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On Total Domination Concept in Chessboard

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Abstract: The chessboard 8×8 is consists 64 squares. It was converted into a graph G = (V, E) with 64 vertices which any two vertices are adjacent by edge. The problem of dominating of the chessboard located within the subjects related to the entertainment mathematics or the puzzles in mathematics. The problem of dominating of the chessboard is to place a certain number of pieces on the chessboard. Let u, v∈V, u is the location of the piece and v is the location to move it, the movement of the piece of chessboard from u to v is the distance between u and v, i.e., d(u, v) = 1. In this study, we applied the domination concept and total domination concept in chessboard pieces (Rook , Bishop, King, Knight and queen). We found graphs for each piece.

Key words: Graph theory, domination, total domination, chessboard, dominating, puzzles

INTRODUCTION

The study of domination in graphs was further developed in the late 1950's and 1960's (Tarr, 2010). Even though the subject has historical roots when De Jaenisch in 1862 studied the problems of determining the minimum number of queens pieces which are necessary to dominate or cover on n×n chessboard (Mahalingam, 2005). In 1958 Claude Berge wrote a book on graph theory in which he introduced the "coefficient of external stability" which is now known as the domination number of a graph. In 1962 Oystein Ore introduced the expressions "dominating set" and "domination number" in his book on graph theory (Freeman, 2014).

In 1977, Cockayne and Hedetniemi were introduced the accepted notation γ(G) to denote the domination number (Maheswari and Meenakshi, 2017). The study of domination in graphs come about a result of the study recreational mathematics or the games. In particular, mathematicians studied how chess pieces of a same kind could be placed on a chessboard in such a way that they would dominate or attack every square on the board (Tarr, 2010).

A graph can be formed from an n×n chessboard by considering the squares as vertices. Edges are added between any two vertices and every two vertices in graph are adjacent. The problem of domination is finding subset of vertices in a graph (the dominating set) that are adjacent to all other vertices in the graph. The vertices in the dominating set can be called keepers and each keeper can dominate itself plus its adjacent vertices. The main problem of domination is to find the minimum number of keepers necessary in order to dominate the entire graph (Burchett, 2005).

MATERIALS AND METHODS

Definition: A subset D of vertices V of a graph G = (V, E). D is called a dominating set of G, if every vertex in V-D is adjacent to some vertex of D. The elements of dominating set are called dominators (Tarr, 2010). In other words, (uV(G)/DvD) such that u and v are adjacent. The size of a smallest dominating set is referred as the domination number of a graph G and denoted by γ(G) (Mahalingam, 2005). Hence, its have 1 ≤(G)≤n where, n is the number of vertices in the graph G (Mahalingam, 2005).

N(v) is the set of all vertices adjacent to v is called the open neighborhood of a vertex v while N[v] = N(v)∪{v} is called the closed neighborhood of a vertex v (Douglas Chatham *et al*., 2008).

Theorm: A dominating set D is a minimal dominating set if and only if for each vertex u in D, one of the following conditions holds:

- u is an isolate of D
- There exists a vertex v in V-D for which N(v)∩D = {u} (Hamid and Balamurugan, 2016)

Definition: A dominating set D is called a total dominating set if every vertex in V is adjacent to some vertex in D and denoted to total dominating set by D^t. The total domination number of G is the minimum cardinality of total dominating set and denoted by $\gamma_t(G)$ (Gupta, 2013). The dominating set differs from the total dominating set. In a dominating set D, the members of D may be either in D itself or adjacent to vertices in D whereas is an total dominating set D^t, the members of D^t are required to them selves be adjacent to a vertex in D^t (Henning and Yeo, 2013).

RESULTS AND DISCUSSION

Applications of the domination concept and the total domination concept on chessboard: Figure 1 is shown the number of squares in the chessboard and the number of vertices and edges in graph after converting the chessboard to graph.

Rook's problem: Every player has two rooks at the beginning of the game. The rook has the freedom to move any number of squares in straight lines up and down or side to side, until it is blocked by another piece or it reaches the end of board. It cannot jump over other pieces as shown in Fig. 2 (Eade, 2005).

