

Applying Simplex Method in a New Way (Addition and Multiplication) on Triangular Fuzzy Number to Find the Optimal Solution

Mohanad R. AL-Janabi, Seif Ali Abdulhussein and Abbas Fadhil Mahdi
Department of Computer Science, Faculty of Computer Science and Mathematics,
University of Kufa, Kufa, Iraq

Abstract: Linear programming is one of the best mathematical methods in scientific management that helps to make the best decisions and is part of the operations research and developed from mathematical programming. Linear programming helps solve problems such as transportation, logistics, agriculture, locations, services, investment, industries, production lines and linear programming reduce the costs of each and maximize the profits that is the outputs must be consistent with the aspirations of the company and its objectives. This problem-solving is based on minimizing costs and maximizing earning limitation by constraints are available resources such as time, staff, raw materials, money, space and quantities. In this study, researcher introduced to study triangular fuzzy number and operation research where the found a new method of operations addition and multiplication of triangular fuzzy number and their application on the numerical example and simplex method to find optimal solution for triangular fuzzy linear programming problem. The results were accurate and better than the classical method.

Key words: Fuzzy sets, fuzzy arithmetic, triangular fuzzy number, function principle, operation, research

INTRODUCTION

Linear Programming (LP) is one of the most frequently used techniques in operations research. In real-world problems, some parameters of LP frequently cannot be precisely determined. We can model imprecise parameters using fuzzy sets which were introduced by Zadeh (1965). The formulation of Fuzzy Linear Programming (FLP) problems by Zimmermann (1978). Modification of subtraction and division by Gani (2012). Use an analytical method to solve linear programming by Gao *et al.* (2009). With the help of similarity measure and ranking function by Sharma (2013). Fully fuzzy linear programming problems with symmetric trapezoidal fuzzy numbers are discussed by Karpagam and Sumathi (2014). Using fuzzy linear programming techniques by Repnik (2016). A computational method for solved Fully Fuzzy Linear Programming Problems (FFLPP) is proposed, based upon the pivot operation by Dhurail and Karpagam (2016). A ranking procedure is proposed based on hexagonal fuzzy numbers which is applied to a fuzzy linear programming problem by Muralidaran and Venkateswarlu (2017). Employ a probability distribution that calculates the probability distribution of a set of attributes by Zadeh (1999). How to deal with the

operations of fuzzy numbers with stage sort by using function standard by Chen (1998). Definition of a positive fuzzy number by Nasser (2008).

In this study, researcher found a new way and a new modification on the two processes addition and multiplication in the triangular fuzzy numbers where he took the first element from the first triangular fuzzy pair addition with the third element from second triangular fuzzy pair and the second element addition with the second element and the third element from the first triangular fuzzy pair addition with the first element from the third triangular fuzzy pair of the same pairs of triangular fuzzy pair for the addition process, also he took the first element from the first triangular fuzzy pair multiplication with the third element from second triangular fuzzy pair and the second element multiplication with the second element and the third element from the first triangular fuzzy pair multiplication with the first element from the third triangular fuzzy pair of the same pairs of triangular fuzzy pair for the multiplication process, also he have put a new definition and the new theorem. The results of the application were more accurate than the previous process. The most important applications of the fuzzy sets in the field of energy engineering and decision making in research operations, business management, etc.

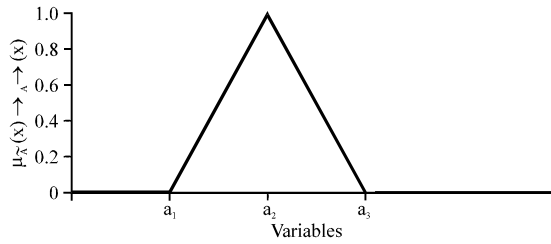


Fig. 1: Triangular fuzzy number ($\tilde{A} = (a_1, a_2, a_3)$)

