= U^{k}_{i,j}+\delta_N U^{k}_{i,j+1}+\delta_S U^{k}_{i,j-1}+\delta_W U^{k}_{i-1,j}\\
S_{2NEW} &= U^{k}_{i+1,j}+\delta_{N1} U^{k}_{i+1,j+1}+\delta_{S1} U^{k}_{i+1,j-1}+\delta_{E1} U^{k}_{i+2,j}
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
T_1 &= \beta_1\left(\delta_S U^{k+1}_{i,j-1}-\delta_S U^{k}_{i,j-1}+\delta_W U^{k+1}_{i-1,j}-\delta_W U^{k}_{i-1,j}\right)+\\
&\delta_E\left(\delta_{S1} U^{k+1}_{i+1,j-1}-\delta_{S1} U^{k}_{i+1,j-1}\right)\\
T_2 &= \delta_{W1}\left(\delta_S U^{k+1}_{i,j-1}-\delta_S U^{k}_{i,j-1}+\delta_W U^{k+1}_{i-1,j}-\delta_W U^{k}_{i-1,j}\right)+\\
&\beta\left(\delta_{S1} U^{k+1}_{i+1,j-1}-\delta_{S1} U^{k}_{i+1,j-1}\right)
\end{aligned}
\tag{17}
$$

RESULTS AND DISCUSSION

The developed linear system from approximation non-linear diffusion equation is then solved by 2-EGAOR. The performance of this iterative method is compared with SOR and AOR methods. The assessment is done by contrasting the quantity of iteration and computational time taken of these iterative techniques in delivering nearly a similar impact of blurred image. In order to produce almost the same blurred image for each methods, we set up the classical Jacobi iterative method as the control method. The iterations of SOR, AOR and 2-EGAOR are stopped when the overall pixels difference to the output image of Jacobi <5%. The blurring rate is controlled by diffusion coefficient function as mention in section 2. So, this experiment used the threshold value at K = 5 for Eq. 2 and time-step or value of λ in Eq. 3 at 1.0. This study used three example pictures as shown in Fig. 2 for the blurring process as arranged accordingly, to the image sizes of 512×512, 1024×1024 and 2048×2048, respectively.

The number of iterations and blurring time are examined to compare the efficiency of 2-EGAOR iterative method against SOR and AOR methods. The iterations are running three times for colour (Red, Green and Blue) channels separately as the algorithm filtered the colour images. This causing each colour recorded different number of iterations k and computational time t. Thus, the average number of iterations k and computational time t of the three channels run for each example, images are taken and recorded in Fig. 1 and 2, respectively. For Jacobi method, the number of iterations used for the control parameter of the final image in this experiment are k = 200, k = 400 and k = 800, for example (a-c), respectively.

By referring to Table 1, 2-EGAOR iterative methods significantly reduce the number of iterations compared to SOR and AOR in producing the blurred images. The 2-EGAOR iterative method requires the high percentage of iterations reduction by 90.75-93.5% against Jacobi method. Meanwhile, the method slightly better in reducing the number of iterations by 23.53-26.58% and 4.91-7.14% against SOR and AOR iterative methods, respectively. Thereby, the result of the blurring time, t in Table 2 also proved that the 2-EGAOR method able to

Table 1: The number of iteration, k for image blurring by SOR, AOR and 2-EGAOR iterative methods

Examples/	Jacobi	SOR		AOR		2-EGAOR	
Methods	k	k	error	k	error	k	error
512×512	200	17	0.0477	14	0.0475	13	0.0468
1024×1024	400	50	0.0483	39	0.0492	37	0.0481
2048×2048	800	79	0.0495	61	0.0494	58	0.0494

Table 2: The computational time, t (milliseconds) for image blurring by SOR, AOR and 2-EGAOR iterative methods

Example/Method	Jacobi	SOR	AOR	2-EGAOR
512×512	12746	1349	1223	1167
1024×1024	188082	19546	16289	16086
2048×2048	2220414	176664	166180	151825

