Study of Even-Even $^{20\text{-}30}$Mg Isotopes using Shell Model Calculations

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Abstract: By utilizing the effective interactions namely, USDA, W, USDB and SU3SD in the shell model code NuShellX@MSU, we had performed shell model calculations on a large-scale for SD-shell for the isotopes $^{20\text{-}30}$Mg. The calculations of binding energy and energy levels are in good agreement with to the experimental data and the best effective interaction is USDA for this mass region.

Key words: Shell model, binding energies, NuShellX@MSU, large-scale, USDA, interaction

INTRODUCTION

The atomic nucleus is an emphatically fermionic quantum many-body systems made out of neutrons and protons. The energies of the discrete single particle are clustered, leading to the structure of shell for the nucleus (Mayer, 1949). The set of single particle orbits have large energy gaps that take place at specific fillings depending on the magic number of both neutrons and protons. Thus, the resulting shell structure and the nuclear potential generate stable nuclei. It is observed a modification to the shell structure in the exotic nuclei that have large imponderables of neutrons and protons. In this case, other magic numbers are involved in the normal shell gaps collapsed (Otsuka *et al.*, 2001). The study of the concatenation of isotopes $^{20\text{-}40}$Mg includes only three magic numbers of shell. They are N = 28, 20 and 8. Thus, it represents outstanding situation to study the development for structure of shell with neutron number, magic numbers evanescence, undermining of shell closures and manifestation of islands inversion (Sorlin and Porquet, 2008). By employing the shell model code OXBASH, Mohammadi *et al.* (2015) calculated the energy levels using spin-parity of valance nucleons in the SD model space for 13 potentials. Majeed (2013) studied symmetric study for 2^{+1} and 4^{+1} energies for even-even $^{20\text{-}32}$Mg by means of the large-scale shell-model computations utilizing the effective interactions USDB and USDBPN with SD and SDPN model space, respectively. The reduced transition B (E2) were also calculated for the chain Mg isotopes. Iwasaki *et al.* (2001) have studied nucleus with rich-high neutron by means of Coulomb excitation utilizing Pb target with a radioactive ^{34}Mg beam at 4.99 MeV/nucleus. In this research for the first time, it was determined the value of B ((E2; 0+62) for the proposed 2^{+}_{1} state in ^{34}Mg to be 631 (126) e^2 fm^4. The last value consort to deformation parameter $_{r}2$ of 0.58 (6)

of the quadrupole suggesting that the disfigurement of ^{34}Mg is anomalously large (Iwasaki *et al.*, 2001). Caurier *et al.* (1998) studied the calculations of shell model of the nuclei with the high neutron. The nuclei were with Zw.14 to O, Ne, Na, F and Al utilizing the same effective interaction and valence space. The predictions based on evidence made out for the location of the line of neutron drip and the energies of neutron separation. It found the isotopes deformation of Na, Ne and Mg was at N>22. For the edge of the drip line of ^{40}Mg, the shell closure N528 does not influence (Caurier *et al.*, 1998). The objective of this research is to investigate the energy levels for the binding energies and positive parity of $^{20\text{-}30}$Mg isotopes. The shell model code NuShellX@MSU for windows is used to exam the capability of the present effective interactions on reproducing the experiment without any restriction imposed the model space with effective interactions W, USDA, USADB and SU3SD.

MATERIALS AND METHODS

Calculations of shell model: The computations were achieved in the SD Model space for even-even nuclei $^{20\text{-}30}$Mg near the doubly magic core ^{16}O by using NushellX@MSU code for windows without any limitation enjoined on the enjoined of the model with four effective interactions W, USDA, USADB and SU3SD. The calculations of excitation energy levels and binding energy compared with the modern available experimental results. The center mean field potential in the nuclear shell model in the model of nuclear shell, the center mean field potential is formed by the particular nucleons. The only nuclear Hamiltonian is formally written by the kinetic energy and potential for two body interaction. Introducing one- body nucleon potential, the Hamiltonian is given by:

$$H = \sum_i^A \frac{p^2}{2m} + \sum_{i>k}^A V_{ik}\left(\vec{r}_i - \vec{r}_k\right) \tag{1}$$

$$H = \sum_{i=1}^A \left[\frac{P^2}{2m} + U_i\left(\vec{r}\right)\right] + \sum_{i>k}^A V_{ik}\left(\vec{r}_i - \vec{r}_k\right) - \tag{2}$$

$$\sum_{i=1}^A U_i\left(\vec{r}\right) = H_0 + H_{res}$$

The nucleon Single Particle Energies (SPE) in a center potential are the solution Schrödinger equation with as it notices in states of the single particle (hole) outside nucleons with a doubly-Closed Shell (CS) on (CS±1). The matrix elements of two bodies for the residual interactions H_{res} symbolize the alternate interaction of the valence nucleons as noticed in the (CS±2) neighbors of the magic nucleons (Goeppert and Jensen, 1995).