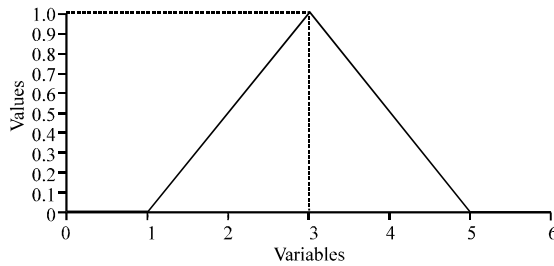


Fig. 2: Numerical example ($\tilde{A} = (1, 3, 5)$)

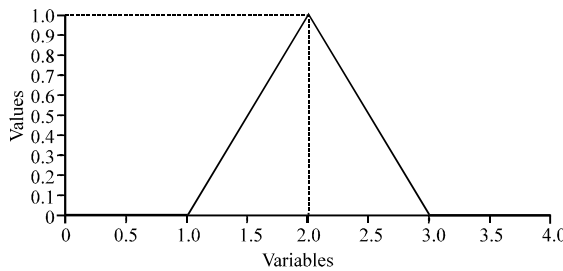


Fig. 3: Numerical example ($\tilde{B} = (1, 2, 3)$)

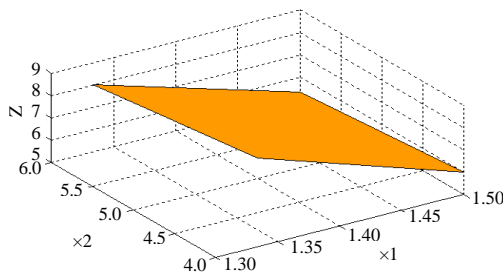


Fig. 4: Simplex method in a new way

Researcher also used MATLAB Version 2014a to program the triangular fuzzy numbers and the new method of addition and multiplication and target function programming to find the optimal solution (Fig. 1-4). In the second section, researcher gave some basic definitions of the fuzzy sets and triangular fuzzy numbers. Where researchers gave the third section to give definitions and new theories after the amendment to the processes of addition and multiplication and the

application of numerical example to satisfy the new conditions. In the fourth section the application of the simplex method to solve the fuzzy linear programming problem to find the optimal solution and in section five, giving some problems to find solutions in the future and after the conclusion.

MATERIALS AND METHODS

Preliminaries: In this study, some necessary backgrounds and notions of the fuzzy set theory are reviewed.

Fuzzy set: Fuzzy set \tilde{A} can have an infinite number on the closed interval $[0, 1]$ of the membership function $\mu_{\tilde{A}}(x) \in [0, 1]$. Then $\tilde{A} = \{x, \mu_{\tilde{A}}(x) : x \in A, \mu_{\tilde{A}}(x) \in [0, 1]\}$.

Membership function:

- A characteristic function, the values assigned to the elements of the universal set fall within a specified range and indicate the membership grade of these elements in the set
- Larger values denote higher degrees of set membership
- A set defined by membership functions is a fuzzy set
- The most commonly used range of values of membership functions is the unit interval $[0, 1]$
- The universal set X is always a crisp set
- Notation, the membership function of a fuzzy set A is denoted by $\mu_{\tilde{A}}(x)$:

$$\mu_{\tilde{A}}(x) : X \rightarrow [0, 1]$$

A fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is said to be the triangular fuzzy number. If its membership function is:

- Increasing function from a_1 - a_2
 - Decreasing function from a_2 - a_3
 - $a_1 \leq a_2 \leq a_3$:
- $$\mu_{\tilde{A}}(x) = \begin{cases} 0, & \text{if } x < a_1 \\ \frac{x - a_1}{a_2 - a_1} & \text{if } a_1 \leq x \leq a_2 \\ \frac{a_3 - x}{a_3 - a_2} & \text{if } a_2 \leq x \leq a_3 \\ 0, & \text{if } a_3 > x \end{cases}$$

The function above shows us the definition a triangular fuzzy number and a code function has been plotted in the MATLAB as shown in Fig. 1.

α -cut and strong α -cut: Given a fuzzy set \tilde{A} defined on X and any number $\alpha \in [0, 1]$, the α -cut and strong α -cut are the Crisp sets: $\tilde{A}^\alpha = \{x | \tilde{A}(x) \geq \alpha\}$, $\tilde{A}^\alpha = \{x | \tilde{A}(x) > \alpha\}$.