Fig. 2: a-c) The input images

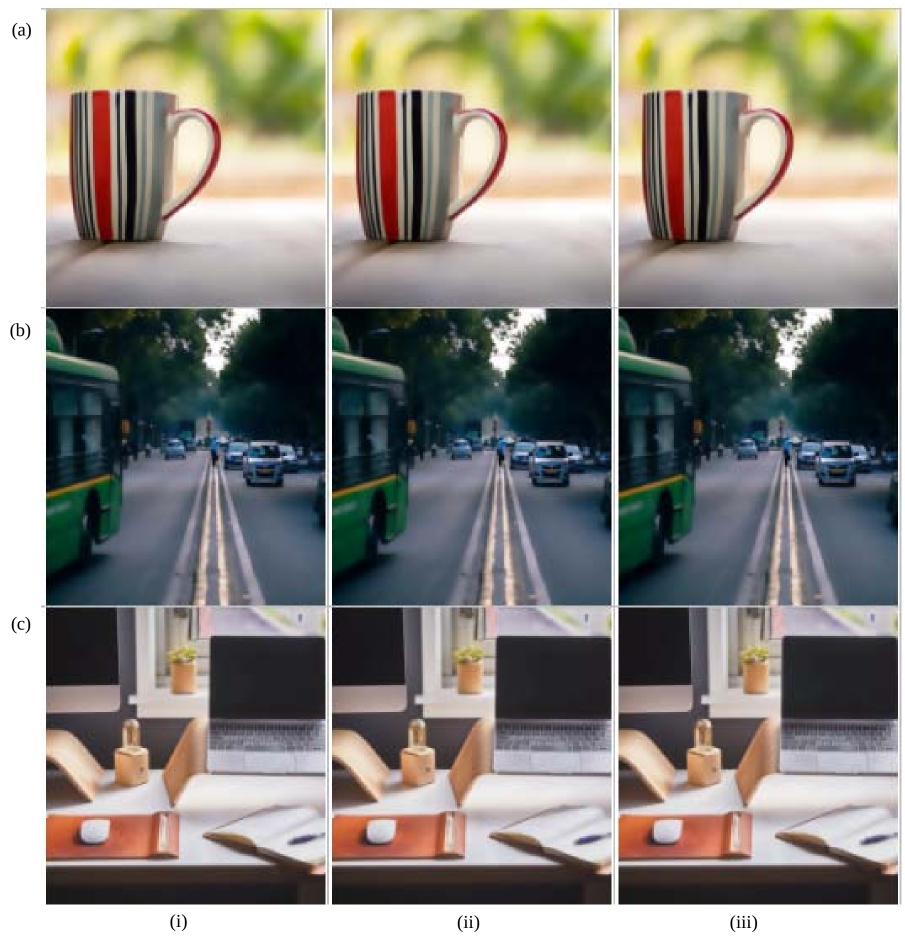

Fig. 3: The output image produced by (i) SOR (ii) AOR and (iii) 2-EGAOR iterative methods using examples (a-c)

produce the faster blurred images as it requires the shortest time compared to SOR and AOR methods. By using 2-EGAOR iterative method, the blurring time taken improved by approximately 90.84-93.16% compared to Jacobi method. By comparing to the others iterative methods, the 2-EGAOR has reduced the blurring time by 13.49-17.70% and 4.58-8.64%, respectively.

Based on the numerical result, the 2-EGAOR iterative method acquires minimum iterations and blurring time compared to other methods. This is because 2-EGAOR is block iterative method where a group of two points is calculated in one iteration. There is no obvious difference for the blurred images produced by all three methods as illustrated in Fig. 3 which the pixels difference (error) between image produced by Jacobi method and those three iterative methods are <5% as shown in Table 1.

CONCLUSION

This study attempt on utilizing the 2-EGAOR technique in image blurring has been obtained and discussed. We have compared the number of iterations and computational time for the image to blur of 2-EGAOR iterative method in solving the proposed application with SOR and AOR iterative methods. The Jacobi has used as a control method with iterations, k = 200, k = 400 and k = 800, for example (a-c), respectively which based on the size of image stated earlier. The iterations have stopped if the pixels difference of the blurred images produce from each methods against output images of Jacobi method is <5%. As expected, the results have showed the 2-EGAOR iterative method is superior to SOR and AOR with the smallest iterations and computational time in producing the same quality of Jacobi image. The reduction percentage for both criteria has slightly reduced approximately by more than 13.00 and 4.00% for SOR and AOR methods, respectively. Besides, the performance of the 2-EGAOR iterative method which is categorized as a family of two point block and two parameter iterative method has been analyse in this study. Then, future research we will be continued to inquire the efficiency of the four point block such as 4-EGSOR (Eng *et al*. 2017a, b; Chew and Sulaiman, 2017), 4-EGAOR (Akhir and Sulaiman, 2015; Dahalan *et al*., 2017) and also half sweep method (Chew and Sulaiman, 2018) that involves half of the grid points in the iteration process.

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