A square 8×8 chessboard, the rooks domination number is 8, i.e., $\gamma(G_R) = 8$. The chessboard is fully covered (domination) by the rooks when the rook's pieces put on a single row or a single column in full or on diagonal form as shown in Fig. 3 (Ruzic, 2003).

It is applied the total domination concept when placing the rook's pieces as one row or one column only as shown in Fig. 4. We cannot apply the concept of total domination when placing the rook's pieces diagonally because there are no edges between the vertices in dominating set.

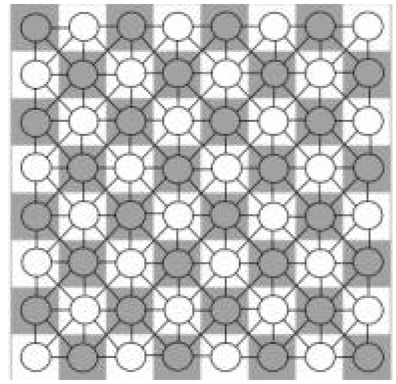

Fig. 1: The graph of chessboard

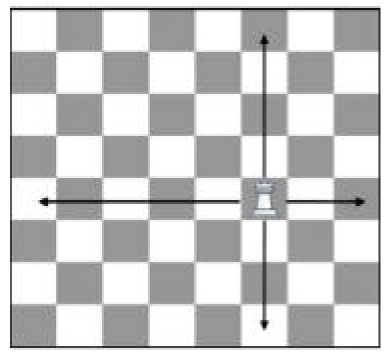

Fig. 2: Movement of rook

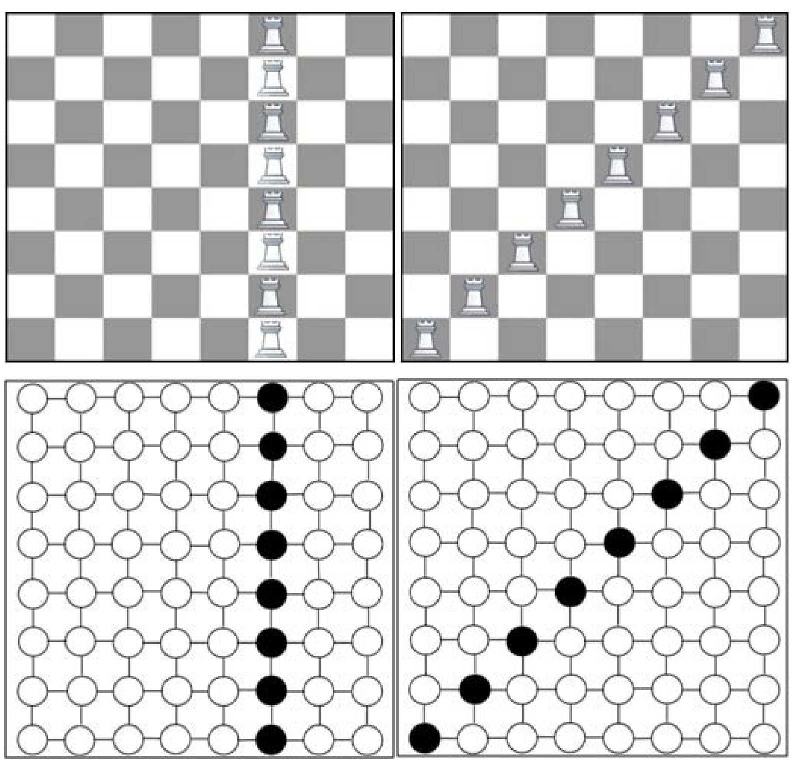

Fig. 3: The rook's domination

Bishop's problem: Every player has two bishops at the beginning of the game. The bishop can move as many squares as wanted until it meets the end of the board or another piece.

It cannot jump over other pieces and it can slide between squares along diagonals. The piece is always stay on the same color squares which started as shown in Fig. 5 (Sowndarya and Naidu, 2018; Eade, 2005).