He α -cut of a fuzzy set \tilde{A} is the Crisp set that contains all the elements of the universal set X whose membership grades in A are greater than or equal to the specified value of α .

The strong α -cut of a fuzzy set \tilde{A} is the Crisp set that contains all the elements of the universal set X whose membership grades in \tilde{A} are only greater than the specified value of α .

A level set of \tilde{A} : The set of all levels $\alpha \in [0, 1]$ that represent distinct α -cuts of a given fuzzy set \tilde{A} :

$$\mu(\tilde{A}) = \{x \mid \tilde{A}(x) = \alpha \text{ some } x \in X\}$$

Sign of a triangular fuzzy number: Let $\tilde{A} = (a_i), \forall i = 1, 2, 3$. \tilde{A} it is said that triangular fuzzy number, \tilde{A} is positive, denoted by $\tilde{A} > 0, \mu_{\tilde{A}}(x)$ membership function, if $\mu_{\tilde{A}}(x) = 0, x \leq 0$.

Let $\tilde{A} = (a_i), \forall i = 1, 2, 3$. \tilde{A} it is said that triangular fuzzy number, \tilde{A} is non-negative, denoted by $\tilde{A} \geq 0, \mu_{\tilde{A}}(x)$ membership function, if $\mu_{\tilde{A}}(x) = 0, x \leq 0$.

Let $\tilde{A} = (a_i), \forall i = 1, 2, 3$. \tilde{A} it is said that triangular fuzzy number, \tilde{A} is negative, denoted by $\tilde{A} < 0, \mu_{\tilde{A}}(x)$ membership function, if $\mu_{\tilde{A}}(x) = 0, x \geq 0$, e.g., $\tilde{A} = (-3, -2, -1)$, \tilde{A} is a negative this can be written as $\tilde{A} = -(1, 2, 3)$.

Let $\tilde{A} = (a_i)$ and $\tilde{B} = (b_i), \forall i = 1, 2, 3$. \tilde{A} and \tilde{B} it is said that triangular fuzzy members, if $\tilde{A} = \tilde{B}$, then $a_1 = b_1, a_2 = b_2, a_3 = b_3$.

Definition: The four operations on triangular fuzzy numbers by the function basic. Let $\tilde{A} = (a_i)$ and $\tilde{B} = (b_i), \forall i = 1, 2, 3$, then:

- Operation (+): $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
- Operation (-): $\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$
- Operation (\times): $\tilde{A} \times \tilde{B} = (\min(a_1 \times b_1, a_1 \times b_3, a_3 \times b_1, a_3 \times b_3), a_2 \times b_2, \max(a_1 \times b_1, a_1 \times b_3, a_3 \times b_1, a_3 \times b_3))$
- Operation (\div): $\tilde{A} / \tilde{B} = (\min(a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3), a_2/b_2, \max(a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3))$

Example: Let $\tilde{A} = (1, 3, 5)$ is fuzzy number and $\tilde{B} = (1, 2, 3)$ b fuzzy number. Then:

- $\tilde{A} + \tilde{B} = (4, 5, 6)$
- $\tilde{A} - \tilde{B} = (0, 1, 2)$
- $\tilde{A} \times \tilde{B} = (3, 6, 5)$
- $\tilde{A} / \tilde{B} = (1, 3/2, 5/3)$

Remark: As mentioned earlier that $\tilde{A} + (-\tilde{A}) \neq 0, \tilde{A} \times 1 / \tilde{A} \neq 1, 0$ and 1 are represented by the fuzzy numbers is $(0, 0, 0)$ and $(1, 1, 1)$, respectively. To solve the \tilde{C} is the fuzzy linear equation $\tilde{A} - \tilde{B} = \tilde{C}$ is not satisfied the condition, $\tilde{A} = \tilde{B} + \tilde{C}$, for this example explain the operation $\tilde{A} - \tilde{B} = (0, 1, 2) = \tilde{C}$. But $(1, 3, 5) = (1, 2, 3) + (0, 1, 2) = (3, 3, 3) \neq \tilde{A}$. Herself the problem appears when solving the fuzzy linear equation $\tilde{A} / \tilde{B} = \tilde{C}$ is