RESULTS AND DISCUSSION

The excitation energies: Utilizing the shell model code NuShellX@MSU and using W, USDA, USDB and US3SD effective interactions we set the core at ^{16}O for the entire isotopes under the investigation with valence nucleon distributed SD valance space. The differentiation of the computed energy level for ^{20}Mg isotopes is given in Fig. 1-6 using W, USDA, USDB and US3SD effective

interactions together with the modern obtainable experimental results. Figure 1 shows the comparison between our computations utilizing the previous mentioned effective interactions with regard to the positive parity state for ^{20}Mg isotope. It is noted that the whole effective interactions appropriately transcribe the spin 0^+ of ground-state. Our calculations for the whole effective interactions agree fairly well with the corresponding experimental results. Figure 2 represents the computations of the excitation energies for the positive state for ^{20}Mg isotope. Obviously, the ground state spin of 0^+ is created with all effective interactions. In addition to that, numerous unproven data of the experimental results are proven in our computation. Thus, our computations for the whole effective interactions are reasonable. Figure 3-5 demonstrates the comparison of the low low-lying energy level that accomplished by utilizing the same effective interactions by taking into account the core at ^{20}O with experimental results. By the use of effective interactions, the ground state for their isotopes are reproduced and again many unconfirmed results are confirmed.

Figure 6 shows the computed energy levels for positive and negative parity for ^{20}Mg isotope obtained using W, USDA, USAB and US3SD together with the experimental data are shown. Clearly, (5^+) state are an unconfirmed value of the experimental results are confirmed in our work.
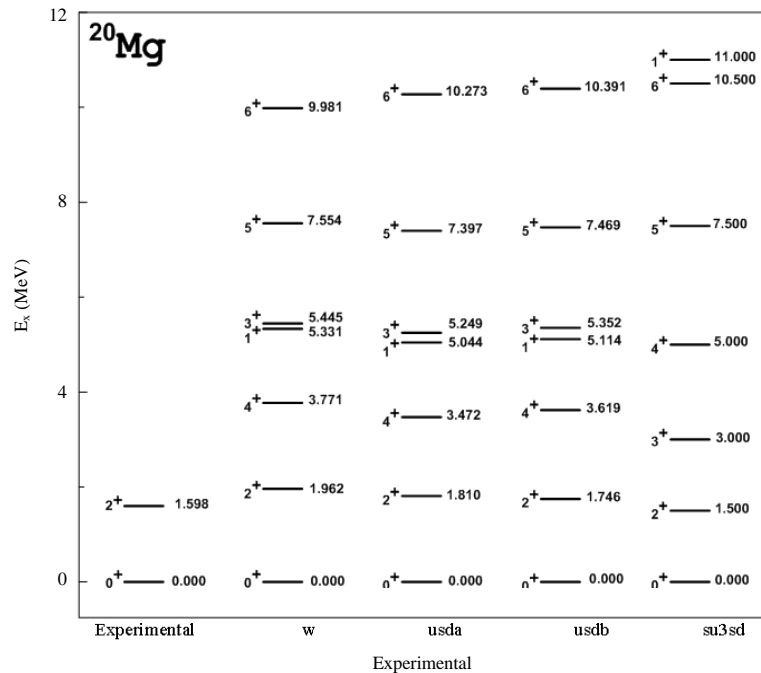

