

## Detection of Image Descriptors and Modification of the Weighting Function for the Estimation of the Fundamental Matrix Using Robust Methods

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**Abstract:** The computer vision is one of the most important specialties which can contribute on development of the computing. The estimation of the fundamental matrix remains a necessary tool to obtain and evaluate the relationships between two different view images. Different methods of segmentation exist to satisfy the specification of elimination of the false correspondence point. In this study, we propose a method based on the image segmentation using the super pixel algorithm. After that, we develop a new modification on the weighting function related with the fundamental matrix. Experimental comparisons were conducted through a simulation between the RANSAC, LMed, M-estimator and our method in order to estimate the projection error. Consequently, the proposed method gives a good performance results with a low error of projection compared to the others robust methods.

**Key words:** Epipolar geometry, estimate fundamental matrix, robust methods, Gaussian noise, evaluate Sampson error, segmentation, weighting function

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### INTRODUCTION

Different estimation methods of fundamental matrix exist and almost all of them use only matching image points as data. These methods are distinguished by the parameterization of the fundamental matrix 'F'. The epipolar geometry plays a very important role in the computer vision and it can be represented as an 'F' which is  $3 \times 3$  and rank-2 singular matrix (Hartley and Zisserman, 2004). In the fact the estimation of the fundamental matrix is primary in almost all matching and reconstruction algorithms. We can elaborate the fundamental matrix from two images captured from different views about one scene. However, the estimation of the fundamental matrix is probably faced on two major problems: incorrect matching of points and lack of precise pixel coordinates. Many researcher's studies have proposed methods of solving these problems (Luong and Faugeras, 1996; Armangue and Salvi, 2003).

Generally, two broad categories of approaches used to estimate the 'F', linear and non-linear methods. The results of the previous research of the linear method have shown two major defects related to the absence of rank constraint of the fundamental matrix and to the absence of normalization of the minimization criterion, resulting in

errors in the estimation of the matrix. To defeat this defect, non-linear methods are proposed to reduce lag and improve accuracy by minimizing the distance between the points and their epipolar lines (Hartley and Zisserman, 2004; Zhang, 1998a, b). The projection error (Bartoli and Sturm, 2004; Kanatani and Sugaya, 2010). Or the sampson error (Zhang, 1998; Migita and Shakunaga, 2007). The robust methods are proposed to reduce the effect of potential aberrant values and have greater tolerance to data noise.

The first technique M-estimator (Torr and Murray, 1997). Leads to a good result in the presence of noise Gaussian at the selected points of the image but it is limited in its capacity to take outliers into account. Two other techniques are classified as robust methods and they are similar, the Random Sampling Consensus method (RANSAC) (Torr and Murray, 1997). And the method Least Median Squares (LMeds) (Zhang, 1998). These two least consist in selecting randomly the set of points used for the approximation of the fundamental matrix. The LMeds method calculates for each estimate of 'F' the Euclidean distance between the points and the pipolar lines and the choice of 'F' corresponds to the minimization of this distance. The RANSAC method for its part, calculates for each value of 'F' the number of points

that may be appropriate (inliers). The matrix 'F' chosen maximizes this number. Once the aberrant points are eliminated, the matrix 'F' is recalculated to obtain a better estimation. Experiments have shown that the LMeds technique gives a better result than the RANSAC method in terms of precision (Armangue and Salvi, 2003). Although, the M-estimator gives a good performance compared to the others robust methods in terms of the projection of point of correspondences.

In this study, we develop a robust and efficient method which depends on the correspondence of the characteristics to estimate the fundamental matrix. We start with the image segmentation by the super pixel algorithm (Ren and Malik, 2003) then we detect the descriptors points using the SURF (Speeded Up Robust Features) method (Bay *et al.*, 2008). After the detection of the matched features, we calculate directly the fundamental matrix using the three methods above. The objective of calculating the 'F' consists in determining the projection error. The research is projected in the modification of the weighting function at the level of the M-estimator method. This new proposition not only makes it possible to estimate the matrix F but also precisely identify the different aberrant level values from the set of point correspondences. The simulation result from the real images shows that our proposed method is robust than the other methods at the level of projection error.