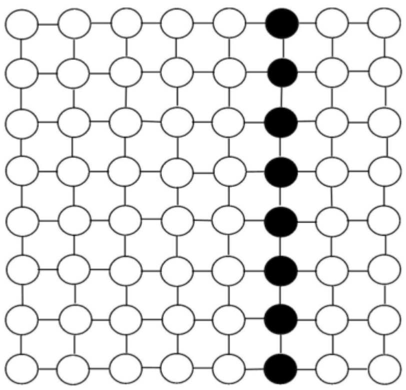

Fig. 4:. The rook's total domination

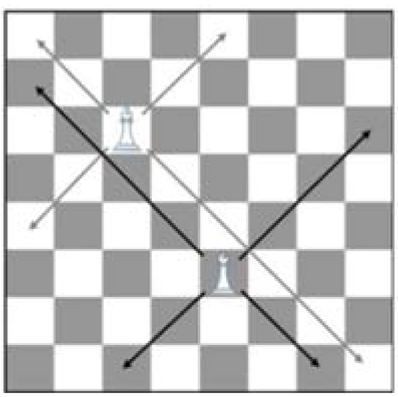

Fig. 5: Movement of bishop

The domination number for bishops on a square 8×8 chessboard is 8, i.e., $\gamma(G_B) = 8$, its can be found by putting the pieces of bishops on a single row or a single column as shown in Fig. 6 (Freeman, 2014). Thus, the chessboard is covered by the regardless of the existence of square dominated by more than one piece and we don't place bishop's pieces diagonally on the chessboard for they are not dominated on the whole chessboard.

We cannot find the total domination in bishop's pieces because there is no connection between the dominating set of vertices in Fig. 6. To find the total domination should be added edges to the graph which represents of the movement of the bishops, so that, the dominating vertices connected with each other and This contradicts the rules of the bishop's movement.

Knight's problem: Every player has two knights at the beginning of the game. Knights are allowed to move two squares in one direction (either vertically or horizontally) and one square in the other direction as long as they don't take the place of a friendly piece (Fig. 7). It can't control the squares right next to it. The full move resembles the Letter L. Knights are unique in that they are the only pieces allowed to jump over other pieces (both enemy and friendly) (Eric, 2003). The knight controls eight squares when positioned in the center of the board as shown in Fig. 7, (Eade, 2005).

The domination number for knights on a square 8×8 chessboard is 12, i.e., $\gamma(G_{Kn}) = 12$. To find the knights domination number is put the pieces as groups as shown in Fig. 8, (Freeman, 2014). To find the total domination of vertices are added to the set of dominate vertices as shown in Fig. 9.

Queens problem: The queen is the most powerful piece in chessboard. A queen's power is directly related to its