Fig. 1: The computed and experimental data comparison for low-lying spectra of ^{20}Mg isotope with W, UADS, USDB and SU3SD effective interactions. Experimental results are taken from (Audi *et al.*, 2003)
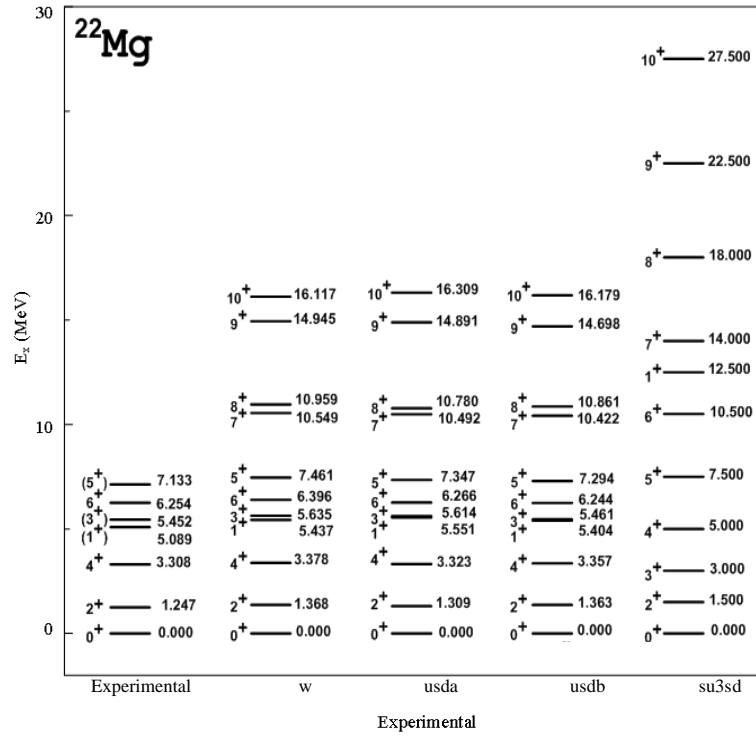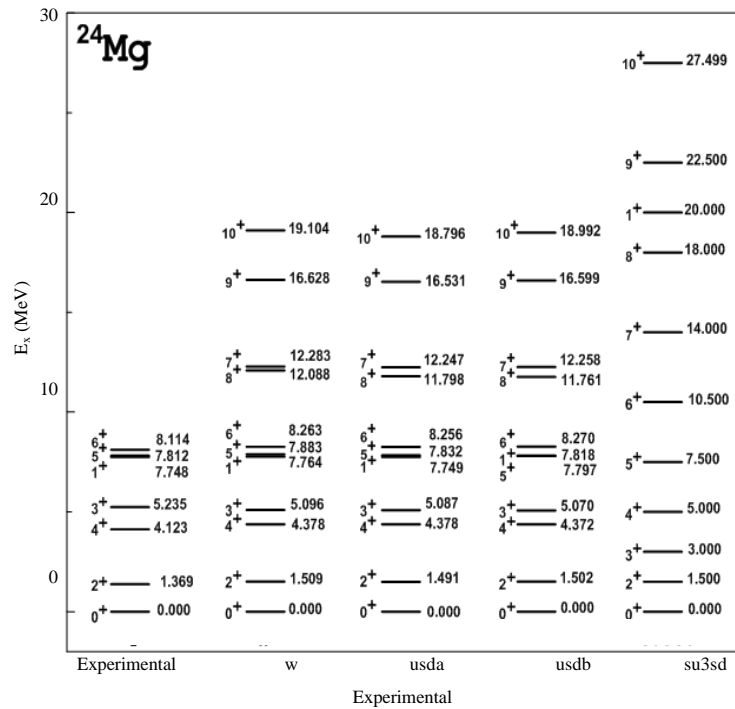
Fig. 2: The computed and experimental data comparison for low-lying spectra of ⁻Mg isotope with W, UADS, USDB and SU3SD effective interactions. Experimental results are taken from (Audi *et al.*, 2003)

Fig. 3: The computed and experimental data comparison for low-lying spectra of ^{24}Mg isotope with W, UADS, USDB, and SU3SD effective interactions. Experimental results are taken from (Audi *et al.*, 2003; NNDC., 2017)

Table 1: Lists the computed and experimental binding energy

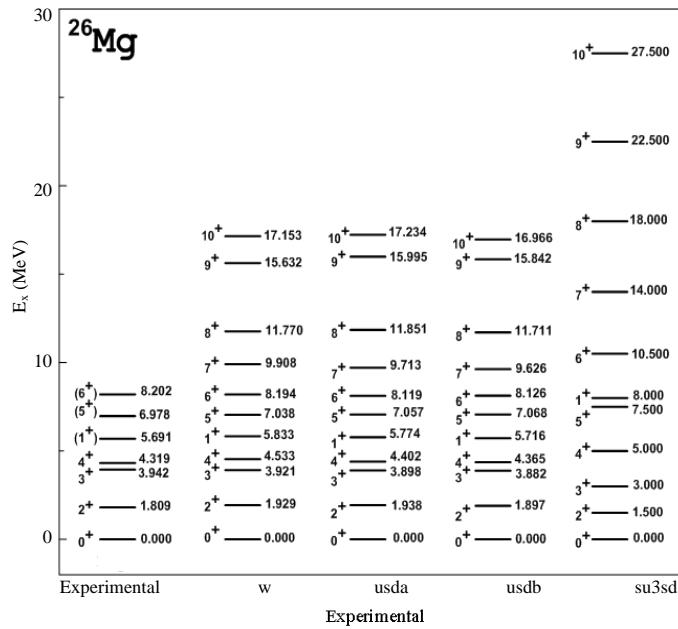| Isotopes	Exp.	W	USDA	USDB	SU3SD
^{20}Mg	134.468	151.446	151.129	151.251	153.619
^{22}Mg	168.570	185.228	185.256	185.197	289.119
^{24}Mg	198.257	214.709	214.753	214.723	367.619
^{26}Mg	216.681	233.155	233.117	233.140	315.119
^{28}Mg	231.627	248.151	248.031	120.500	208.619
^{30}Mg	241.662	258.551	257.920	258.093	204.118