### Epipolar geometry

**Relation existing tow different image:** The epipolar geometry defines the set of relations existing between two different image views of the same scene captured using a camera. Figure 1 presents a stereoscopic system composed of two image sensors arranged at positions  $O_l$  and  $O_r$  in the scene. A point  $P$  of space is projected, respectively into  $p_l$  and  $p_r$  in both left and right images. The point  $P$  and the two optical centers form a plane  $\pi$ , called the epipolar plane which intersects the two image planes according to two straight lines  $l_l$   $e_l$   $l_r$  called epipolar lines. These lines cut the baseline  $o_l$  and  $o_r$  a constant of the device if it is assumed that the arrangement of the two sensors is rigid, respectively at two points  $e_l$  and  $e_r$  called epipoles. These two points are therefore also constants of the stereoscopic system.

The relationship between the points ( $o_l$ ,  $o_r$ ,  $P$ ,  $p_l$ ,  $p_r$ ) which is expressed from two left and right images to calculate the fundamental matrix of rank equal to 2 and the points  $p_l$ ,  $p_r$ , Eq. 1:

$$p_l^T F p_r = 0 \quad (1)$$

where, T: transposed.

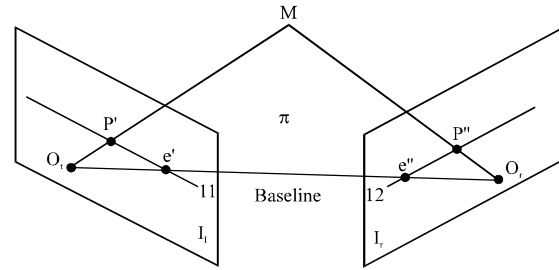


Fig. 1: Epipolar geometry

**Segmentation of the source image:** The objective of image segmentation is to eliminate all points of false matches by an affine transformation in the uniform region.

First, we have to separate an image into different areas and merge them by paired points in two images. To perform the segmentation, we choose to use the super pixel algorithm (Ren and Malik, 2003) which consists in segmenting the first image to be processed and after this operation we must find homogeneous regions (gray level, color, etc.) and well limited between them, then we take each region of the reference image and look for the correspondence in Fig. 2.

**Establishment of correspondence points:** Correspondence is an approach inspired by epipolar geometry and it is based on the calculation of the criterion of similarity. The epipolar geometry is a basic tool of computer vision systems not limited only to the search of the corresponding points but also to the reduction of the points in epipolar lines, however, the epipolar geometry cannot detect the corresponding points if the matched point is located in an incorrect position on the corresponding epipolar line. To manage this situation, we add an affine constraint to assembly the uniform regions and eliminate the false correspondents on the epipolar line. This constraint is based on arbitrary point correspondences of scene images. Our method is based on the use of the descriptor extractor SURF (Speeded Up Robust Features) which is chosen according to a comparative study between two descriptors, SIFT (Scale Invariant Feature transform) (Lowe, 2004) and SURF (Bay *et al.*, 2008). The results of the SURF study give good results in terms of speed (processing time) and number of detected points. From the points detected by the SURF method between two images of the same scene. We can find the accurate fundamental matrix 'F' from  $l_l$  and  $l_r$  which connects the corresponding pairs of image points for each of them  $\{P_{li}\}$  and  $\{P_{ri}\}$ . ( $P_{li}$ ,  $P_{ri}$  are the corresponding pixels for the two images).

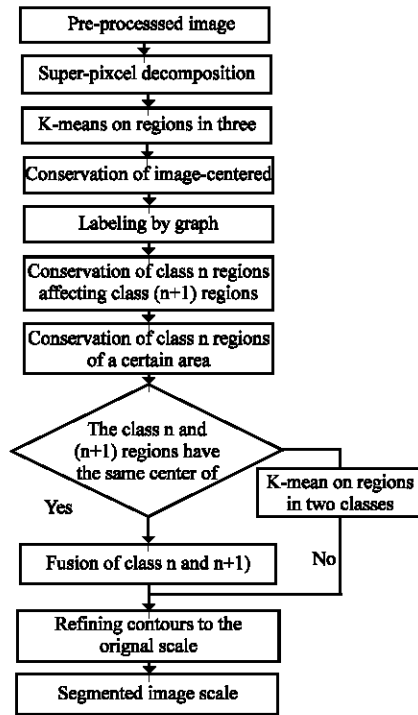


Fig. 2: Segmentation using super-pixel

## MATERIALS AND METHODS

**Fundamental matrix:** The estimation of the fundamental matrix relies on the knowledge of a certain number of corresponding pairs of points. This estimation can be represented as the following equation:

$$p^T F p = 0$$

where,  $F$  is a matrix of dimension  $3 \times 3$  and of rank-2 and determined from 'F' and zero, the Eq. 1 is the relationship which relies the points of the left imaged  $P_{li} = (u_{li}, v_{li}, 1)$  and points of the right imaged  $P_{ri} = (u_{ri}, v_{ri}, 1)^T$ . This equation can be rewritten in the following linear form:

$$U_f = 0 \quad (2)$$

With:

$$f = [F_{11}, F_{12}, F_{13}, F_{21}, F_{22}, F_{23}, F_{31}, F_{32}, F_{33}]^T$$

And:

$$U = \begin{bmatrix} u_{l1} & u_{r1} & u_{l1} v_{r1} & u_{l1} & v_{l1} & u_{r1} & v_{l1} v_{r1} & v_{l1} & u_{r1} & v_{r1} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_{lN} & u_{rN} & u_{lN} v_{rN} & u_{lN} & v_{lN} & u_{rN} & v_{lN} v_{rN} & v_{lN} & u_{rN} & v_{rN} & 1 \end{bmatrix}$$

It can be noted from the decomposition of the Eq. 2 that there are 9 unknown and 7 independent parameters. But it is still possible to estimate this matrix from only 7 pairs of points (Zhang, 1998). The major advantage of this method consists on his simplicity and speed. However, the quality of the results deteriorates rapidly when a few points are poorly localized. Moreover, the solution is not always unique and the result depends on the choice of the 7 points in the set of available matches. This approach has been improved (Hartley, 1997). This researcher has proposed also a more robust algorithm called eight standardized point. This approach greatly improves the result of the seven point method. In this research, we took the last algorithm of Hartely (eight standardized point).

Equation 2 is a starting point for most methods of determining the fundamental matrix. Which can be solved for up to a scale factor if  $N = 8$  and if  $N$  is greater than that it will be solved uniquely in a way that minimizes Eq. 2. In general, to solve for  $N$  equations, the Singular Value Decomposition (SVD) of  $U$  is taken so that (Hartley and Zisserman, 2004):

$$[FU, FS, FV] = \text{SVD} (U) \quad (3)$$

Where:

$$U = [FU \quad FS \quad FV^T]$$

The estimated 'F' should be a rank-2 matrix in order to model the epipolar geometry with all the epipolar lines intersecting in a unique epipole. Although, the rank-2 constraint is not imposed in most of the surveyed methods, there is a mathematical method which transforms a rank-n square matrix to the closest rank-(n-1) matrix.

The 'F' is decomposed in  $F = USV^T$  by using singular value decomposition where,  $S = \text{diag} (s_1, s_2, s_3)$  the component with the smallest weight is removed obtaining  $S = \text{diag} (s_1, s_2, s_3)$   $F$  is recalculated in the following way (Hartley and Zisserman, 2004):

$$\hat{F} = U \begin{bmatrix} s_1 & 0 & 0 \\ 0 & s_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} V^T \quad (4)$$

## Robust methods

**We present three robust methods:** RANSAC, LMedS and M-estimator. The first method for its part, calculates for each value of 'F' the number of points that may be suitable (inliers). The matrix 'F' chosen is that which maximizes this number. Once the aberrant points are eliminated, the matrix 'F' is recalculated to obtain a better estimate. The LMedS method calculates for each estimation of 'F' the Euclidean distance between the points and the epipolar lines and the choice of 'F' corresponds to the minimization of this distance.