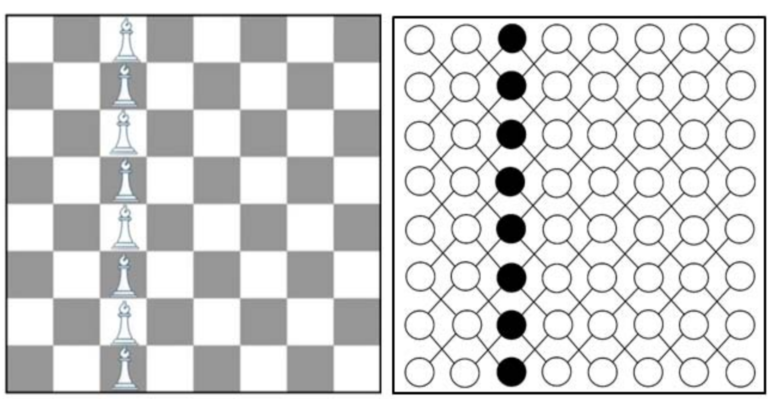

Fig. 6: The bishop's domination

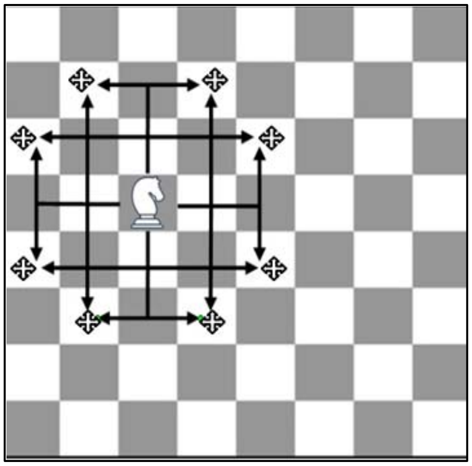

Fig. 7: Movement of knight

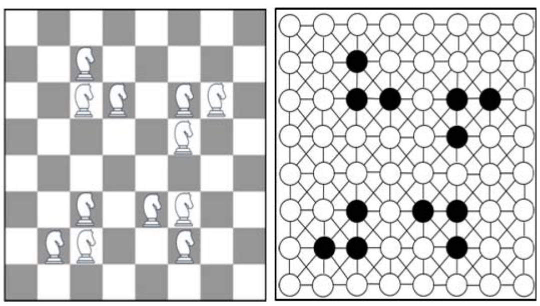

Fig. 8: The knight's domination

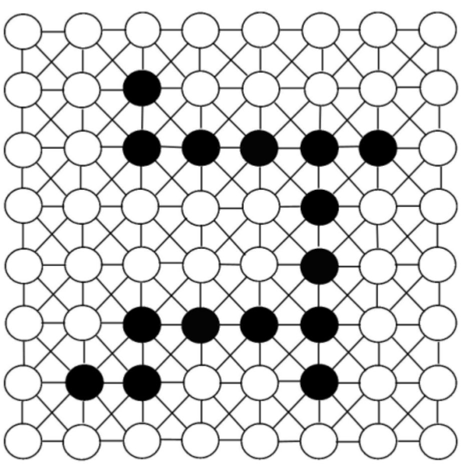

Fig. 9: The knight's total domination

moves. The queen's moves are simply the combination of the bishop's diagonal moves and the rook's side-to-side, down-to-up moves. Her only restriction is that she can't jump over pieces (Eade, 2005). Any vertex (or square) to which a queen is able to move is adjacent to the same vertex (or square) containing the queen. Therefore, there is an edge between those two squares or vertices of the graph. Figure 10 shows the squares that a queen can attack or dominate (Burchett, 2005).

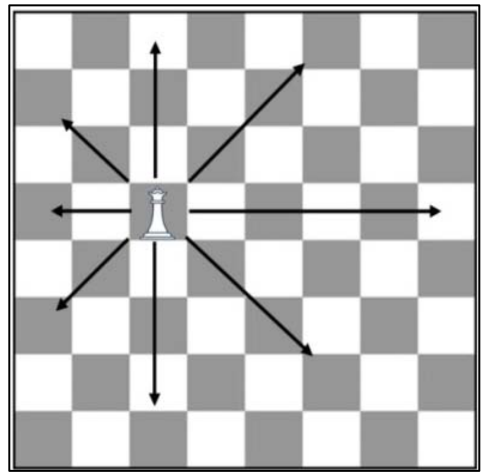

Fig. 10: Movement of queen

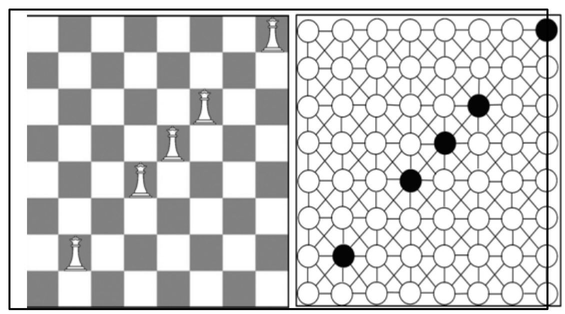

Fig. 11: The queen's domination

To find the minimum number of queens that can be put on a board such that any square is either occupied by a queen originally or can be occupied by a queen in a single step. In Fig. 11 five queens are shown who dominate all the chessboard and this know by the (five queens problem) (Kenareh, 2006; Seinn, 2016). The domination number for queens on a square 8×8 chessboard is 5, i.e., $\gamma(G_Q) = 5$ (Seinn, 2016).

To find the total domination is added one queen to the dominant queens shown in Fig. 11 and thus, Fig. 12 illustrates the total domination concept to the chessboard. Thus, the dominating vertices are connected.

King's problem: Kings are allowed to move exactly one square in any direction as long as they do not take the place of a friendly piece as shown in Fig. 13 (Eade, 2005; Freeman, 2014).