not satisfied the condition $\tilde{A} = \tilde{B} \times \tilde{C}$ for this example explain the operation, $\tilde{A} / \tilde{B} = (1/1, 3/2, 5/3) = \tilde{C}$. But $\tilde{A} = \tilde{B} \times \tilde{C} \Rightarrow (1, 3, 5) = (1, 2, 3) \times (1/1, 3/2, 5/3) = (5/3, 3, 1) \neq \tilde{A}$. So, when researcher take the inverse addition we do not give zero and inverse multiplication also doesn't give one, so, researcher suggested a new method to solve this problem.

RESULTS AND DISCUSSION

A new way (addition and multiplication) to triangular fuzzy number: In this section, the topical is to improve new way (addition and multiplication) operations to the triangular fuzzy number.

Addition: Let $\tilde{A} = (a_i)$ and $\tilde{B} = (b_i), \forall i = 1, 2, 3$, then $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$. The new addition operation exists only the following condition is satisfied $d_p(\tilde{A}) \geq d_p(\tilde{B})$, then $d_p(\tilde{A}) = a_3 - a_1/2$ and $d_p(\tilde{B}) = b_3 - b_1/2$, the symbol d_p approach to different point of a triangular fuzzy number.

'Addition' process properties:

- Inverse operation (-): $\tilde{A} - (\tilde{A} + \tilde{B}) = (\tilde{A} + \tilde{B}) - \tilde{A} = \tilde{B}$
- Neutral element: $(\tilde{A} + 0) = (0 + \tilde{A}) = \tilde{A}$
- Operation (+) is a commutatively: $(\tilde{A} + \tilde{B}) = (\tilde{B} + \tilde{A})$
- Associative: $\tilde{A} + (\tilde{B} + \tilde{C}) = (\tilde{A} + \tilde{B}) + \tilde{C}$
- Multiplication by scalar: $w(\tilde{A} + \tilde{B}) = (w\tilde{A} + w\tilde{B})$
- Inverse addition: a: i.e., $\tilde{A} + (-\tilde{A}) = (-\tilde{A}) + \tilde{A} = \tilde{0} = (0, 0, 0)$
- Regularity: $\tilde{A} + \tilde{B} = \tilde{A} + \tilde{C} \Rightarrow \tilde{B} = \tilde{C}$
- Pseud distributive with regard to: $(\tilde{A} - \tilde{B}) + (\tilde{C} - \tilde{W}) = (\tilde{A} + \tilde{C}) - (\tilde{B} + \tilde{W})$

Definition: Let $\tilde{A} = (a_i)$ and $\tilde{B} = (b_i), i = 1, 2, 3$. \tilde{A} and \tilde{B} it is said that triangular fuzzy numbers, then: by the def. 1 and 2 are straightforward when 3 satisfies:

- Apply the condition: $\tilde{A} + \tilde{B}$, then $a_1 + b_1 \leq a_2 + b_2 \leq a_3 + b_3 \Rightarrow a_1 + b_1 \leq a_3 + b_3$ if $a_1 \leq b$ and $b_3 \leq a_3$ then $a_1 + b_3 \leq a_3 + b_1$
- The new way addition: $\tilde{A} + \tilde{B} = (a_1 + b_3, a_2 + b_2, a_3 + b_1)$
- Subtraction: $\tilde{A} - \tilde{B} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$

Needful presence condition for addition

Properties: A new way of addition exists if $d_p(\tilde{A}) \geq d_p(\tilde{B})$ is satisfied.

Proof: By the def. 3.3. (i) I get to $a_1 + b_3 \leq a_3 + b_1$:

$$\begin{aligned} &\Rightarrow (m_p(\tilde{A}) + d_p(\tilde{A})) + (m_p(\tilde{B}) - d_p(\tilde{B})) \leq \\ &\quad (m_p(\tilde{A}) - d_p(\tilde{A})) + (m_p(\tilde{B}) + d_p(\tilde{B})) \\ &\Rightarrow -2d_p(\tilde{A}) \geq -2d_p(\tilde{B}) \Rightarrow -d_p(\tilde{A}) \geq -d_p(\tilde{B}) \Rightarrow \\ &\quad d_p(\tilde{A}) \geq d_p(\tilde{B}) \end{aligned}$$

$|-d_p(\tilde{A}) \geq -d_p(\tilde{B})|$. The triangular fuzzy number must be positive and then after placing the absolute value on the condition as stated in the def. 2.3. $d_p(\tilde{A}) \geq d_p(\tilde{B})$ this is the needful presence condition for addition operation.