Although, M-estimator is inspired by the two preceding methods it consists in dividing the detected points into two sets, inliers and quasi-inliers. The latter method is based on solving the following expression:

$$\min_F \sum_i w_i r_i^2 \quad (5)$$

$w_i$  is the weighting function. M-estimator considers the residual of each point on the epipolar line and affects it for each outlier. Suppose that the  $r_i$  is the residual of Where  $P_n = (u_n, v_n, 1)^T$ ,  $p_{li} = (u_{li}, v_{li}, 1)$  and:

$$F = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

$$r_i = (u_{li}, v_{li}, 1) \begin{bmatrix} f_{12} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} (u_n, v_n, 1)^T \quad (6)$$

$$r_i = f_{11}u_{li}u_{ir} + f_{12}u_{li}v_{ir} + f_{13}u_{li} + f_{21}v_{li}u_{ir} + f_{22}v_{li}v_{ir} + f_{23}v_{li} + f_{31}u_{ir} + f_{32}v_{ir} + f_{33}$$

Huber *et al.* (1981) has proposed the following expression for  $w_i$ :

$$w_i = w_i(p_i', p_i'') = \begin{cases} 1 & |r_i| \leq s \\ \frac{s}{|r_i|} & s \leq |r_i| < 3s \\ 0 & 3s < |r_i| \end{cases} \quad (7)$$

According by Zhang (1998) the robust standard deviation  $\sigma$  can be expressed as  $\sigma = 1.48 (1+5 \text{ median}(r_i)/n-q)$ . Where  $n$ , the number of points detected by the method SURF at each image.

And  $(1+5/n-q)$  finite sample correction factor with the total number of parameters  $q = 8$  in this case. Experiments have shown that the technique Lmed gives better result than RANSAC method in terms of accuracy (Armangué and Salvi, 2003). Lmeds and RANSAC are considered similar, they consist to select randomly the set of points used for the approximation of the fundamental matrix. The difference exist between this two methods in the way use to determinate the chosen 'F'. LMed calculate the 'F' from the distance between the points and the epipolar lines where it seeks to minimize the median. RANSAC calculate the matrix 'F' from the number of inliers. However, M-estimator leads to a good result in the

presence of a Gaussian noise at the selected points of the image, the robustness of this method is manifested in the reduction of aberrant values.

**Multilevel weighting function:** In the literature (Zheng *et al.*, 2011) it exists two conditions to minimize the Eq. 5, one is detailed by Hartley and Zisserman (2004) and another formula will be used to evaluate Sampson error it should be noted that this error is calculated using an accurate matrix 'F':

$$E_{\text{Sampson}} = \sum_i w_i \frac{(p^T F p'')^2}{(F p')_1^2 + (F p')_2^2 + (F p'')_1^2 + (F p'')_2^2} \quad (8)$$

where,  $w_i$  is weighting function.  $(Ep_j)^2$ ,  $J = 1, 2$  the square of the  $j$ th entry for the vector  $(Ep_i)$  according to Liqiang Wang and referring to the documentation (Hartley, 1997), we choose to use the weighting function which makes it possible to calculate the weight of each point, we clarify that this function aims to improve the precision of the fundamental matrix. The weighting function is given in Eq. 9 (Wang and Zhang, 2016):

$$w_i = w_i(p_i', p_i'') = \begin{cases} 1 & r \leq \phi_i \sigma \\ \theta & \phi_i \sigma < \sigma \\ \frac{\theta \sigma}{|r_i|} & \sigma \leq r_i < \varphi \sigma \\ 0 & \varphi \sigma < r \end{cases} \quad (9)$$

Where:

- $\phi$  = Factor to ensure the boundary of the inliers and quasi inliers
- $\sigma = \frac{\text{median}(r_i)}{\lambda}$  = Scale of the error
- $\theta$  = Median  $(r_i)$  scale of the error  $\theta$ : Proportional factor whose scope is (0, 1)
- $\varphi$  and  $\lambda$  = Constants Torr (Torr and Murray, 1997)

This new weighting Eq. 9 shows clearly that the corresponding weight of points should be updated in four cases. This function carries more functionality than Torr method (Torr and Murray, 1997). Which deals with three sets of dense cloud points, inliers, quasi-inliers and aberrant values. To summarize, this feature can overcome the dense descriptor point problem and can contribute to improving accuracy. Before to calculate the fundamental matrix, we affected the factor  $\phi$  the value 1, after in the first iteration, we calculate the factor  $\phi$  which equals the