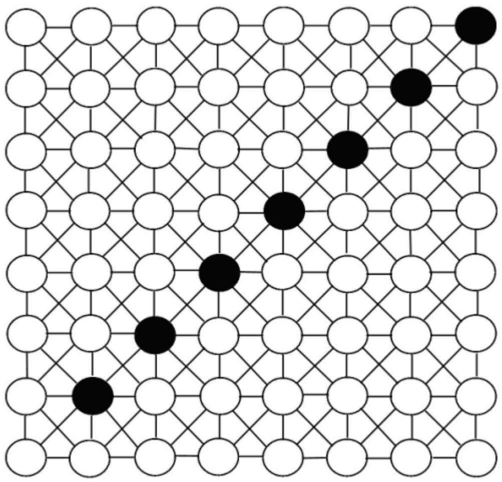

Fig. 12: The queen's total domination

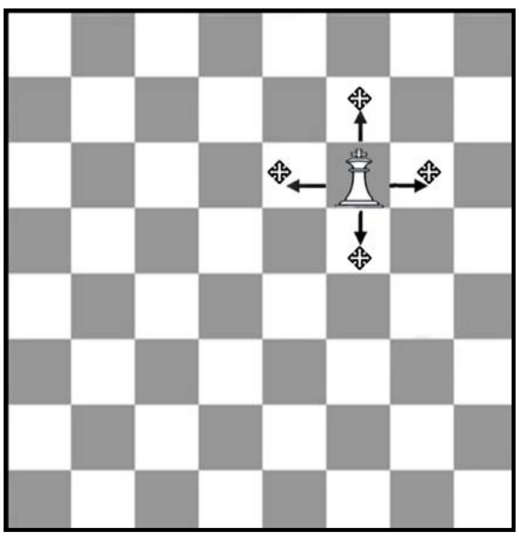

Fig. 13: Movement of king

Table 1: Domination number and total domination number for each piece in cheeseboard

Pieces of cheeseboard	The domination number	The total domination number
Rook	$\gamma(G_R) = 8$	$\gamma_t(G_R) = 8$
Bishop	$\gamma(G_B) = 8$	Not found
Knight	$\gamma(G_{Kn}) = 12$	$\gamma_t(G_{Kn}) = 15$
Queen	$\gamma(G_Q) = 5$	$\gamma_t(G_Q) = 7$
King	$\gamma(G_k) = 9$	$\gamma_t(G_K) = 25$

The domination number for king on a square 8×8 chessboard is 9, i.e., $\gamma(G_K) = 9$. So that, it covers full the chessboard as shown in Fig. 14, (Eade, 2005).

To find the total domination is added vertices to the set of dominated vertices in Fig. 14 as shown in Fig. 15 and Table 1.

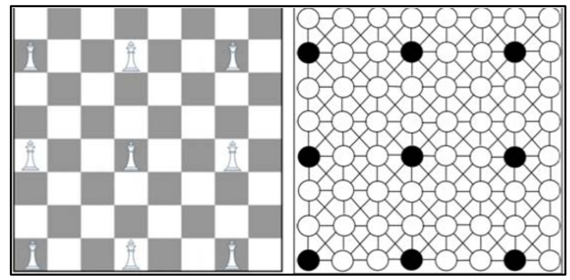

Fig. 14: The king's domination

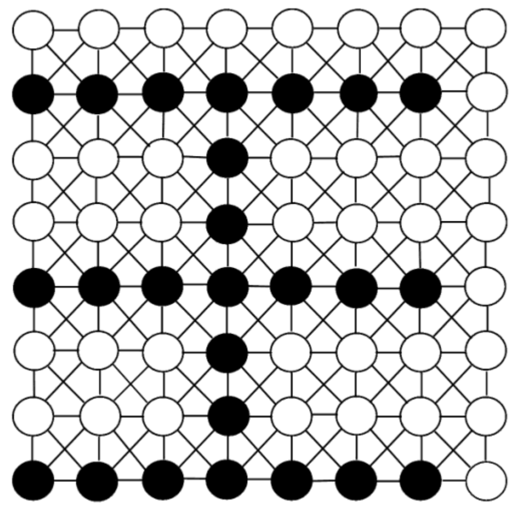

Fig. 15: The king's total domination

CONCLUSION

In this research, we discussed the concept of domination in the chessboard as well as applying the concept of total domination on the chessboard 8×8.

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