Multiplication: Let $\tilde{A}=(a_i)$ and $\tilde{B}=(b_i)$, $\forall i=1, 2, 3$. Then:
 $\tilde{A} \times \tilde{B}=(a_1 \times b_1, a_2 \times b_2, a_3 \times b_3)$. The new multiplication operation exists only the following condition is satisfied $|(D_p(\tilde{A}) \times M_p(\tilde{B}))| \geq |(M_p(\tilde{A}) \times D_p(\tilde{B}))|$. The value of the triangular fuzzy number must be positive and then after placing the absolute value on the condition as stated in the def.2.3. Hence:

$$d_p(\tilde{A}) = \frac{a_3 - a_1}{2} \text{ and } d_p(\tilde{B}) = \frac{b_3 - b_1}{2}$$

$$m_p(\tilde{A}) = \frac{a_3 + a_1}{2} \text{ and } m_p(\tilde{B}) = \frac{b_3 + b_1}{2}$$

Where:

m_p = Approach to midpoint

d_p = Approach to difference point of a triangular fuzzy number

'Multiplication' process properties:

- $\tilde{A} \times \tilde{B} / \tilde{A} = \tilde{B} / \tilde{A} \times \tilde{A} = \tilde{B}$ divide is the inverse of the multiplication process
- The neutral element of the multiplication operation is $\tilde{I}(1, 1, 1)$, i.e., $\tilde{I} \times \tilde{A} = \tilde{A} \times \tilde{I} = (1, 1, 1) \times (a_1, a_2, a_3)$
- The multiplication operation is a commutatively: $(\tilde{A} \times \tilde{B}) = (\tilde{B} \times \tilde{A})$
- Inverse multiplication is \tilde{I} / \tilde{A} : i.e., $\tilde{A} \times \tilde{I} / \tilde{A} = \tilde{I} / \tilde{A} \times \tilde{A} \Rightarrow (a_1, a_2, a_3) \times (1, 1, 1) / (a_1, a_2, a_3) = (1, 1, 1) / (a_1, a_2, a_3) \times (a_1, a_2, a_3) = (1, 1, 1) = \tilde{I}$
- Regularity: $\tilde{A} \times \tilde{B} = \tilde{A} \times \tilde{C} \Rightarrow \tilde{B} = \tilde{C}$
- Pseud distributive with regard to +: $(\tilde{A} + \tilde{B}) \times (\tilde{C} + \tilde{W}) = (\tilde{A} \times \tilde{C}) + (\tilde{A} \times \tilde{W}) + (\tilde{B} \times \tilde{C}) + (\tilde{B} \times \tilde{W})$

Definition: Let $\tilde{A}=(a_i)$ and $\tilde{B}=(b_i)$, $i=1, 2, 3$. \tilde{A} and \tilde{B} it is said that triangular fuzzy number, then: by the def. 2.3 1 and 2 are straightforward when (3) satisfies:

- Apply the condition: $\tilde{A} \times \tilde{B} \Rightarrow a_1 \times b_1 \leq a_2 \times b_2 \leq a_3 \times b_3 \Rightarrow a_1 \times b_1 \leq a_3 \times b_3$ if $a_1 \leq b_1$ and $b_3 \leq a_3$ then $a_1 \times b_3 \leq a_3 \times b_1$
- The new way multiplication: $\tilde{A} \times \tilde{B} = (a_1 \times b_3, a_2 \times b_2, a_3 \times b_1)$
- $\tilde{A} / \tilde{B} = (a_1 / b_1, a_2 / b_2, a_3 / b_3)$