ratio of the quantities of the inliers and the integer points which are judged using the initial fundamental matrix. In our case study we found 273 inliers in the set of correspondences of 450 points when the fundamental matrix was estimated for the first time. Thus from the second iteration, the limit factor became  $\phi = 273/450$ . We apply the Torr method to define the value:  $\phi = 3$  and  $\lambda = 0.6745$ , the values of these parameters can be referred by Torr and Murray (1997). After having assured the factors, the residue  $r_i$  can decide the weighting value  $w_i$  of each point in each iterative process. If  $\sigma \phi_i$  is greater than  $\sigma \phi$ , the corresponding point can be used as inlier and  $w_i$  is equal to 1. If  $r_i$  is  $< 3\sigma$  and greater than  $\sigma$ , the corresponding point can be considered as a quasi-inlier and  $w_i$  is 0.6. If  $r_i$  is  $< 3\sigma$  and greater than  $\sigma$ , the corresponding point is an overrun and the weighting  $w_i$  is calculated by the expression  $0.6\sigma/|r_i|$ . However, if  $3\sigma$  is less than  $r_i$  the weighting function corresponding to estimate the fundamental matrix is 0.

**Normalizing data and calculates fundamental matrix by three methods:** The normalization of the coordinates of each point correspondence follows the determination of the weight of each point correspondence. The formula for standardizing data consists to use a scaling matrix  $T_s$  and a translating  $T_t$  (Armangue and Salvi, 2003):

$$T = T_t T_s \quad (10)$$

Where:

$$T_s = \begin{bmatrix} \frac{1}{c_x} & 0 & 0 \\ 0 & \frac{1}{c_y} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad T_t = \begin{bmatrix} 1 & 0 & -c_x \\ 0 & 1 & -c_y \\ 0 & 0 & 1 \end{bmatrix}$$

with  $c_x = \sum w_i x_i$  and  $c_y = \sum w_i y_i$  are transformation parameters. Noting that the coordinate of the correspondence point are designated as,  $p_i = (x_i, y_i, 1)^T$ . And the standardized point is defined as follows: Then, the corresponding fundamental matrix  $F_T$  can be estimated from the normalized points  $P_i$ .

After normalization of the points detected by the SURF method, the fundamental matrix is calculated using the following three robust method algorithms: RANSAC, LMed, M-estimator and proposed method. Finally, the fundamental matrix 'F' has to be restored as the following Eq. 11 (Fig. 3):

$$F = T^T F_T \quad (11)$$

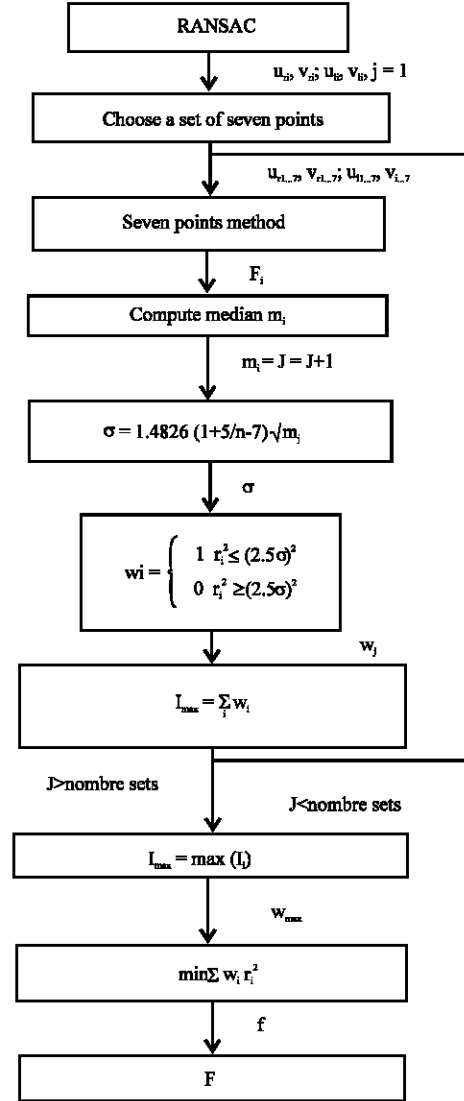


Fig. 3: Flow schemes of RANSAC

The process of the new weighting function is described in the following algorithm:

**Algorithm 1; Multilevel weighting for Estimate fundamental matrix:**

Input: a set of point correspondences  $\{p_i, p_i'\}$

Output: the fundamental matrix  $F$  begin:

Step 1: Nitilaise the fundamental matrix  $F_0$  by using eight point method:

Step 2: Estimate fundamental matrix by multilevel weighting algorithm:

1. Repeat
2. If  $n = 1$  then
3.  $F^1 = F_0, w_i = 1, I = 1, 2, \dots$
4. Find the number of inliers by using the matrix  $F_0$
5. Calculate the boundary factor
6. Else
7. If  $\max(\max(F - F^i)) < \text{error}$  then
8. Break
9. Else

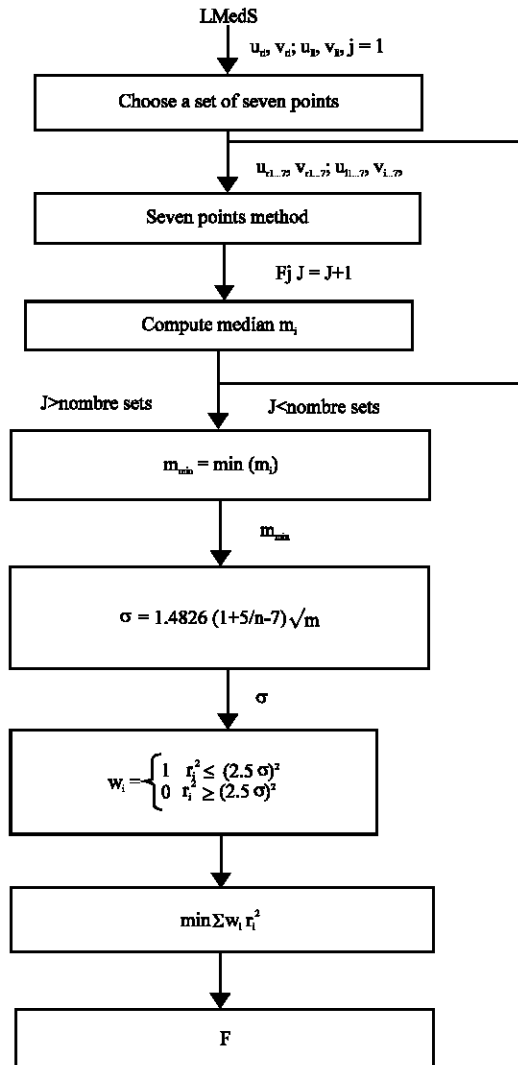


Fig. 4: Scheme of LMed

10.  $F = F'$
11. End if
12. End if
13. Calculate the residual of each point  $r_i$  Eq. 6
14. Estimate the weight of each corresponding the point  $w_i$  Eq. 9
15. Normalise the weighted point and estimating the matrix:  $F$
16. Compute the fundamental matrix Eq. 11
17. Until path retrace
18. Return the fundamental matrix

After calculating the fundamental matrix “F” by the methods that have already processed, we take this matrix and we replace it in Eq. 8 in order to calculate the sampson error. Then, we add the Gaussian noise on the two source images and we compare the noise influencing by the algorithm (Fig. 4 and 5).

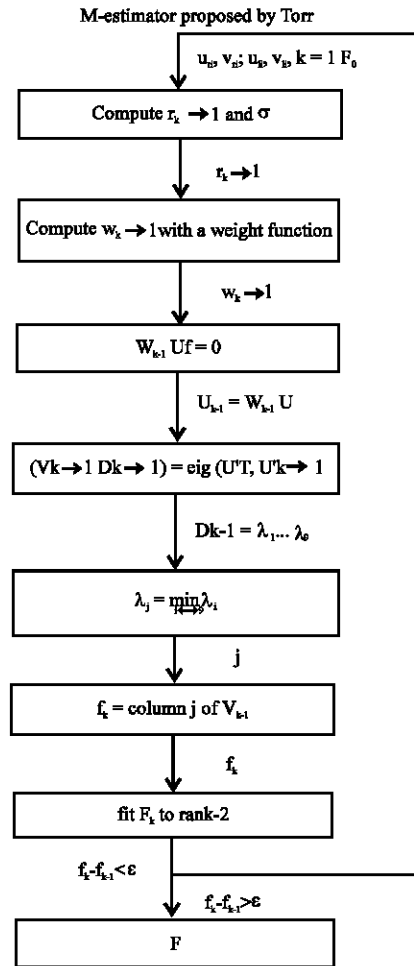


Fig. 5: Flow scheme of M-estimators proposed by Torr

## RESULTS AND DISCUSSION

In this study, experiments were carried out on two real images Fig. 6 and 7 from two different positions by the same camera. First, we segment the main image (Fig. 8) by super-pixel method and search the matching points by region in the second image. Second, we apply the SURF method to detect the point of the characteristics (Fig. 9 and 10). Then, we identify the correspondences between the scene images, Fig. 11 whose purpose is to determine the fundamental matrix and calculate the projection sampson error.