Fig. 4: The computed and experimental data comparison for low-lying spectra of ˙Mg isotope with W, UADS, USDB and SU3SD effective interactions. Experimental results are taken from (Audi *et al.*, 2003; NNDC., 2017)
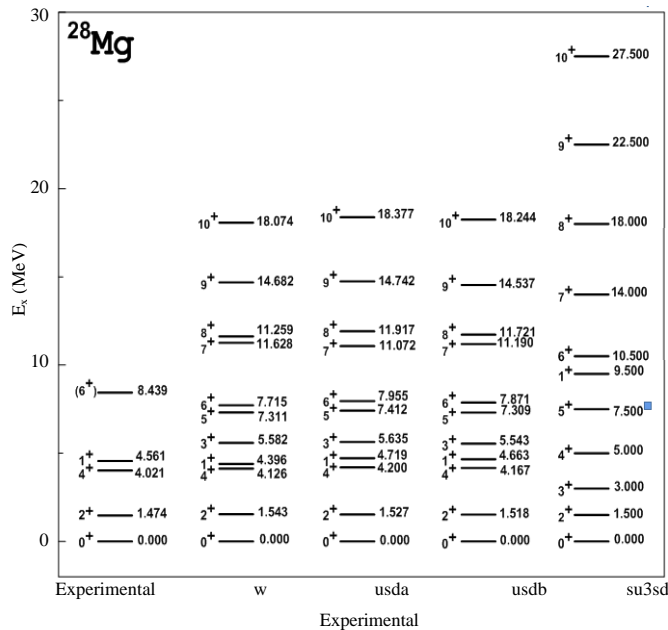

Fig. 5: The computed and experimental data comparison for low-lying spectra of ˙Mg isotope with W, UADS, USDB and SU3SD effective interactions. Experimental results are taken from (Audi *et al.*, 2003; NNDC., 2017)

Fig. 6: The computed and experimental data comparison for low-lying spectra of ·Mg isotope with W, UADS, USDB and SU3SD effective interactions. Experimental results are taken from (Audi *et al.*, 2003; NNDC., 2017)
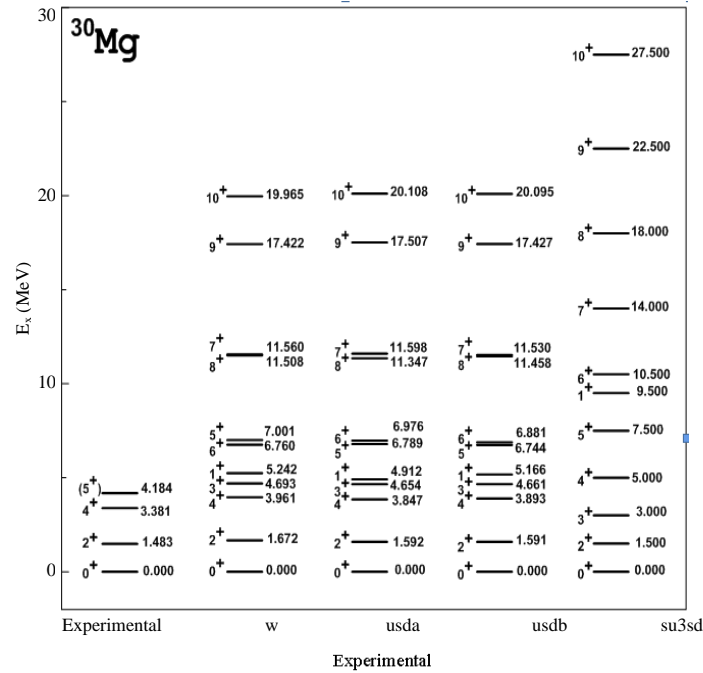
Binding energy: Table 1 lists the computed and experimental binding energy for the whole isotope under consideration. The effective interactions utilized in this research are qualified for precisely predicting the binding energies for the whole isotope under consideration.

CONCLUSION

Unrestricted large-scale shell-model computations are achieved in the present work to investigate the binding energies and energy levels and for the $^{20-30}$Mg isotopes. This work employs the shell model code NuShellX@MSU by using the model space SD with W, USDA, USDB, SU3SD effective interactions. We take into account that there is no restriction imposed on the entire basis and on valance nucleons in the computations. We conclude that the efficient interactions are sufficient option to the nuclei that located in this region of mass. We found that the efficient interactions USDA is exceedingly proportionate to reproduce the experimental results for nuclei under the investigation. Further, the current shell model computations reproduce the experimental binding energies.

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