Needful presence condition for multiplication

Properties: The new multiplication process exists if the following condition is satisfied:

$$|(d_p(\tilde{A}) \times m_p(\tilde{B}))| \geq |(m_p(\tilde{A}) \times d_p(\tilde{B}))|$$

Proof: By the def. 3.9 (i) I get to $\Rightarrow a_1 \times b_3 \leq a_3 \times b_1$:

$$\Rightarrow (m_p(\tilde{A}) + d_p(\tilde{A})) \times (m_p(\tilde{B}) - d_p(\tilde{B})) \leq (m_p(\tilde{A}) - d_p(\tilde{A})) \times (m_p(\tilde{B}) + d_p(\tilde{B}))$$

$$\Rightarrow (m_p(\tilde{A}) \times m_p(\tilde{B})) - (m_p(\tilde{A}) \times d_p(\tilde{B})) + (d_p(\tilde{A}) \times m_p(\tilde{B})) - (d_p(\tilde{A}) \times d_p(\tilde{B})) \leq (m_p(\tilde{A}) \times m_p(\tilde{B})) + (m_p(\tilde{A}) \times d_p(\tilde{B})) - (d_p(\tilde{A}) \times m_p(\tilde{B})) - (d_p(\tilde{A}) \times d_p(\tilde{B}))$$

$$\Rightarrow -2(d_p(\tilde{A}) \times m_p(\tilde{B})) \geq -2(m_p(\tilde{A}) \times d_p(\tilde{B})) \Rightarrow$$

$$|-(d_p(\tilde{A}) \times m_p(\tilde{B}))| \geq |-(m_p(\tilde{A}) \times d_p(\tilde{B}))|$$

$$\Rightarrow -2d_p(\tilde{A}) \geq -2d_p(\tilde{B}) \Rightarrow |-d_p(\tilde{A}) \geq -d_p(\tilde{B})| \Rightarrow d_p(\tilde{A}) \geq d_p(\tilde{B})$$

The value of the triangular fuzzy number must be positive and then after placing the absolute value on the condition as stated in the def. 2.3:

$$|-(d_p(\tilde{A}) \times m_p(\tilde{B}))| \geq |-(m_p(\tilde{A}) \times d_p(\tilde{B}))|$$

$$|(d_p(\tilde{A}) \times m_p(\tilde{B}))| \geq |(m_p(\tilde{A}) \times d_p(\tilde{B}))|$$

This is the needful presence condition for multiplication operation. m_p approach to midpoint and d_p difference point of a triangular fuzzy number.

Where:

$$d_p(\tilde{A}) = a_3 - a_1 / 2 \text{ and } d_p(\tilde{B}) = b_3 - b_1 / 2$$

$$m_p(\tilde{A}) = a_3 + a_1 / 2 \text{ and } m_p(\tilde{B}) = b_3 + b_1 / 2$$

Numerical examples: Let $\tilde{A}=(1, 3, 5)$ and $\tilde{B}=(1, 2, 3)$ are triangular fuzzy numbers:

$$m_p(\tilde{A}) = \frac{a_3 + a_1}{2} = \frac{5 + 1}{2} = \frac{6}{2} = 3$$

$$d_p(\tilde{A}) = \frac{a_3 - a_1}{2} = \frac{5 - 1}{2} = \frac{4}{2} = 2$$

$$m_p(\tilde{B}) = \frac{b_3 + b_1}{2} = \frac{3 + 1}{2} = \frac{4}{2} = 2$$

$$d_p(\tilde{B}) = \frac{b_3 - b_1}{2} = \frac{3 - 1}{2} = \frac{2}{2} = 1$$

For a numerical example which represents the application of the new way was drawn in MATLAB using a triangular fuzzy number as shown in Fig. 3 and 4.

Now condition $d_p(\tilde{A}) \geq d_p(\tilde{B}) \Rightarrow 2 \geq 1$ this condition satisfied the condition for the addition operation $|(d_p(\tilde{A}) \times m_p(\tilde{B}))| \geq |(m_p(\tilde{A}) \times d_p(\tilde{B}))| \Rightarrow 2 \times 2 \geq 3 \times 1 \Rightarrow 4 \geq 3$. This condition satisfied the condition for the multiplication operation.