Finally, we calculate the mean, the standard deviation and the Sampson error are calculated as a function of Gaussian noise with zero aberrant values and constant aberrant values 10%. Figure 11-14 the results of the various simulations.

In this study, we present some experiments results of our methods. We compare the proposed method with addition of aberrant values and the noise. We can define



Fig. 6: Left image original of 350×400 pixel



Fig. 7: Right image original of 350×400 pixel



Fig. 8: Segmentation using super-pixel left image

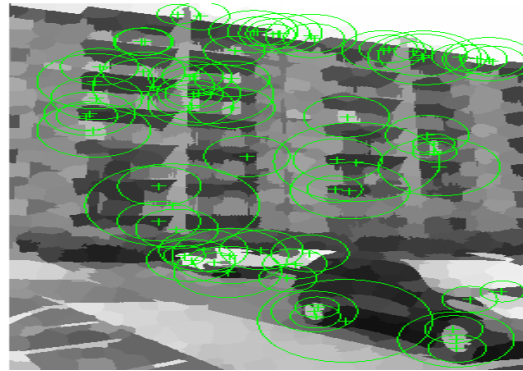


Fig. 9: Detector SURF of left image



Fig. 10: Detector SURF of right image



Fig. 11: The points of correspondence

a vector  $(a, b)$  which signify that the random noise added to the correspondence points is a Gaussian distribution  $N(0, a)$ . We apply two cases studies in our experiments to test the robustness of our proposed method and the three other methods. The first case consists to vary the Gaussian noise from 0-1 ( $V = 0$ ;  $V = 0, 1$ ;  $= 0, 5$  and  $V = 1$ ) and to fix the aberrant value as 0%. The second case

consists to use an aberrant value of 10% and vary the Gaussian noise from 0-1. Figure 12-15 show all cases studies of our simulation.

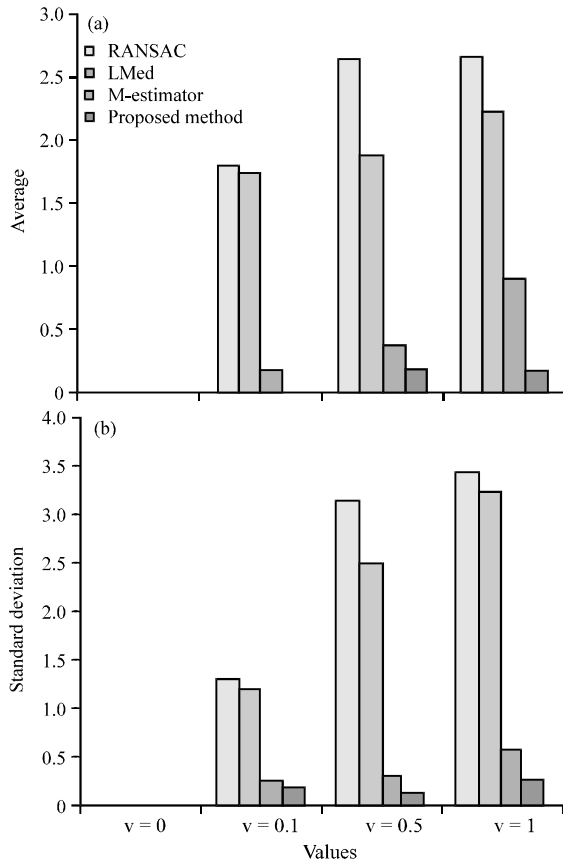


Fig. 12: Mean and standard deviation of the corresponding data as a function of Gaussian noise

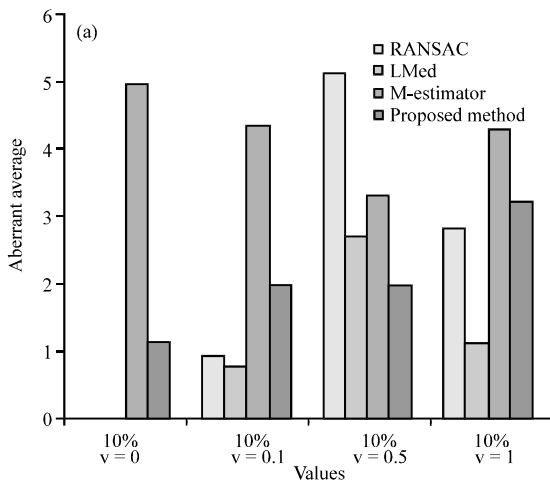


Fig. 13: Countinue

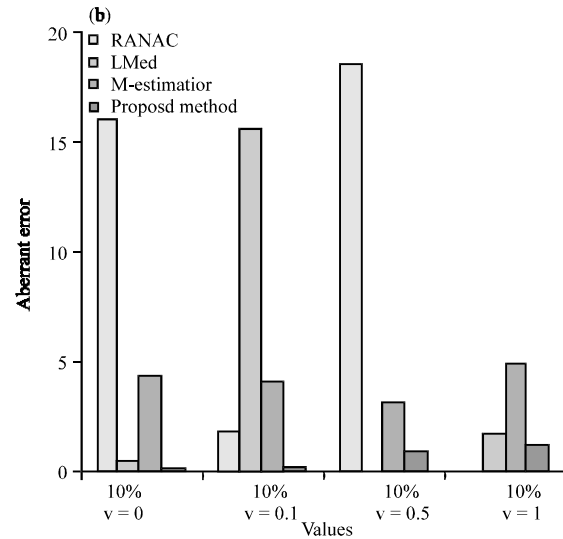


Fig. 13: Mean and standard deviation of the correspondence data according to Gaussian noise and the outliers

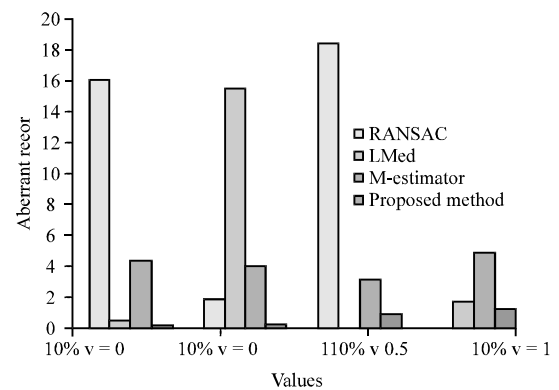


Fig. 14: Projection error as a function of Gaussian noise and aberrant values

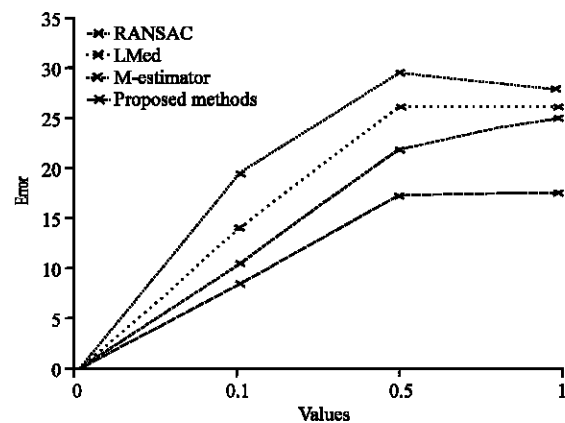


Fig. 15: Comparison of the proposed method projection Sampson error as a function of Gaussian noise

## CONCLUSION

In this study, we have presented a robust method based on the modification of weighting function of M-estimator in order to estimate the fundamental matrix and to calculate the projection error. Then, we compare the proposed method with the methods that already exist in the literature as RANSAC, LMed and traditional M-estimator method. Our method is based on the segmentation and the distribution of the pairs of the point detected by the SURF method in different sets which are inlier, quasi-inlier and outlier.

We have two added values in this research, first, delete the descriptors of low contrast and bad correspondence correlated to the epipolar line. Second, incorporate a weighting function to organize the aberrant values allows to increase the accuracy of the fundamental matrix and consequently to improve the calculation of the projection error.

The experimental results on the whole of the simulation data applied to the real images show that our proposed method gives a better performance for the estimation of the fundamental matrix and the calculation of the projection error compared to the other methods.

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