Table 1: A new way for a simplex method on a triangular fuzzy number

Maximize	$\tilde{x}(\tilde{C})$	\tilde{x}_1 (6, 4, 4)	\tilde{x}_2 (4, 4, 4)	\tilde{s}_1 (0, 0, 0)	\tilde{s}_2 (0, 0, 0)	\tilde{B}	In this column take less value
\tilde{x}_0	\tilde{C}_0						
\tilde{s}_1	(0,0,0)	(4, 2, 2)	(2, 2, 2)	(1, 1, 1)	(0, 0, 0)	(10, 6, 6)	(10, 6, 6)/(4, 2, 2) = (2.5, 3, 3)
\tilde{s}_2	(0,0,0)	(6, 6, 6)*	(8, 6, 6)	(0, 0, 0)	(1, 1, 1)	(8, 4, 4)	(8, 4, 4)/(6, 6, 6) = (1.33, 0.66, 0.66)
$\tilde{C}_0^T \tilde{A} - \tilde{C}^T$		-(6, 4, 4)*	-(4, 4, 4)	(0, 0, 0)	(0, 0, 0)	(0, 0, 0)	$\tilde{C}^T \tilde{B}$

Table 2: Find the optimal solution

Maximize	$\tilde{x}(\tilde{C})$	\tilde{x}_1 (6, 4, 4)	\tilde{x}_2 (4, 4, 4)	\tilde{s}_1 (0, 0, 0)	\tilde{s}_2 (0, 0, 0)	\tilde{B}	$\tilde{C}^T \tilde{B}$
\tilde{x}_0	\tilde{C}_0						
\tilde{s}_1	(0,0,0)	(0, 0, 0)	(-2, 0, -2/3)	(1, 1, 1)	(-4/6, 2/6, 2/6)	(4, 3, 10/3)	
\tilde{x}_1	(0,0,0)	(1, 1, 1)	(4/3, 1, 1)	(0, 0, 0)	(1/6, 1/6, 1/6)	(4/3, 3/2, 3/2)	-(6, 6, 6) $R_2 + R_1$
$\tilde{C}_0^T \tilde{A} - \tilde{C}^T$		(0, 0, 0)	(2, 0, 4/3)	(0, 0, 0)	(1, 4/6, 4/6)	(9, 6, 16/3)	-(6, 4, 4) $R_2 + R_1$

Applying simplex method in a new method (addition and multiplication): In this section, researcher will solve triangular fuzzy linear programming problem using the simplex method:

$$\text{maximization } \tilde{Z} = (6, 4, 4)\tilde{x}_1 + (4, 4, 4)\tilde{x}_2$$

Subject to constraint:

$$\begin{aligned} (4, 2, 2)\tilde{x}_1 + (2, 2, 2)\tilde{x}_2 &\leq (10, 6, 6) \\ (6, 6, 6)\tilde{x}_1 + (8, 6, 6)\tilde{x}_2 &\leq (8, 4, 4), \tilde{x}_1 \text{ and } \tilde{x}_2 \leq 0 \end{aligned}$$

Rewrite as:

$$\text{Max } \tilde{Z} = (6, 4, 4)\tilde{x}_1 + (4, 4, 4)\tilde{x}_2 + (0, 0, 0)\tilde{s}_1 + (0, 0, 0)\tilde{s}_2$$

Subject to constraint :

$$\begin{aligned} (4, 2, 2)\tilde{x}_1 + (2, 2, 2)\tilde{x}_2 + (1, 1, 1)\tilde{s}_1 + (0, 0, 0)\tilde{s}_2 &= (10, 6, 6) \\ (6, 6, 6)\tilde{x}_1 + (8, 6, 6)\tilde{x}_2 + (0, 0, 0)\tilde{s}_1 + (1, 1, 1)\tilde{s}_2 &= (8, 4, 4) \\ \tilde{x}_1, \tilde{x}_2, \tilde{s}_1 \text{ and } \tilde{s}_2 &\geq 0 \end{aligned}$$

In Table 1 shown apply a new way of using the simplex method to find the optimal solution for the triangular fuzzy number.

Select the most negative triangular fuzzy elements in the last row and the corresponding column to become the working column. -(6, 4, 4)* is the most negative triangular fuzzy element in the last row. Hence, the pivot triangular fuzzy element must lie in the column above.

Form the ratio in the last column and decide the smallest. (8, 4, 4)/(6, 6, 6) = (1.33, 0.66, 0.66) is the smallest one. Divide (6, 6, 6)* by (6, 6, 6) to change it to one by dividing the whole row by (6, 6, 6). Use matrix operations and properties to make all other triangular fuzzy elements in the working column equal to zero.

In Table 2, we found the optimal solution using the new way of the triangular fuzzy number and the results were very accurate after the solution was checked.

Finding the maximum of \tilde{Z} : Since, the last row in the new table has non-negative triangular fuzzy elements. We get the optimal solution at the end of the column \tilde{B} . Hence:

$$\tilde{x}_1^* = \left(\frac{4}{3}, \frac{3}{2}, \frac{3}{2}\right), \tilde{s}_1^* = \left(4, 3, \frac{10}{3}\right) \text{ and } \tilde{x}_2^* = \tilde{s}_2^* = 0 \text{ with } \tilde{Z}^* = \left(9, 6, \frac{16}{3}\right)$$

Choice point:

$$\begin{aligned} \tilde{Z} &= (6, 4, 4)\tilde{x}_1 + (4, 4, 4)\tilde{x}_2 \Rightarrow (6, 4, 4) \times \\ &\left(\frac{3}{4}, \frac{3}{2}, \frac{3}{2}\right) + (4, 4, 4) \times (0, 0, 0) \Rightarrow \tilde{Z} = \left(9, 6, \frac{16}{3}\right) \end{aligned}$$

This study shows us the application of the new way using triangular fuzzy numbers and the results were plotted in the MATLAB using 3-D drawing as shown in Fig. 4.

CONCLUSION

The fundamental objective of this study is to insert a new process of addition and multiplication. The characteristic of this process is to subject reverse processes of subtraction and division.

RECOMMENDATIONS

This process might be helpful to solve many of the optimal problems by modifying the four processes of addition, subtraction, multiplication, division and application to fuzzy linear programming methods to find the optimal solution.

REFERENCES

- Chen, S.H., 1998. Operations of fuzzy numbers with step form membership function using function principle. *Inf. Sci.*, 108: 149-155.
- Dhurai, K. and Karpagam, 2016. A new pivotal operation on the triangular fuzzy number for solving fully fuzzy linear programming problems. *Intl. J. Appl. Math. Sci.*, 9: 41-46.
- Gani, N.A., 2012. Modification of subtraction and division. *J. Math. Sci.*, 6: 525-532.
- Gao, S., Z. Zhang and C. Cao, 2009. Multiplication operation on fuzzy numbers. *J. Software*, 4: 331-338.
- Karpagam, A. and P. Sumathi, 2014. Innovative method for solving fuzzy linear programming problems with symmetric trapezoidal fuzzy numbers. *Intl. J. Latest Res. Sci. Technol.*, 3: 95-98.
- Muralidaran, C. and B. Venkateswarlu, 2017. Accuracy ranking function for solving hexagonal fuzzy linear programming problem. *Intl. J. Pure Appl. Math.*, 115: 215-222.
- Nasseri, H., 2008. Fuzzy numbers: Positive and nonnegative. *Intl. Math. Forum*, 3: 1777-1780.
- Repnik, M., 2016. The application of fuzzy linear programming methods in energy planning/uporaba metod mehkega linearnega programiranja v energetskem nacrtovanju. *J. Energy Technol.*, 9: 35-42.
- Sharma, U., 2013. A new method for solving fully fuzzy linear programming problem. *Intl. J. Sci. Res.*, 4: 428-430.
- Zadeh, L., 1965. *Fuzzy Sets, Information and Control*. Vol. 8, CRC Press, New York, pp: 338-353.
- Zadeh, L.A., 1999. Fuzzy sets as a basis for a theory of possibility. *Fuzzy Sets Syst.*, 100: 9-34.
- Zimmermann, H.J., 1978. Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets Syst.*, 1: 45-55.